

C O C K E R's D E C I M A L A R I T H M E T I C K,

Wherein is shewed the Nature and Use of Decimal Fractions in the usual Rules of Arithmetick, and the Mensuration of Plains and Solids.

Together with Tables of Interest and Rebate for the valuation of Leases and Annuities, Present, or in Reversion, and Rules for Calculating those Tables.

Whereunto is added

His Artificial Arithmetick, shewing the Genesis or Fabrick of the Logarithms, and their use in the Extraction of Roots, the solving of Questions in Anatocism, and in other Arithmetical Rules in a Method not usually practised.

A L S O

His Algebraical Arithmetick, containing the Doctrine of Composing and Resolving an Equation ; with all other Rules requisite for the understanding of that mysterious Art, according to the Method used by Mr. *John Kersey* in his Incomparable Treatise of *ALGEBRA*.

Composed by *EDWARD COCKER*, late Practitioner in the Arts of *Writing, Arithmetick* and *Engraving*.

Perused, Corrected and Published

By *JOHN HAWKINS*, Writing Master at *St. Georges Church* in *Southwark*.

*Cum tua non edas cur hæc mea Zoile Carpis;
Carpere vel noli nostra, veleda tua.*

Μαμαίσθαι μὲν ῥᾶδ' ἔστι μιμᾶσθαι δὲ χαλεπὸν.

The Third Edition.

L O N D O N,

Printed for *George Sawbridge*, at the *Three Flower de Lucies* in *Little-Britain* : And *Richard Wellington*, at the *Dolphin* and *Crown* at the West end of *St. Paul's Church-Yard*. 1703.



To the Right Worshipful
Sir *PETER DANIEL*, Kt.

A N D

PETER RICH, Esq;

Aldermen of the

C I T Y

O F

L O N D O N.

THOMAS LEE, Esq;

A N D

JAMES READING, Esq;

Justices of the Peace for the COUNTY of

S U R R Y.

JOHN HAWKINS Humbly Dedi-
cateth this Treatise of ARITHMETICK.

To the Right Worshippful
SIR PETER DAVENANT
AND
HIS HEIR RICHARD
Admiral of the
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OF
LONDON
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Justices of the Peace for the COUNTY of
SURRY.
JOHN HAWKINS Humbly Desires
to publish this Treatise of ARITHMETICK.

TO THE READER.

Courteous Reader,

IN the year of our Lord 1677, I published Mr. Cocker's *Vulgar Arithmetick*; and therein gave an account of the speedy publication of his *Decimal, Logarithmical and Algebraical Arithmetick*; but other extraordinary occurrences intervening, occasioned its not seeing the light before this time:

By the *Vulgar* part, the *Ingenious Learner* may be qualified with so much of that most necessary *Art of Arithmetick* as is sufficient for the management of business in the greatest concerns of *Trade and Commerce*; and for those *Ingenious Souls*, whose active fancies lead them to a further scrutiny into the study of the *Arts Mathematical* was this *Treatise* composed, which will fairly lead them by the hand, without any other *Guide or Company*, into the *Contemplation* of those most sublime speculations, an inheritance entailed only upon the *ingenious, and industrious sons of Art*.

The *Method* throughout the whole is plain, perspicuous, and clear, and I hope will prove satisfactory

To the Reader.

tisfactory to those who shall seriously apply themselves to the Rules, Precepts and Examples therein contained.

The use of Decimals (in the solution of questions Arithmetical, and such Geometrical as are necessary in the mensuration of the most usual planes and solids) is as plainly laid down as the Author or my self could possibly contrive it, and particularly in all the varieties of Interest both Simple and Compound, with Tables, and Rules for the Calculation thereof, according to the Method of several Famous Authors (who have bestowed much pains in the management thereof,) and especially of that most Famous, and no less laborious Mathematician of our Age and Nation, Mr. *John Kersey*, whose Memory deserves highly to be honoured by all the Professors of this Science.

The Genesis or Fabrick of the Logarithms, and their use in Arithmetick is laid down after a different but more intelligible manner than hitherto hath been used by other Authors, and I hope the studious Reader will receive that satisfaction therein which our Author earnestly aimed at, or himself can expect.

And as for the Algebraical part I think there is nothing therein expressed that is superfluous, nor any thing omitted that could be thought necessary to render it plain, perspicuous and clear; so that what other Authors treating upon this subject have left intricate, and difficult to be understood is here made obvious (by clear demonstration) to the meanest Capacity; therefore, Courteous Reader, if thou intendest to be a proficient in the Mathematicks, begin chearfully, proceed gradually, and with resolution; and
the

To the Reader.

the end will crown thy endeavours with success; and be not so sloathfully studious as at every difficulty thou meetest withal to cry out, *Ne plus ultra*, for pains and diligence will overcome the greatest difficulty: To conclude, That thou mayest so read as to understand, and so understand, as to become a proficient, is the hearty desire of him who wisheth thy welfare and the progress of Arts.

*From my School at St. George's
Church in Southwark,
Octob. 27 1684.*

JOHN HAWKINS.

*Anixo guo Anamfiggino Jorammi Lehkeg Lofoxrofe-
bolru Torgiemgig Im Xonifafu Disohemiengi Pawi-
nafig feha.*

G I H,

IT you Lèpeag fo zegfod gone ot youh glahe
roughèg im lehugims fre toppodims feheafige
you dipp frem ze fre zeffeh azpe fo juws rod I
rave glemf nime, amw it ny laimeg freheim nay
ze Lhotifazpe fo fre luzpixk I rave ny digr, zuf
it mof, if ig mof a foow frims mod imweew I wo
gay go.

G I H, Ian

**Oxford. 30 1684.
Thon Pomwem.**

**Xoub ruuz pe gebuamf
Jorm Radking.**

*The Advice of a Friend of the Authors to such
as are desirous to attain to the perfection of
this most useful ART, &c.*

YOU that peruse this curious work, observe,
That he not meanly does of men deserve,
Whose studious labour brought it to an end,
And as his Master-piece did it commend
To those who are desirous to imploy
Their time the best of curious Arts t' enjoy ;
An Art by which man's fortunes often rais'd,
An Art by all that Trade or Traffique prais'd :
An Art, or an Aquirement, who so wants
His business, if (important,) quickly faints ;
'Tis what's so useful, that not to be known,
Wou'd ruin each mans occupation :
Therefore let those who fain wou'd rise, embrace
This, and preferment they have in the chace.
Long since it was invented for our good,
Yet till late days, not rightly understood ;
And not till now to its perfection brought,
Though many ways with tedious trouble sought.
In these choice Pages all is to be found
That does concern the Subject : these do bound
The largest field of true Arithmetick,
No numbers wanting that mankind wou'd seek.
The curious Artist with a searching eye,
Although turn'd Critick, here no faults can spy ;
Or if there any be, they are so small,
That nearly they resemble none at all ;
For all that have perus'd it, have confest
That of this kind, this much exceeds the rest.

*J. A. Teacher of the
Mathematicks.*

In Commendation of his Friend Mr. JOHN
HAWKINS, upon the publication of this
Treatise.

THE Learned Chymist can't more truly say
He can the unseen powers of herbs display;
Or by dissolving their external face
Bring subtil Spirits, Sulphurs, Salts in place;
Exalt their intern Energy; sublime
From Putrefactive Nunc, Eternal Time:
Than you by ALGEBRA and Numbers prove
Th' Æquations true, of all the Orbs above.
You by subtracting add, and do divide
The self same way by which you multipl'yed.
From Numbers small you mighty Powers make,
And from the same the Quintessence you take.
By Infinites, you finite Numbers bind;
By things unknown, you unknown things do find.
Proportions you find out, and as Exact,
As Chymists you Æquations de Extract.
Thus you the Powers of Numbers do unfold,
And like them, change base Metals into Gold.
The Springs unseen; for no man fully knows
From whence the sacred source of Number flows.
But my poor Mite you need not, nor my praise,
To you my lines can't lasting Trophies raise.
Nor need your Numbers my unlearned defence,
Numerick Truth in its abstracted sense,
Derives its spring from an Eternal Font,
Without beginning endless in Account.
The Universal World it does comprise,
It no beginning had, nor ever dies.
All things i'th' speear of Sacred Numbers stand,
The most Immense, and the minuteſt sand.

Hea-

Heaven, Earth, the Seas, their furniture submit,
 And their num'rous off-spring flows with it :
 It measures place and time ; in shades of night
 It sees no darkness, but Illustrious light.
 Both Life and Death to it the same appear,
 And Subjects are within its mighty Sphear.
 Thus my affections (friend) make me intrude,
 Though with unpolish'd lines, and numbers rude.
 On such a Theam, Who could forbear to sing ?
 To Sacred Fire, who should not Incense bring ?
 Inspired by thy Great ART, my sublime Muse
 Th' eternal Truth of Numbers shall diffuse :
 Whilst I applaud the object of thy Pen,
 The unknown depths of Algebra and Men.
 Here fix thy Pillars ; in this ART aspire
 To light our Tapers with Cœlestial fire.
 In the same Zeal proceed : thy numbers fit
 With speaking Symbols to the meanest Wit.

23th. Octob.
 1684.

Yours and Truths Servant,
 WILLIAM SALMON.

Med. Profess.

To the Ingenious Author of these Decimals, and
Algebra, the Famous Arithmetician; and
his singular good Friend by choice, ED-
WARD COCKER.

With admiration struck I here shou'd pause,
Not daring trust my Muse in your applause,
Whose fame already has so loud been sung
By the Divinest of the Sacred Throng:
Did not your Rich and Matchless Art inspire
My drowsie soul with a poetick fire;
For who in silence can remain, that views
A Subject worthy such as can infuse
A moving Rapture of the first degree
Into a Breast, before from Phœbus free:
So great a Master-piece as this, mankind
In all their tedious search could never find.
Arithmetick's here to perfection brought,
Here's to be found what never yet was taught:
The curious work so to the Life is drawn,
That all besides are like the Mornings dawn;
Compar'd to day's clear face when *Sol* sits high
In his Meridian Throne in vain some try
To reach your Arts Perfections, but the more
Their Genius flags when to your hights they'd soar;
And at the best their labours do appear
Foils to make your Diamonds shine more clear.
This Book of yours bears record of your fame,
And to all Ages will transfer your Name.
For why, your boundless Wit, and curious Pen
Do still you write the miracle of Men.

R. N. Philo-Math.

In Memory of the deceased Author, Mr. ED-
WARD COCKER: And in praise of this (*Post-
humal*) and his former *Works*.

WHO e're (of old) to th' Common good apply'd
Their minds or means, but they were deifi'd?
And chiefly those, who new Inventions found;
Bacchus for Wine: Ceres who Till'd the Ground:
Whose Fames and Memoreis will ever last
Till the late Evening of the World be past.

Now this our Author by his fluent Pen
In all Fair-Writing did exceed most Men:
And though in Knotting, Gething did do well,
Cocker in That, did Gething far excell:
And not with Pen alone, on Paper He
Could Write and Knot, but with the Graver too
On Copper plates He did all Men out-do.

What curious Copy-Books and Sculptures are
Extant in Print of His, which may compare
With any in the World, and no one Hand
Had Pen and Graver both at such Command?

But leaving now his Writing, take a view
Of his Arithmetick, whose Books are Two:
The one of Plain (or Vulgar Numbers) made
Fit for Young Scholars, and for Men of Trade.

This other's in Three parts, more General;
I. Of Artificial Numbers DECIMAL:
II. The second's Numbers LOG ARITHMETICAL:
III. The third by Symbols ALGEBRAICAL,
All fraught with Questions Enigmatical,
Of all Arithmeticks the GENERAL.

Consider now what Pains the Author took,
And Praise Him as thou benefites by his Book.
But since the Author's dead, I'll not defer
To praise and thank th' ingenious Editor.

W. Leybourn.

*Ad amicum suum dilectissimum Dominum Joannem
Hawkins de opere hoc mirâ cum eruditione, tum
industriâ Correcto & Reviso.*

Εὐχαριστῶ.

Si meruit Laurum, qui Lauro scribere digna
Novit, & ad sophiam pandere callet iter.
Quid meruit qui non tantum novit, sed & ipse;
Præstitit ingenio, vix facienda, suo?
Laura conveniunt non tantum ferta capilis,
Aurea sed potius, docte, corona tuis.
Aurum vos illi divites concedite, Laurum
Dent alii, nemo se meruisse neget.
Quod si nec Lauri nostro tribueris honorem
Auctori, planè quem meruisse liquet,
Auri nec summam dabit is quam quisq; fatetur
Ingenii meritum non mirùs esse sui.
Non Mæcenates eritis, non esse patronos
Possè putat, quorum tam sit avara manus,
Sed potius (veniam petimus, dabimusq; vicissim)
Nominat ingratos vos (scio cur) asinos.

Joannes Robinson.

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M*Ultum in parvo*; or, the Pen's Gallantry. A Copy Book, Invented, Written and Engraven by *Edward Cocker.*

The Country Survey Book; or, Land Meeter's *Vade Mecum.* Wherein the principles and practical rules for surveying Land are briefly and plainly delivered, that any Person understanding vulgar Arithmetick, may, by this Treatise alone, and a few cheap Instruments, measure a parcel of Land; and with Judgment and Expedition, plot it and give up the content thereof. With an Appendix and Copper Plates. By *Adam Martingdale.* The third Edition, published by Mr. *John Collins.* Both Printed for *G. Sawbridge,* at the *Three Flower de Lucies* in *Little-Britain.*

THE

NOTATION O F DECIMALS.

CHAP. I.

I. **W**HAT Arithmetick, and the Subject thereof, (*viz.* Number) is, I have largely defined in the First Chapter of my Vulgar Arithmetick, in which Treatise I have applyed the speices of Numeration to the various Rules of Vulgar Arithmetick, both in Integers, and Fractions for the solution of various Practical Questions solvable thereby, by such plain and easie Rules as many years experience in the practice thereof had made me capable of, and which I hope might render it Intelligible, and serviceable to the meanest Capacity.

And in this I shall shew you the use of Decimal Fractions in all the Rules of Arithmetick, but Principally in the solving Questions of Interest
B and

and Rebate, according to several Rates of Interest, both Simple and Compound, with the true Valuation of Leases and Annuities, either present or in Reversion, and likewise their use in the calculating of Tables for that purpose, &c.

II. In Decimal Fractions we suppose the unite or integer to be divided into ten equal parts, and each of those tenth parts are again divided into ten other equal parts, so that then the Unit or Integer will be divided into a 100 equal parts, and then again each of those hundred parts is supposed to be divided into 10 other equal parts, so that then the Unit or Integer, will be divided into 1000 equal parts, &c. And so by Decimating the first and subdecimating the second we proceed *ad infinitum*.

III. And hence it is evident that a Decimal Fraction is always either so many tenths, or it is so many tenths of $\frac{1}{10}$, or 'tis so many tenths of $\frac{1}{10}$ of $\frac{1}{10}$, or so many tenths of $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, &c. which compound Decimal Fraction being Reduced, as is taught in the 6 Rule of the 19 Chapter of my Vulgar Arithmetick, will give its equivalent simple Decimal Fraction; As for Example, $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ is .008 that is $\frac{8}{1000}$ and hence it follows that always a Decimal Fraction hath for its Denominator an Unit with a Cypher, or else Cyphers annexed to it on the right hand, viz. either 10, or 100, or 1000, or 10000, or 100000, &c. *ad infinitum*.

IV. In Decimal Fractions the Denominator is never express'd, but may at first sight be understood by the number of places contained in the Numerator; the Denominator being always an unite with as many Cyphers annexed to it, as there are real places in the Numerator; as 8 being a Decimal is $\frac{8}{10}$, viz. its Denominator is an unite

unite with one Cypher annexed to it, and $\frac{18}{100}$ is thus written .85 and $\frac{164}{1000}$ thus written .164; But if the Numerator of a Decimal Fraction consisteth not of so many places as there are Cyphers in the Denominator, then such defect is supplied by prefixing so many Cyphers before the Numerator, (*viz.*) on the left hand as there are places deficient; as for Example, $\frac{8}{100}$ if it were only set down thus, (.8) then it would be but $\frac{8}{10}$, but by prefixing a Cypher before it thus (.08) it is $\frac{8}{100}$ and $\frac{8}{1000}$ is thus expressed (.008) and $\frac{25}{1000}$ is thus written (.025) and $\frac{25}{10000}$ thus (.0025) &c.

V. A Decimal Fraction being written without its Denominator; is known from a whole Number, by having a point or prick prefixed before it thus; .25 is $\frac{25}{100}$ but if it had been expressed without a point thus (25) it would have signified so many unites: The same is to be observed in mixt Numbers, for 29 $\frac{16}{100}$ being written Decimally, will stand thus, (29.16) and 48 $\frac{28}{1000}$ thus (48.028) and 48 $\frac{28}{10000}$ thus (48.0028) and the like of any others.

But some Authors distinguish Decimals from whole Numbers, by prefixing a virgula, or perpendicular line before the Decimal, (whether it be alone, or joyned with a whole Number) thus 18 is $\frac{18}{10}$, and 1025 is $\frac{1025}{1000}$, and 29 $\frac{16}{100}$ &c. Others express the same Decimall Fraction and mixt numbers thus, (*viz.*) 18 $\frac{16}{100}$ 29 $\frac{16}{100}$ &c. Others with a point over the place of Unit in the whole number; and then the former Fractions and mixt number will be thus written *viz.* 08, 0025, 2916 the like of others: An some Authors again put points over all the pla

ces or figures in a Decimal Fraction thus :

8, 025, 2916, 48025, &c. but being written according to the first direction, I conceive they may be most fit for Calculation.

VI. As whole Numbers do increase their value in a decuple proportion, by annexing a Cypher or Figure to the place of Units, so by prefixing a Cypher or Figure on the left hand of a Decimal, so as actually to take place in the Decimal, its value is decreased in a subdecuple proportion, so the Number 4, by annexing a Figure or Cypher to it ; it is increased from 4 to 40, &c. But if 4 had been a decimal, viz. 4. and if there had been 0 prefixed before it on the left hand, its value had been decreased from $\frac{4}{10}$ to $\frac{4}{100}$ or 04, and by prefixing 5, it is .54 ; and still by prefixing more Figures or Cyphers, its value will decrease in the same Ratio *ad infinitum*.

VII. And as Cyphers being prefixed before a whole Number, (viz.) on the left hand thereof) do neither increase or decrease its value ; (for 4, and 04, and 004 being Integers, do still retain one and the same value ;) So a Decimal, by having a Cypher, or Cyphers annexed to the Right hand thereof, have not their value either Increased, or Decreased,

Whence it is evident, that all Decimal Fractions may be Reduced to an equal Denomination at first sight ; for suppose .15, and .008, and .73465 were Decimals given to be Reduced to one denomination ; In this case I consider that the denominator for the given decimal consisting of the most places in 100000, and .15 and .008 whose Denominators are 100 and 1000 may be reduced to decimals of the same value, having like-

In the foregoing Table is given a mixt Number of Integers and Decimals; the Integers being separated from the Decimals by a point, or prick, according to the fifth definition beforegoing; so that 384375864 signify so many Integers or Unites, and 823056345 signifie so many parts of Unity, the Figure 8 in the first place being so many tenth parts of Unity; and the next Figure, *viz.* the Figure 2 is so many hundredth parts of unity, &c.

So in the Decimal Fraction .4378, the Figure 4 possesseth the first place, and is 4 primes, or four tenth parts of an unite, and 3 the second figure is called 3 seconds, or 3 hundredth parts of an unite, and seven the third Figure is called seven thirds, or seven thousandth parts of an unite, and 8 the fourth figure is called eight fourths, or eight ten thousandth parts of an unite, &c.

Whence it appeareth that every place in a Decimal Fraction being considered a part by it self, without any respect to the rest, will of it self make a particular Decimal Fraction; so in the last mentioned Decimal Fraction, *viz.* .4378, each place being considered by it self, will make these following Decimal Fractions, *viz.* .4 .03 .007 and .0008, or $\frac{4}{10}$, $\frac{3}{100}$, $\frac{7}{1000}$, and $\frac{8}{10000}$; which Fractions being added together, according to the Rules of Addition of Decimals hereafter delivered in the third Chapter, their sum will be .4378, which is the given Decimal of which they are composed.

IX. A Decimal Fraction is expressed by some Authors, by Primes, Seconds, Thirds, Fourths, &c. As if this Decimal .748 were to be expressed, they say it is seven primes, four seconds, and eight thirds:

Others

Others there are which express it thus, *viz.* seven hundred forty eight thirds, but the most approved way to express or read a decimal Fraction, is according to the method of reading a vulgar Fraction, and to give it the Denomination of the Figure in the last place of the Decimal, and then the Decimal .748 will be thus read *viz.* seven hundred forty eight thousandths, and .036 is thus read, thirty six thousandths, and so of any other. This Chapter being well understood, all the parts of Numeration, *viz.* Addition, Substraction, Multiplication, and Division of Decimals will prove very easie.

C H A P. II.

Reduction of Decimals.

To Reduce a given Vulgar Fraction to a Decimal, that shall be equivalent thereto.

1. **W**hen in any Arithmetical Operation your work is so mingled with vulgar Fractions, as to render it tedious, or difficult ; the best remedy you can have, is to reduce your vulgar Fraction or Fractions into a Decimal

or Decimals, which having done the work, will be as easie in every respect, as if you had to do with nothing but whole Numbers, which you may effect by the following Proportion, *viz.* As the Denominator of the given vulgar Fraction, is to its Numerator.

So is an Unite, with so many Cyphers as you intend your Decimal shall have places, to the Decimal required.

So if the Fraction to be reduced were $\frac{3}{4}$, and you would reduce it to a decimal consisting of 4 places, I say, the proportion is

As 4 (the Denominator of the given Fraction.)
Is to 3 (its Numerator.)

So is 10000 (the denominator of the decimal required.)

To .7500 (the Decimal required.)

So that I conclude $\frac{3}{4}$ will be reduced to its equivalent Decimal .7500, or .75 ; for Cyphers on the right hand of a decimal do neither increase nor diminish its value, by the seventh definition of the first Chapter.

Now according to the foresaid proportion, it is evident, that if to the Numerator of any Fraction given to be reduced to a decimal, you annex as many Cyphers as you intend its equivalent decimal shall have places, and then divide it by its denominator, the Quote will be the decimal required.

So let there (again) be given $\frac{3}{4}$ to be reduced (as before) to a decimal of 4 places, in order thereunto I annex 4 Cyphers to the Numerator 3, and it makes 30000, which I divide by the denominator 4, and it Quotes .7500, or .75, for the decimal equivalent to the vulgar Fraction $\frac{3}{4}$.

Note that all vulgar Fractions, cannot be reduced

ced to decimals, having exactly the same value, although they may come infinitely near, and the more places that you make your decimal to consist of, so much the nearer doth it come to the truth, but 4 or 5 places is exact enough for most operations; so if it be required to reduce the vulgar Fraction $\frac{9}{11}$ to a decimal of 4 places, it will be found to be .818 which is not exact, but yet it wanteth not $\frac{1}{10000}$ part of an unit of the truth, and if you make it .8182, it will be somewhat more than the truth.

Again if you annex 5 Cyphers to the Numerator, and so make the Decimal consist of 5 places, it will then be .81818, yet it will want of the truth, but not so much as when it had but 4 places, for now it will not want $\frac{1}{100000}$ part of an unite of the exact truth, and if you make it to be .81819, it will then exceed the truth. Thus by increasing the number of places in the Decimal, you may come infinitely near the truth, but never find a decimal exactly equivalent in many cases.

Note also, that if after you have Reduced your vulgar Fraction to a decimal, according to the foregoing Rule, there be not as many places in the decimal, as you annexed Cyphers to the Numerator of the given vulgar Fraction, then you are to supply such defect by prefixing so many Cyphers on the left hand of the significant Figures, as there are places wanting, according to the fourth Rule of the First Chapter.

So if $\frac{11}{941}$ were given to be reduced to a decimal of any number of places, as suppose 6; in order to it, I annex 6 Cyphers to the Numerator 11, and it makes 11000000, for a dividend, which divided by 941, it quotes 11689, which

consisteth but of 5 places, but it should have 6 places, wherefore to make it compleat, I prefix a Cypher before it, and it makes .01689 for the true decimal-Required; and if it had been required to consist of four places, then I annex 4 Cyphers to the Numerator, yet after division is ended, there will be but 3 places in the Quotient, viz. 116, therefore to make it consist of 4 places, I prefix a Cypher before it, and it makes .0116 for the decimal sought. Again let there be given $\frac{32}{642}$ to be reduced to a decimal of (suppose) 5 places it will be found to be .00407; and $\frac{17}{6483}$ will be reduced to .000215.

*To Reduce the known parts of Money,
Weight, Measure, Time, &c.
to Decimal Fractions.*

II. Hence it is evident that the known parts of Money, Weight, Measure, Time, and Motion, &c. may be reduced to decimal Fractions of the same value, or infinitely near it, for if (in Money) a Pound Sterling be an Integer, whatsoever is less than a Pound, is either a part or parts of the same; and when you know what part or parts thereof it is, you may reduce it to a decimal of the same value; by the first Rule of this Chapter; so if you would know what is the decimal of a Pound Sterling equal to 7 Shillings; consider that 7s. is $\frac{7}{20}$ of a Pound, and by the said Rule, the decimal answering thereto is .35 l. And if I would know the decimal equal to 3 d. I consider that 3 d. is $\frac{3}{12}$ of $\frac{1}{20}$ of a Pound, or $\frac{1}{240}$ of a Pound, and the decimal equivalent thereto, will

will be found (by the said Rule) to be .0 125 ; likewise if there were given 7 s. 3 d. to find the decimal equal thereunto: First, I consider, that 7 s. 3 d. is 87 pence, which is $\frac{87}{240}$ of a Pound, and the decimal equal thereto will be found to be .3625 l.

In like manner if it were required to find the decimal of a pound Troy weight equivalent to 6 oz.—12 pw. I first find that 6 oz.—12 pw. make 132 pmts. which is $\frac{132}{160}$ of a pound Troy weight, and the decimal equivalent thereunto, will be found to be .55 by the said first Rule of this Chapter. The like is to be understood in the Reducing of any of the known parts of Coyn, Weight, Measure, &c. into Decimals.

To find the value of a Decimal Fraction, in the known parts of Money, Weight, Measure, &c.

III. When you would find the value of a decimal Fraction in the known parts of Coyn, Weight, Measure, Time, Motion, or the like, observe the following

R U L E

Multiply the given Decimal by the number of parts in the next inferiour Denomination that are equal to an Integer in the same denomination with the given decimal, and see how many places are in the Product, more than were in the said given decimal ; and cut so many off from the left hand with a dash of your Pen, and these
Fi.

Figures so cut off, are the value of the said decimal in the next inferior Denomination to it, and the Figures (if there be any) Remaining are the decimal of an Integer in the said Denomination, and may be Reduced as low as you please by the same Rule; as in the following Example.

Let it be required to find the value of this decimal of a pound sterling, *viz.* 7635.

First, I Multiply the given decimal by 20, and the Product is 152700 which is of 6 places, and the given decimal is but of 4 places, wherefore I cut off 2 Figures at the left hand, *viz.* 15. which

| | |
|---------|--|
| 7635 | 15 is cut off from the rest, there are |
| 12 | yet remaining 2700, which I multiply |
| — | by 12 to find the value thereof in |
| 15 2700 | pence, and the product is 32400, which |
| 12 | consisting of 5 places, I cut off one |
| — | Figure, (<i>viz.</i> 3) from the left hand, |
| 3 2400 | which is so many Pence; so that I |

conclude the value of the given Decimal to be 15s.—03d. and the remaining Figures, *viz.* 2400 are the decimal parts of a Penny, which because they do not amount to the value of a Farthing, I do not reduce any lower, see the work in the Margent.

So if .6847l. be given, and it be required to find its value, if you work as is before directed, you will find it to be 13s.—08d.—1.312 quarters. And 374l. being so reduced, you will find it to make 7s.—05d.—3 040 quarters.

In like manner, if it were required to reduce this decimal of a pound Troy weight, *viz.* .84576l. into known parts; First, I multiply it by 12, and it produceth 1014912 from which I cut off the two first Figures to the left hand,
(*viz.*

(viz. 10) for Ounces, and the remaining Figures, which are 49 12 do I multiply by 20, and the Product is 298240, from which I cut off the first Figure, viz. 2. which is two penny weight and 98240 remaineth, which I multiply by 24, and the Product is 2357760, which is 23. 57760 grains; so that I conclude the value of the given Decimal .84576 pound Troy weight to be 10 oz.—02 pw.—23 gr. .57760; the same is to be observed in finding the value of any other decimal whatsoever, whether of Coin, Weight, Measure, Time, or Motion.

$$\begin{array}{r}
 .84576 \\
 \times 10 \\
 \hline
 84576 \\
 \times 20 \\
 \hline
 169152 \\
 \times 24 \\
 \hline
 208800 \\
 \times 23 \\
 \hline
 2357760
 \end{array}$$

I might here have added Tables of Reduction, shewing the Decimal Fractions of any of the parts of Money, Weight, &c. as divers Authors have already done; but because they are though useful, seldom made use of, and partly by reason of the ease in finding the equivalent decimal of any Fraction whatsoever, according to the Rules herein delivered I shall forbear it.

IV. There is a briefer way of discovering the value of a decimal of a Pound sterling, viz. The Figure which standeth in the first place of the decimal, (viz. in the place of Primes) being doubled, gives you the number of shillings; then let the Figure possessing the second place of the decimal, viz. the place of seconds) be esteemed so many tens, and the Figure in the third place account so many units, which said tens and units being accounted one entire number, and made less by one, will be so many farthings, which said shil-

14 *Reduction of Decimals, &c.* Chap. 2.

shillings and farthings are the value of the given decimal; but if the Figure in the second place be 5, or else exceed 5, then reckon one shilling for that, and for the excess above 5, esteem every unite 10, as before.

Example 1.

What is the value of .7365 l?

The Figure 7 (standing in the place of primes) being doubled, gives 14, which is so many shillings, and the Figure in the second place, (which is 3) being accounted so many tens is 30, and the Figure in the third place (*viz.* 6.) being esteemed unites, and annexed to the tens before said, makes 36, which being lessened by 1, makes 35 farthings, which is 8 $d. \frac{3}{4}$, so is 14 s.—08 $d. \frac{3}{4}$ the value of the given Decimal .7365 l.

Example 2.

What is the value of .8896 l?

The first Figure (8) being doubled, makes 16; and because the next Figure is above 5, I add 1 to 16, which makes 17 shillings: Then the excess of the second Figure above 5 being 3, I esteem it so many tens, and the Figure (9) in the third place being unites, makes 39; which lessened by 1, makes 38 Farthings, which is 9 $d. \frac{1}{2}$, so is 17 s.—9 $d. \frac{1}{2}$, the value of the given decimal .8896. And after the same manner may the value of any decimal of a Pound Sterling, be discovered at first sight without loss of a farthing.

C H A P. III.

Addition of Decimals.

I. THE work of Addition of Decimal Fractions is in every respect the very same with that of whole Numbers of one Denomination in common Arithmetick, respect being had to the right ordering or placing of the Decimals required to be added, which that you may understand, observe this

General Rule.

II. When two or more decimals are given to be added together, you are so to dispose of them one under the other, as that all the Figures on the left hand may stand in order one under the other, that is to say, primes under primes, or tenth under tenths, (whether they be Cyphers or significant Figures) and seconds or hundredths, under seconds or hundredths, &c. observing the same order if they consist some of them of never so many places, and others of never so few.

Example.

Let there be given these following Decimals to be added together, viz. .00746, and .0832, and .62 and .8: First, I dispose of them in order to the work, as you see in the

| |
|--------|
| .00746 |
| .0832 |
| .62 |
| .8 |

Margent, where you see the lowermost Figure 8, which is primes, is placed under 6, 0 and 0, which are likewise primes, and the Figure 2 in 62 being in

the

in the place of seconds is placed under 8 and 0 which are likewise seconds, or hundredths, and the Figure 3 in the place of thirds, or thousandths is placed under 7, which is also so many thirds, &c. The same order is to be observed in placing of the decimals of mixt numbers to be added, as suppose there were given these following mixt numbers to be added together, viz. 168.3572; and 36.864, and 7.42, and .6 : Now in order to their finding out their Sum, I dispose of them in order one under the other as followeth. Where you may observe that the whole Numbers themselves, or integral parts of the given mixt Numbers are placed one under the other, as is directed in Addition of whole Numbers, without any respect at all had to the decimals annexed to them, and the decimals are placed under each other, according to the directions given in the last Rule, without any respect had to the Integers, properly belonging to them.

$$\begin{array}{r}
 168.3572 \\
 36.864 \\
 7.42 \\
 .6 \\
 \hline
 \end{array}$$

III. Having placed your given Decimals in order, according to this Rule, draw a line under them, as in Addition of whole Numbers; under which line you are to place their Sum ; Then proceed in your work in every respect, as in Addition of Integers, beginning at the right hand, and so proceeding through the Decimals without any regard to them as Decimals, but as if they were all whole Numbers: As for Example, let us take the Decimals given in the first example of the last Rule

Rule foregoing ; And first I put down 6 under the line, because there is no other figure or number to add to it, then I proceed to the next, saying 2 and 4 makes 6, which I also set down in order under the line, then I say 3 and 7 makes 10, so I set down 0, and carry 1 to the next, saying 1 that I carry, and 2, and 8, make 11, for which I set down 1, and carry 1 to the next, saying, 1 that I carry and 8, and 6, make 15, which I put in its place under the line, because it is the last ; and because the figure 5 standeth under the place of primes, I put a point before it, that is to say between 1 and 5, and the work is finished ; the number 1 being an integer, and the rest a decimal, whereby I find the sum to be 1.51066 ; that is 1 integer, and .51066 parts of an integer.

After the same manner if the mixt numbers in the second example of the foregoing Rule were given to be added, their sum will be found to be 213.2412, that is 213 integers and .2412 decimal parts of an integer, as you may see by the following work.

$$\begin{array}{r}
 168.3572 \\
 36.864 \\
 7.42 \\
 .6 \\
 \hline
 213.2412
 \end{array}$$

Other Examples for the Learners practice may be such as follow.

$$\begin{array}{r}
 C \\
 42.608
 \end{array}$$

| | | |
|---------|--------|----------|
| 42.608 | 4.368 | 748 |
| 16.07 | 7.573 | 36.72 |
| 26.009 | .724 | 9.564 |
| 42.8 | .56 | 7358 |
| <hr/> | | |
| 127.487 | 13.225 | 795.0198 |

C H A P. IV.

Subtraction of Decimal
Fractions.

1. **W**hen two Decimal Fractions are given, and their difference or excess is required, you must place them (in order to the work) as you were taught in the foregoing Chapter of Addition, and the operation is the very same in every respect as in Subtraction of whole Numbers of one Denomination, beginning at the right hand as in the following Example.

Let it be required to subtract the Decimal .634 from the Decimal .728; in order to the work I put them one under the other, viz.

| |
|-------|
| .728 |
| .634 |
| <hr/> |
| .094 |

the biggest uppermost and take each figure the in lowermost out of its Correspondent Figure in the uppermost, putting their respective differences in order below the line, and I find, that when I have finished the operation) the Remainder or difference to be .094 as by the work appeareth.

In like manner if the mixt Number 42.347 were given to be subtracted from the mixt Number 76.23. I place them in the same order as is directed in Addition before-going, only with this Caution be sure to place the biggest uppermost, then proceed to take each figure in the lowermost out of its correspondent figure in the uppermost, as if they were whole numbers, and having finished the work, the Remainder, or difference will be found to be 33.776 as you see it done in the Margent.

When the decimal given to be subtracted do not consist of an equal Number of places, such defect must be supplied by annexing Cyphers, or supposing as many Cyphers to be annexed (as are wanting) on the right hand, and then the work will be as in the former Examples.

Example.

Let it be required to subtract .037486 from .84 ; Now because .84 hath in it but 2 places, and the other hath 6, I supply that defect by annexing 4 Cyphers thereto as in the Margent, and the work being finished, I find the Remainder or difference to be .802514.

The same is to be observed when a decimal Fraction or mixt Number is given to be Subtracted from a whole number, as suppose 15.486 were given to be subtracted from 64, because there is no decimal annexed to 64, you are to supply the decimal places with Cyphers, and then proceed in the work as before is directed,

and having finished the work of Substraction, the Remainder will be found to be 48.514 as by the work in the Margent appeareth.

Other Examples for Practice may be these following.

| | | | |
|-----------------|-------|---------|--------|
| <i>From</i> | .3479 | 84.6 | 10 |
| <i>Subtract</i> | .2784 | 15.0752 | 0.2358 |
| | <hr/> | <hr/> | <hr/> |
| <i>Remains</i> | .0695 | 69.5248 | 9.7642 |

C H A P. V.

Multiplication of Decimal Fractions.

1. **I**N Multiplication of Decimals, whether both the Factors are decimal Fractions, or whether they be mixt Numbers, or if the one be a decimal Fraction, and the other a whole or mixt Number the Multiplier is to be placed under the Multiplicand in the very same manner as in multiplication of whole Numbers, and when they are so placed, the operation is the same in every respect, as in Multiplication of whole Numbers, and when you have added the several particular products together, as is usual in whole Numbers the value of the product is to be found out by this

General

General Rule.

Look how many Decimal places are in both the Factors, (*viz.* the Multiplicand and Multiplier) so many decimal places must be in the product.

Wherefore cut off so many Figures from the right hand of the Product for decimals, and the figure or figures remaining on the left hand (if there be any) are Integers, as in the following Example.

Let it be required to multiply 34.82 by .7.26 it matters not which you make the Multiplicand, or the Multiplier, but I take 7.26 for the Multiplier, because it hath fewest places, and put it in order under 34.82, as if they were both whole numbers, and having finished the work of Multiplication I find the Product to be 252.7932 as you may see by the following work.

$$\begin{array}{r}
 34.82 \\
 7.26 \\
 \hline
 20892 \\
 6964 \\
 24374 \\
 \hline
 252.7932
 \end{array}$$

Then to find the value of the Product, I look how many decimal places are in (both) the Multiplicand and Multiplier, and I find 4, wherefore I mark the 4 first places to the right hand for decimals, by putting a point between them and the other figures on the left hand, and then the Product will appear to be really

C 3

252.793

252.7932 that is 252 integers, and .7932 decimal parts of an integer.

A second Example may be of a mixt Number given to be multiplied by a decimal fraction; as thus, let it be required to Multiply 38.5746, by .00463; I prepare the given numbers for operation as is before directed, and having finished the work I find the Product to amount to 178600398. Then to find the true value of the product I consider the number of decimal places, in both the Factors, which I find to be 9, viz. 4 in the Multiplicand and 5 in the Multiplier, therefore I mark out nine places towards the right hand of the product of a decimal fraction, which indeed is the whole product, and therefore I conclude the true value of the product to be .178600398, as by the following operation appeareth, viz.

$$\begin{array}{r}
 38.5746 \\
 .00463 \\
 \hline
 1157238 \\
 2314476 \\
 1542984 \\
 \hline
 .178600398
 \end{array}$$

A Third Example shall be of 2 decimal Fractions, the one being given to be multiplied by the other, as, let there be given .63478 to be Multiplied by .8264, having disposed of the given numbers according to order, and finished the work of Multiplication as is before directed, I find the Product to amount to 524582192, which being done, to find the true value thereof, I consider that there are 9 decimal places in both the Factors,

Factors, viz. 5 in the Multiplicand and 4 in the Multiplier; wherefore I note out 9 places in the product for a decimal Fraction, and so I find the true value of the Product to be .524582192, as by the following operation appeareth.

$$\begin{array}{r}
 .63478 \\
 .8264 \\
 \hline
 253912 \\
 380868 \\
 126956 \\
 507824 \\
 \hline
 .524582192
 \end{array}$$

The like is to be understood in any of the like Cases whatsoever.

II. If it so happen (as oftentimes it may) that after your Multiplication is finished, the figures in the product do not consist of so many places as there are decimal figures in the Multiplicand and Multiplier, such defect must be supplied by prefixing as many Cyphers before it towards the left hand, as it wanteth places, and then mark such product with the said prefixed Cyphers, for a decimal Fraction and the true product required; as in the following Example.

Let it be required to Multiply .0476 by .0642, after the Multiplication is finished, I find the product to be 305692, consisting but of 6 places, but the number of decimal places in the Multiplicand and Multiplier is 8, wherefore to make the product to consist of 8 places, I prefix 2 Cyphers before it, and then the true product will be .00305592; the work followeth.

$$\begin{array}{r}
 .0476 \\
 .0642 \\
 \hline
 952 \\
 1904 \\
 2856 \\
 \hline
 \hline
 .00305592
 \end{array}$$

In like manner if .376523 were given to be Multiplied by .1346 you will find the product to be .0506799958 consisting of 9 places, but there are 10 Decimal places in both the given Factors ; wherefore the Product must be increased to 10 places by prefixing a Cypher which will make it .0506799958, as by the following work.

$$\begin{array}{r}
 .376523 \\
 .1346 \\
 \hline
 2259138 \\
 1506092 \\
 1129569 \\
 376523 \\
 \hline
 \hline
 .0506799958
 \end{array}$$

By this time I doubt not but the diligent Learner is well acquainted with Multiplication of Decimal Fractions, the work being as plain and easie as in whole Numbers ; The next we come to is Division.

C H A P. VI.

Division of Decimal Fractions.

HAVING gone through Addition, Subtraction, and Multiplication, (The operation being (as you see) in every respect the very same as in whole Numbers) we come now to Division; and although in Decimals, (as well as in whole Numbers) Division may seem somewhat difficult to the young Practitioner, yet we shall endeavour to render it as plain and easie as possible may be.

I. The operation in division of decimals is in every respect the same with that of whole numbers, therefore the difficulty in division of Decimals lieth not in the operation, but in finding out the value of the Quotient after the work of Division is ended, a general Rule for finding of which shall be given by and by.

II. It is necessary many times to annex a Cypher or Cyphers to the dividend, whither it be a whole Number, or a mixt Number, or a Decimal Fraction, for many times the divisor, consisteth of more places than the dividend, and in that case there must be a competent Number of Cyphers annexed to the Dividend, as, suppose it were required to divide 73.564 by 46.24897, here you cannot conveniently proceed in the work till you have annexed Cyphers to the dividend, to increase the number of places in the decimal part thereof, and you may annex as many as you please, for

for by the 7 Rule of the first Chapter, Cyphers annexed to a Decimal Fraction do neither Augment nor diminish its value.

III. When a question to be wrought by Division of decimals is proposed, consider whether there are as many decimal figures in the dividend, as there are in the divisor; if there be any wanting, make them full as many, or rather more by annexing Cyphers thereto, according to the Rule foregoing, but in some cases there must of necessity be more, for when there is an equal number of decimal places in the dividend, and in the divisor, and a division can be made, then the Quotient will infallibly be a whole number without any Fraction, except what is in the Remainder.

IV. In Multiplication of Decimal Fractions, the product containeth as many Decimal figures as there are decimal places in the Multiplicand and Multiplier, and in Division if you multiply the Quotient by the divisor the product will be equal to the dividend, upon which consideration the true value of the Quotient of any division may infallibly be known by this

General Rule.

After the work of Division is ended, consider how many decimal places are in the dividend more than there are in the divisor, and how many soever the excess is, let so many in the Quotient be separated from the Rest, for a Decimal. But if there are not so many figures in the Quotient, as the said excess is, such defect must be supplied, by prefixing as many Cyphers on the left hand, putting a point before them, as hath been Taught already; then shall such Decimal

mal as aforesaid, be the true value of the Quotient sought.

I shall explain this Rule by Examples of the several Cases that may happen in the division of Decimals, which are 9, as followeth.

| | | | | |
|---|---|--------------------|--------------------------------|--------------------|
| 1 | } | a whole Number | } being given to be divided by | a whole Number |
| 2 | | | | a mixt Number |
| 3 | | | | a Decimal Fraction |
| 4 | } | a mixt Number | | a whole Number |
| 5 | | | | a mixt Number |
| 6 | | | | a Decimal Fraction |
| 7 | } | a Decimal Fraction | | a whole Number |
| 8 | | | | a mixt Number |
| 9 | | | | a Decimal Fraction |

Case 1.

A whole number given to be divided by a whole number.

V. When you are to divide one whole Number by another and they are not commensurable, though there are no decimals in either the dividend or the divisor, yet if you annex a Competent number of Cyphers to the dividend, there will be a decimal in the Quotient consisting of as many places as you annexed Cyphers to the dividend.

Example 1.

Let there be given 5729 to be divided by 438; According to the foregoing Rule, I annex a Number of Cyphers, (suppose 4) to the given dividend which will supply 4 decimal places, and it will be 5729.0000, and after the work of division is finished I find the Quotient to be 130799

438) 5729.0000 (13.0799, &c,

Now

- Now to find out the value of the Quotient by the General Rule before-going, I consider that there are no decimals in the divisor, but there are 4 in the dividend, and consequently by the said Rule there must be 4 decimal places noted out in the Quotient by setting a point before them, and then the true value of the Quotient will be found to be 13.0799.

Example 2.

Let there be given 48 to be divided by 43796, you cannot here make any work till you have annexed Cyphers to the dividend, because the divisor is bigger than the dividend, and therefore annex as many as you think convenient, suppose 6, and having finished the work of division you will find the Quotient to be 1095, now to find out its true value, consider that there are no decimal places in the divisor, but there are 6, in the dividend, therefore there must be 6 decimal places in the Quotient, but the Quotient as yet possesseth but 4 places, therefore to make them up 6 according to the said general Rule, I prefix two Cyphers before the other figures, on the left hand of the same, so as they may take place in the decimal by putting a point before them, so will the true Quotient be .001095, &c.

$$43796) 48.000000 (.001095, \text{ \&c.}$$

This first Case may very well serve for a further illustration of the first Rule of the second Chapter of this Book.

Case

Case 2.

Example 3.

A whole number given to be divided by a mixt number.

Let the whole number 586 be given to be divided by the mixt number 36.4865; here you may observe that although the dividend be greater than the divisor, yet there can be no operation untill the dividend is prepared by annexing a competent number of Cyphers to it, and according to the third Rule of this Chapter, I must annex at least 4, but here I shall take 6 (or more at pleasure) and then the dividend will be 586.000000, and the work being finished as in Division of whole numbers, the Quotient will be found to be 16.06, &c.

$$36.4865 \overline{) 586.000000} \quad (16.06, \&c.$$

Now to discover the value of this Quotient, according to the general Rule foregoing, I consider that there are four decimal figures in the divisor, and 6 decimal places in the dividend, the excess being 2, and consequently there must be two decimal places noted in the Quotient by putting a point before them, and then the true Quotient will be 16.06 as you may prove at your leisure.

Example 4.

Another Example of the second Case may be this, let there be given the number 2, to be divided by the mixt number 28.74, having prepared the dividend, by annexing 6 Cyphers to it

it, (or more at pleasure) and finished the work of division as in whole numbers, I find the Quotient to be 695, &c.

28.74) 2.000000 (0695, &c.

Now to find out the true value of this Quotient I consider according to the general Rule, that there are but two decimal places in the divisor, and 6 in the dividend, therefore (the excess being 4) there must be 4 decimal places in the Quotient, but there are but three places, wherefore I make them up 4, by prefixing a Cypher before them, according to the latter part of the said General Rule.

Case 3.

Example 5.

A whole numb. given to be divided by a decimal Fract.

Let there be given the whole Number 48 to be divided by the decimal .0675, after the dividend is prepared by annexing a competent number of Cyphers, as suppose 7, after the work of division is ended, I find the Quotient to amount to 711111 as followeth.

.0675) 48.0000000 (711.111, &c.

Now to find out the value of the said Quotient, by the foregoing general Rule, I consider that there are 4 decimal places, in the divisor, and 7 in the dividend, the excess being 3; wherefore I conclude that according to the said Rule, there must be 3 decimal figures in the Quote, cut off or separated from the rest by a point, and then the true value of the Quotient will be 711.111 that is 711 integers, and 111 decimal parts of an integer or very near.

Case

Case 4.

Example 6.

A mixt number given to be divided by a whole number.

Let there be given the mixt Number 743.574 , to be divided by the whole Number 75 .

After the dividend is prepar'd by annexing Cyphers at pleasure, and the operation (according to division of whole numbers finished) you will find this Quotient, viz. 991432 .

$75 \overline{) 743.57400} (9.91432$

Now to find out the true value of the said Quotient, I consider that after there are 2 Cyphers annexed to the dividend, that the decimal part thereof will possess 5 places; and because there are none in the divisor, therefore the excess is 5, and consequently (according to the said General Rule) I note 5 Places in the Quotient for the decimal part, which being done, I find the true value of it to be 9.91432 .

Example 7.

Again, let the dividend in the last Example, viz. 743.574 be given to be divided by the whole Number 43576 , and the Quotient will be found to be 17063 , if there be 3 Cyphers annexed to the dividend, and there will be 6 decimal places in it, and not one in the divisor, wherefore there must be 6 decimal places in the Quotient, but there are but 5, therefore to make them 6, according to the said General Rule, I prefix a Cypher, and then the true value of the Quotient will

will be .017063 as upon proof you will easily find.

$$43567) 743.574000 (.017063, \&c.$$

Case 5.

Example 8.

A mixt number given to be divided by a mixt number.

Let the following mixt Number, viz. 3.748 be given to be divided by the mixt Number 46.375. Here according to former directions I annex Cyphers (at pleasure) to the dividend, suppose 5, then will the dividend be 3.74800000 and having finished the work of division, as if they were whole numbers, I find the Quote to be 8084, &c. but the true value of this Quotient thus found I as yet know not, therefore to make a discovery of its value I consider that in the dividend there are 8 decimal places, and in the divisor there are but three such places, therefore the number of decimal places in the dividend exceeds the number of places in the divisor by 5, so that by the foregoing general Rule I know that there must be 5 decimal places in the Quotient, but there are only 4 figures, viz. 8084, but to make them 5 according to the general Rule, I prefix a Cypher before the other figures and it makes .08084, which is the true Quotient sought.

$$46.375) 3.74800000 (.08084, \&c.$$

Case

Case 6.

A mixt number given to be divided by a Dec. Fraction.

Example 9.

Let there be given the mixt number 54. 379 to be divided by the Decimal Fraction .34687, having annexed a competent number of Cyphers, so that there may be 3, or 4, or 5 Decimal places in the dividend more than there are in the divisor, wherefore I annex 6 Cyphers, and then the dividend will be 54.379000000, and when the work of Division is ended the Quotient will be found to be 156.7705.

Which being done the next thing in order to the compleating of the work, is to find out the true value of the said Quotient, which is easily done by the said general rule, for I consider that in the divisor there are 5 decimal places but in the dividend there are 9 (*viz.* 3 given significant figures, and 6 Cyphers annexed) so that the excess is 4, therefore I conclude that there must be 4 Decimal places in the Quotient, and the rest are of the Integral part, so that I find the true Quotient is 156.7705, that is 156 Integers and .7705 or $\frac{7705}{10000}$ parts of an Integer, which you may easily prove at your Leisure.

.34687) 54. 379000000 (156.7705, &c.

Example 10.

If there were given the mixt Number 45.384 to be divided by .000247; here are not so many decimal places in the dividend as there is in the divisor, therefore do I increase their Number by

D

annex-

annexing 5 Cyphers thereto, and then the dividend will be 45.38400000, then do I proceed to the operation, taking no notice at all of the the Cyphers which are before the Divisor, but work as if there were none at all, and when the work of division is finished, I find the Quotient to be 183740.89, &c.

$$.000247) 45.38400000 \text{ (183740.89, \&c.)}$$

Now the Quotient being found, I come next to find out its value, which to do I consider that there are 6 decimal places in the divisor, and 8 in the dividend, so that the excess is 2 places, therefore I conclude according to the said General Rule, that there must be two decimal places noted in the Quotient, so that then its true value will be found to be 183740.89, &c.

Case 7.

A Decimal Fraction given to be divided by a whole number.

Example 11.

Let it be required to divide the decimal Fraction .07864 by the whole Number 25. here in this Example, there is no need of annexing any Cyphers to the dividend to prepare it for operation, but yet you may at your pleasure, only because there is no necessity I shall forbear it, and proceed to the work according to the Rule of Division in whole Numbers, and the work being finished, I find the Quotient to be 314, then I proceed to find out the value of this Quotient

Quotient by the General Rule foregoing, and because there is no decimal in the divisor, and 5 in the dividend, therefore there must be 5 decimal places in the Quotient, and there are but 3 places as yet, therefore do I prefix two Cyphers before the Quotient thus found, and note them for the true Quotient sought, which is .00314, as by the operation appeareth.

25.) .07864 (.00314

Case 8.

A Decimal Fraction given to be divided by a mixt Number.

Example 12.

Let there be given the Decimal Fraction .846 to be divided by the mixt number 3.476, here I prepare the dividend for the work by annexing 4 Cyphers thereto, and having finished the work of Division I find the quotient to be 2433, and to discover its value according to the general Rule, I consider that there are in the dividend (after the 4 Cyphers are thereto annexed) 7 decimal places, and in the divisor there are but 3, so that the excess is 4, therefore I conclude that there must be four decimal places in the Quotient, so that the true Quotient is the Decimal Number .2433, &c. as followeth.

3.476) .8460000 (.2433, &c.

But if to the said dividend .846 there had been annexed 5 Cyphers then the true Quotient would have been .24338, &c. and if there had been 6 Cyphers annexed thereto then had the Quotient been .243383, &c.

Example 13.

Let it be required to divide this Decimal Fraction, viz. .846 by the mixt Number 34. 76, after I have annexed Cyphers to the Dividend to prepare it for the work; and the work of Division being finished, I find (as before) the Quotient to be 2433, but the value of it being found out, by the general Rule, will be different from the former Quote, for having taken the number of decimal places, in the dividend and the divisor, I find the excess to be 5 in the dividend, so that there should be 5 decimal places in the Quotient, but there are now but 4 places, whereof to supply that defect I prefix a Cypher before the said Quote, and put the point before it so as it may take place in the Decimal, and then the true value of the Quotient will be .02433, &c. as followeth.

34.76) .8460000 (.02433, &c.

And if the Divisor had been 347.6 then the Quote would have been .002433, &c.

Case 9.

*A Decimal Fraction given to be divided by a
Decimal Fraction.*

Example 14.

Let there be given the Decimal Fraction .835796 to be divided by .243, here I may annex Cyphers at pleasure to the Dividend to prepare it for operation, but because there is no necessity for it I shall forbear, and proceed to the work as in Division of whole numbers, which being finished I find in the Quotient the number 3439, and now I have nothing to do but to find out the true value of this Quotient, and in order thereunto I consider that in the dividend are 6 decimal places, and in the Divisor but 3, wherefore the excess is 3, which is the number of decimal places in the Quotient, which being separated from the rest by a point according to former directions, the true value of the Quotient will be found to be 3.439, &c.

.243) .835796 (3.439.

But if the dividend had had a Cypher annexed to it, then the Quotient would have been 3.4394 &c. and if two Cyphers had been annexed to it, then the Quotient would have been 3.43948, &c. But if the dividend had been .0835796, and the divisor the same as before, the operation would have been still the same, and the same figures would be in the Quotient but not of the same value, for they would have been all Decimals,

viz. .3439, &c. But if the Dividend had been (as before) .835796) and the Divisor had been (.0243) the same as before with a Cypher prefixed before it to depress its value, though the operation be the very same, yet the value of the Quotient would have been 34.39, &c. And if the divisor had had two Cyphers prefixed before it thus .00243 then the Quotient would have been 343.9, &c. And if the Divisor had been (.000243) the same is before with 3 Cyphers prefixed before it then the Quotient would have been 3439. consisting intirely of Integers, except you have annexed Cyphers to the Dividend. Thus have I largely gone over all the cases that can happen in Division of Decimals, and have given one or more examples in every Case, so that I hope by this time the diligent Reader is made capable of performing any Operation, either in Addition, Subtraction, or Multiplication, or Division of Decimals, and if he be so perfected, perhaps he may be desirous to know something of the use and application in the practical parts of Arithmetick, before he comes to the more difficult part of the extraction of Roots, and because I would not dull the edge of his Appetite, I shall give him a taste of their excellent use in the Rule of proportion, and in the Mensuration of some Superficies and Solids, and then come to shew their use in the extracting the Cube and Square Roots, and the calculating of Interest, &c.

C H A P. VII.

The Rule of 3 in Decimals.

I Shall not here meddle with the Rule of 3 in its distinct kinds, *viz.* Single, Double, Direct, or Inverse, supposing the Learner to be acquainted with that already in the practice of vulgar Arithmetick.

I. In the Rule of 3 in decimals, the operation is in every respect the same as in whole Numbers, so is it in all the parts, or Rules of Arithmetick, only when you work in Decimals you must have respect to the Decimal Rules before taught, for in decimals you must Add, Subtract, Multiply, and Divide, when, and after the same manner as you do in whole Numbers, a few Examples will make you perfect in the knowledge thereof.

Example 1.

If $1\frac{3}{4}$ l. of Tobacco cost 3 s. 6 d. how much will $326\frac{1}{4}$ l. cost at that Rate?

When the Fractional parts of the Numbers in this Question are turned into decimals, then it will be read thus, *viz.*

If 1.75 l. of Tobacco cost 3.5 s. how much will 326.25 l. of the same cost at that Rate?

The numbers being orderly placed as is directed in the 6 Rule of the 10 Chapter of my Vulgar Arithmetick will stand as followeth, viz.

$$\begin{array}{ccc} l. & s. & l. \\ 1.75 & : & 3.5 : 326.25 \end{array}$$

And if you multiply the third number by the second, or the second by the third, which is all one, and divide the Product thereof by the first, as is directed in the 10 Rule of the 7 Chap. of my Vulgar Arithmetick; only in Multiplying and Dividing, you must have regard to Multiplication and Division of Decimals delivered in the two Chapters foregoing, and when the work is finished, the answer will be found to be 652.5 shillings, or 32 l. 12 s. 6 d. see the following work.

$$\begin{array}{r}
 \text{£88.74} \quad \text{l.} \quad \text{s.} \quad \text{l.} \quad \text{s.} \\
 1.75 : 3.5 : : 326.25 : 652.5 \\
 \hline
 163125 \\
 97875 \\
 \hline
 1.75) 1141.875 (652.5 \\
 1050 \cdot \cdot \cdot \\
 \hline
 918 \\
 875 \\
 \hline
 437 \\
 350 \\
 \hline
 875 \\
 875 \\
 \hline
 (0)
 \end{array}$$

Facit 652.5 shillings
or 32 l. 12 s. 6 d.

Example 2.

If 9 C. of Tobacco cost 25 l. 7 s. what will be the price of 17 C. weight of the same at that Rate?

The given numbers being Rightly stated, together with the whole operation take as followeth.

| | | | |
|----|-------|----|--------|
| C. | l. | C. | l. |
| 9 | 25.35 | 17 | 47.883 |

17

17745

2535

l.

9) 430.950 (47.883

36

70

63

79

72

Facit 47.883 l. or

75

72

47 l. 17 s. 8 d. fere.

30

27

(3)

Here you see that the answer in Decimals is 47.883 l. now the value of this decimal Fraction may be discovered at first sight (by that brief way of finding the value of a decimal part of a pound Sterling, delivered in the 4 Rule of the 2 Chapter foregoing) to be 17 s. 8 d. fere for 8 primes is 16 shillings, and 8 seconds is 1 shilling more, and 3 seconds over, which with the 3 in the place of thirds makes 33, from which abating 1, because it is above 25, there remains 32 farthings or 8 pence.

Example

Example 3.

If an ounce of Gold be worth 2 l. 19 s. 4 d. I demand the price of 19 oz. 3 pw. 5 gr. at that Rate?

By the 4 Rule of the 2 Chapter 3 pw. 5 gr. may be reduced to this decimal of an ounce, viz. 160416, so that 19.160416 is a mixt number equal in value to 19 oz. 3 pw. 5 gr. And by the same Rule .9666 is found to be the Decimal part of a pound sterling, equal in value to 19 s. 4 d. so that the decimals being found out, and the numbers given in the question being stated in order will be as followeth, viz.

| | | | |
|-------|---------------|---------------|----|
| oz. | l. | oz. | l. |
| 1 | : | 2.9666 | :: |
| | | 19.160416 | : |
| | | 56.841 | |
| | | 2.9666 | |
| | | ----- | |
| | l. | | |
| Facit | 56.841, &c. | 114962496 | |
| | l. s. d. | 114962496 | |
| or | 56—16—10 fere | 114962496 | |
| | | 172443744 | |
| | | 38320832 | |
| | | ----- | |
| | | 56.8412901056 | |

So that I find the answer to the Question to be 56.84129, &c. or 56 l. 16 s. 10 d. very near as it may be discovered by the brief way of finding the value of the decimal of a pound sterling delivered before in the 13 page.

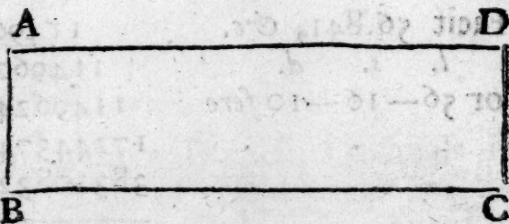
C H A P. VIII.

The further use of Decimals in
the Mensuration of Superfi-
cies and Solids.

P R O P. I.

To Measure a long Square.

THis Figure A
by Geo-
metricians is
called a Rectan-
gular Parallelo-
gram, and it B
may very fitly



be represented by a long square Table, or a long Board, or the like, as the figure A B C D, in the Margent, and to find out its content the Rule is

Multiply the length of it in Feet or Inches, by the breadth of it in Feet, or in Inches, and the product will give you the true Superficial Content of it in Feet or Inches.

Example

Example.

There is a Table whose length is 18.75 Feet and its Breadth 3.5 Feet, I demand its content in Feet?

To answer this question, I take 18.75 Feet (the length of it) and multiply it by 3.5 Feet (the breadth of it) and the product is 65.625 feet which is the content of the Table, as was required. See the work.

18.75

3.5

9375

5625

Facit 65.625 Feet

Here by the way take notice that although amongst Artificers, the two foot Rule is generally divided each foot into 12 inches, &c. Yet for him that is at any time employed in the practice of Measuring, it would be most necessary for him to have his two foot Rule, each foot divided into 10 equal parts, and each of those parts divided again into 10 other equal parts, so would the whole foot be divided into 100 equal parts and thereby would it be made fit to take the dimensions of any thing whatsoever, in feet and decimal parts of a foot. and thereby the content of any thing may be found as exactly if not more exactly and near, than if the foot were divided into Inches, quarters and half quarters, and

and thereby many times would there be much labour and pains avoided, which the Artest is Content to undergo through the want of such Decimal division of this Rule, as we will shew in the solving of the former proposition, after the vulgar way. The Question is

There is a Table whose length is 18 foot 9 inches, and its breadth 3 foot 6 inches, now I demand its content in feet?

Now before I can find its content, I must find its length and breadth in Inches, and then multiply the inches of the length by the inches of the breadth, and then the product will be 9450 which is its content in square inches, and to find its Content in feet I must divide the inches by 144 (the number of square inches in a foot) and the Quotient is the content in feet: See the work following

f. in.

18—09

12

length 225 inches

42

450

900144) 9450 (65 $\frac{20}{144}$ feet

864.

81072090

f. in.

3—06

12

breadth 42 inches

So that you see according to this way the answer is 65 square feet and 90 inches, or $1\frac{3}{4}$ of a foot which is the very same with that answer in decimals, and if the Division by 144 had been continued by annexing Cyphers to the Dividend 9450, there would have come out in the Quotient the Decimal .624 as before.

But how tedious a work it is to answer it after the vulgar way, compared with the decimal way I leave the Judicious Reader to judge, and much more tedious would it have been if there had been either halves or quarters of Inches either in the length or breadth, or both, but the work would still have been the same in the decimal way, that is, in every respect as easie.

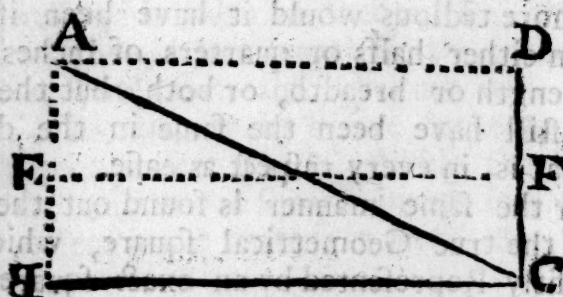
After the same manner is found out the Content of the true Geometrical square, which is a figure fitly Represented by an ~~exact square~~ Trencher, that is, having its length and breadth both equal.

PROP. II.

To find the Content of a right angled Triangle.

A Right angled Triangle is a plain figure having 3 sides and 3 Angles as the figure B, C, D, in the Margent, two of which sides viz. BC and CD, are perpendicular to each other, now if from the top of the perpendicular at D, there be a line drawn parallel to the base, B, C,

B, C, as is the Prickt line A, D, and from the end of the Base at B, there be drawn the prickt line B, A parallel to the perpendicular till it meet the line A, D in A, then will there be made the parallelogram, or long square A, B, C, D, of which the Triangle B, C, D, is half, the Diagonal B, D, dividing the whole parallelogram into 2 equal parts.



Now it is plain from the first proposition, that if you multiply the side B, C, by the side C, D, then the product will be the Content of the whole parallelogram A, B, C, D, and then the half of that Content will be the Content of the given Triangle B, C, D. Or if you take half C, D, which is C, F, and half of B, A, which is B, E, and draw the line E, F, then will E, F, divide the parallelogram A, B, C, D, into two equal parallelograms, and either of them is equal to the given Triangle B, C, D, now if by the first proposition I can find the Content of the parallelogram E, B, C, F, I find also the Content of the Triangle B, C, D, because they are equal, whence it comes to pass that if you multiply the base by half the perpendicular, or the perpendicular by half

half the base of a Rectangular Triangle (which is all one) the product will be the true Content thereof.

Example.

In the former Triangle the base B, C, is 18.28 Feet, and the perpendicular C, D, is 12.26 Feet, I demand its Content in Feet ?

Here I first find the Content of the whole Parallelogram, by multiplying the sides together, and the Product is 224.1128 Feet ; and the half of that product is the Content of the Triangle B, C, D, which is 112.0564 Feet. See the following work.

18.28 the side B, C,
12.26 the side C, D.

10968
3656
3656
1828

2) 224.1128 (112.0564 Feet

The Content would have been the same, if I had multiplyed the one side by half the other, which is indeed the shortest way, and most practical, See the work.

the whole side B C, 18.28

$\frac{1}{2}$ the side C D 6.13

5484

1828

10968

Facit 112.0564 Feet

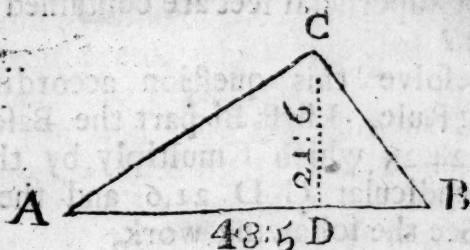
The answer would have been the same, if I had taken the whole side C D, and Multiplied it by half the side B C.

P R O P. III.

To find the Content of any plain Triangle, not Rectangular.

THE best and easiest way is to let fall a perpendicular, upon the longest side from the angle that is opposite to it, which will divide it into two right-angled plain Triangles, as suppose there were given the plain Triangle A B C, as followeth.

Here



Here the side AB , being the longest side, I let fall a perpendicular from its opposite angle at C , which falleth upon D , in the line AB , so is the line CD the nearest distance between the angular point C , and the line AB , and divideth the given Triangle ABC , into two Right-angled Triangles, (*viz.*) ADC and CDB ; and if you find the content of these two Right-angled Triangles (according to the directions in the second proposition) and add them together, their sum will be the content of the given Triangle ABC . But it may be more artificially found out thus,

Multiply half the line CD , into the whole line AB , the Product will give the Content of the Triangle which was sought, or if you multiply the whole line CD , into half the line AB , the product will be the Content of the given Triangle, which is very plain from a due consideration of the method used in solving the second proposition.

Example.

Let the base or longest side AB be 48.5 Feet long, and let the length of the perpendicular CD be 21.6 Feet, I desire to know how many

square, or superficial feet are contained in the said Triangle?

To resolve this question according to the foregoing Rule, I first Bi-part the Base A B 48.5 which is 24.25 which I multiply by the length of the perpendicular C D 21.6 and the Product is 523.8 : See the following work,

| | | |
|-----------------------|---------|--------|
| 24.25 | or thus | 48.5 |
| 21.6 | | 14.8 |
| <hr/> | | <hr/> |
| 14550 | | 3880 |
| 2425 | | 4850 |
| 4850 | | <hr/> |
| | | 523.80 |
| 523.800 <i>facit.</i> | | |

So that you see the content is the same which of the foresaid ways soever you work ; observe the same method in finding the content of any oblique Triangle given.

P R O P. IV.

To find the Content of a Trapezium.

A Trapezium is a plane Figure having four unequal sides, and as many unequal Angles it matters not how unequal they are, and to find out its content observe the following directions, viz.

Divide

Divide it into two oblique Triangles, by drawing a line from any one of the angles, to the angle that is opposite thereto, which line shall be a common base to both the Triangles.

Then if you find out the content of both these Triangles, according to the method prescribed in the third proposition, the sum of their contents, is the content of the given Trapezium.

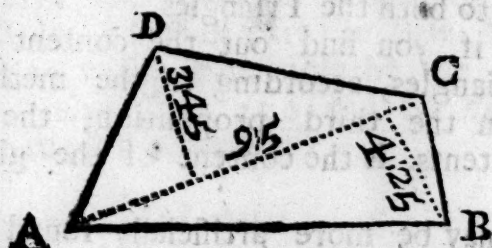
Or it may be more artificially found out thus, viz.

Multiply the length of the common base by half the sum of the perpendiculars let fall from the angles opposite to the said common base, and the product will be the content of the whole Trapezium: or else

Multiply the sum of the said perpendiculars by half the said common base, and it will produce the same effect.

Example.

In the following Trapezium A B C D, draw the base A C, which suppose to be 9.5. Feet, then let fall the perpendicular at D, which let be 3.45 Feet, and that at B 4.25 Feet, the sum of the said perpendiculars is 7.7 half of which is 3.85; by which if the common base A C be multiplied, the product (which is) 36.575 is the content of the Trapezium required. Or if you multiply 7.7 the sum of the perpendiculars by half of the common base 9.5, which is 4.75 the product will be the same. See the following work.



3.45 the perpendicular at D
4.25 the perpendicular at B

7.70 their Sum

3.85 their half Sum
9.5 A C

1925
3465

facit 36.575

4.75 half A C
7.7

3325
3325

facit 36.575

P R O P.

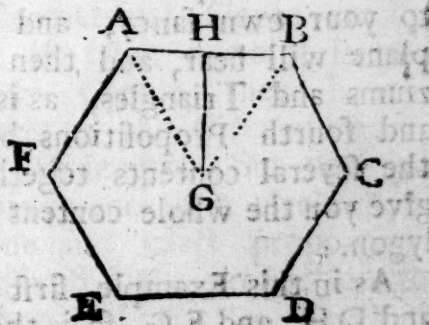
PROP. V.

To find the Content of any regular Polygon.

A Regular Polygon is a plane Figure consisting of equal sides and equal Angles, viz. a Pentagon, consisting of 5 equal sides, and 5 equal Angles; an Hexagon, consisting of 6 equal sides, and 6 equal Angles; an Heptagon of seven equal sides, an Octogon of eight, &c. and to measure any one of these regular planes, do thus draw a line from the Centre of the Figure to the middle of any one of the sides, and multiply that line into half the sum of the sides, and the Product thence arising is the content of the given plane.

I shall give you an Example of this in the Mensuration of an Hexagon, or plane of 6 equal sides.

Let there be given the Hexagon A B C D E F, having the length of each side 30 then will the length of the perpendicular G H be 26 fere, now there being in all 6 sides, and each of them in length 30, the sum of them all is 180 the half of which is 90, which being mul-



E 4

tplied

tiplied by 26, the product will be 2340, which is the content of the given Hexagon.

The reason of this manner of working is very plain, if from the Centre you draw the lines GA , and GB , thereby making the Triangle GAB , whose content (by the third proposition) is found by multiplying the perpendicular GH into half the side AB , viz. into HA , or HB , but there are 6 such Triangles in the given Polygon; therefore GH , multiplied into 6 times HB , produceth the content required.

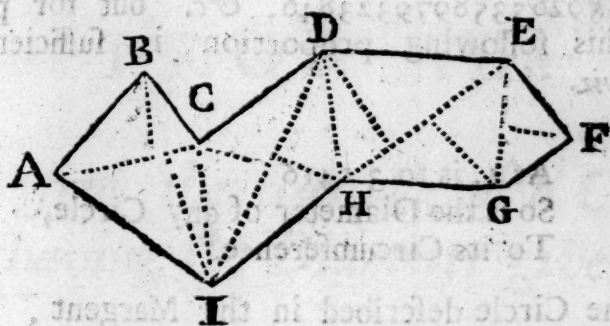
PROP. VI.

To find the Content of any Irregular Polygon.

LET it be required to measure the following Figure $ABCDEFGHI$.

First take care that the whole plane be divided in Trapeziums and Triangles according to your own fancy, and as the nature of the plane will bear, and then measure those Trapeziums and Triangles, as is directed in the third and fourth Propositions before going, and add the several contents together, so will the sum give you the whole content of that Irregular Polygon.

As in this Example, first I draw the lines AC , and DH , and EG , so is the whole figure divided into



into the Trapeziums ABCI, and CIHD, and DHGE, and the Triangle EGF, the contents of which being severally found out by the third and fourth Proposition, the sum of them will be the content of the whole Figure.

PROP. VII.

To find the circumference of a Circle having the Diameter given.

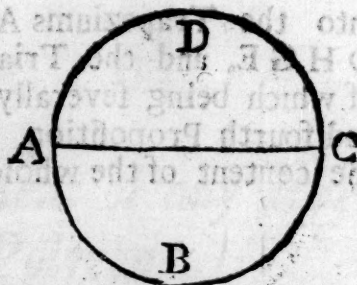
A Circle is a Geometrical Figure exactly round, so that if from a point in the middle of it called the Centre, there be never so many lines drawn to the Circumference, they will all be of equal length. But between the diameter and circumference of a circle there cannot be found a true and exact proportion. *Archimedes* hath demonstrated the proportion to be near as 7 is to 22 ; but that of *Van Ceulen* is the

the most exact, who makes it to be as 1, is to 3. 14159265358979323846, &c. but for practice this following proportion is sufficiently exact, viz.

As 1. is to 3.1416
So is the Diameter of any Circle,
To its Circumference.

In the Circle described in the Margent, the Diameter A C is 28, I demand what is the circumference A B C D?

To answer which I say by the proportion foregoing; As 1 is to 3.1416, so is 28 the diameter to 87.9648, which is the circumference A B C D. The work followeth.



As 1 to 3.1416, so is 28 to 87.9648

28

251328

62832

87.9648

PROP.

P R O P. VIII.

*To find the content of a Circle
having the Diameter given.*

First find out the circumference, by the last Proposition, then multiply half the circumference by half the Diameter, and that product is the Content.

Example.

There is a Circle whose Diameter is 14 inches, I demand how many square Inches are the content of that Circle ?

By the foregoing Proposition, I find the circumference to be 43.9824 Inches, the half of which is 21.9912 which being multiplied by 7, (half the given Diameter) the product is 153.9384 which is the content required. See the work.

21.9912

7

facit 153.9384 square Inches.

P R O P.

PROP. IX.

To find the solid content of a square piece of Timbtr, Stone, &c. Whose bases are equal, that is, whose ends are of the same bigness.

Such a solid piece by Geometricians is called a Parallepipedon, and its content is thus found out, *viz.*

First find out the superficial content of the base or end, (by the first proposition) then multiply that content by the whole length, and that Product is the solid content of the whole piece.

Example.

There is a square piece of Timber, the two contiguous sides at the end of which are 2.5 Feet, and 1.8 Feet, and its length is 22 Feet, I demand how many solid Feet are in that piece of Timber.

First I multiply 2.5 by 1.8 the sides of the base, and that produceth 4.5 for the content of the base or end, and that Product I multiply by 22 the length, and that produceth 99 Feet, and so many is there contained in that piece of Timber. As you may see in the following work.

$\begin{array}{r} 2.5 \\ 1.8 \end{array} \}$ the 2 sides at the end

—

200

25

—

4.50 the superficial content of the end

22

—

900

900

—

facit 99.00 Feet for the content Required.

Here note, if the sides of the end, or base, be given in Inches, and its length in Feet, then Reduce the sides of the base into the Decimal parts of a Foot, and proceed as before, or you may find out the content of the base in Inches, and multiply that content by the length in Feet, and that product divided by 144, will give you the content in Feet, or else reduce the length into Inches, and multiply the content of the base thereby, and divide that product by 1728 (for there are so many Cubical Inches in a Foot) and the Quotient will give you the solid content in Feet. But the Decimal way is preferred.

PROP.

PROP. X.

To find the Solid content of a Cylinder, having the Diameter of its Base given.

A Cylinder is a solid whose bases are Circular, equal, and parallel, and may fitly be represented by a round pillar, or a Rolling-stone of a Garden, and to find the solid content of such a body this is

The Rule.

First find the plane of the base, by the 7 and 8 propositions foregoing, and then multiply that by the length thereof, which product will give you the solid content of the given Cylinder.

Example.

There is a Cylinder, (suppose a Rolling-stone) whose length is 8.75 Feet, and the Diameter of its base 2.8 Feet, I demand the solid content thereof?

As 1

Is to 3.1416

So is 2.8 the given Diameter.

To 8.79648 the Circumference of the base, half of which (*viz.* 4.39824) being multiplied by 1.4 the semi-diameter will produce 6.157536 for the content of the Base, which being

being multiplyed by 8.75 (the length) it produceth 53.87844 for the solid content required.

If there had been given the circumference of the Cylinder, then the Diameter of the base must have been found out by the converse of the seventh Proposition, as suppose there had been given 8.75 the length of the Cylinder, and 8.79648 its circumference to find the solidity thereof. First I find out the Diameter by the following proportion, viz.

As 3.1416

Is to 1

So is 8.79648 the given Circumference
To 2.8 the required Diameter.

And then the rest of the work is the same with that before.

PROP. XI.

To find the Solid Content of a Cone.

A Cone is a Solid Body, having a Circle for its base, and its superficies Circular, decreasing its equidistant Diameters from the base proportionably, till it remaineth in a point over the Centre of its base, and may fitly be represented by a Sugar-loaf, or a round Spire Steeple; and to find its solid Content this is

The Rule.

By the 7 and 8 Propositions foregoing find out the plane of its base, and multiply that by $\frac{1}{3}$ of its

its height, and that product is the Solid content of the Cone Required.

Example.

There is a Cone the circumference of whose base is 22.5 and its height is 16, I demand the solid content of such a Cone ?

As 3.1416

Is to 1.

So is 22.5 the circumference of the base

To 7.162 the diameter of the base.

Then I multiply half 22.5 which is 11.25 by half 7.162 which is 3.581, and it produceth 40.28625 which is the superficial content of the base ; then I take $\frac{1}{3}$ of the height of the Cone (16) which is 5.333 very near, by which I multiply 40.28625 (the superficial content of the base) and it produceth 214.84657125. See the work as followeth ;

40.28625

5.333

12085875

12085875

12085875

20143125

Solid Content is 214.84657125

The same is to be observed in the mensuration of any other Cone.

But here you are to observe that the slanting side of the Cone, (*viz.*) the length from the vertex

vertex to the extremity of its Base) is not to be taken for its true height, but a perpendicular let fall from its vertex or centre of its Base is its true height; and how you may find out that perpendiculars length shall be shown you in the work of the fourth Proposition of the eleventh Chapter.

PROP. XII.

To find the solid Content of a Pyramid.

Between the Cone and Pyramid, this is the Difference, as the Cone hath a circular base and superficies, the Pyramid hath a Polygon for its base, so that its base and superficies are Angular, its vertex terminating in a point just over the Centre of its base, and to find out its solidity, here followeth

The Rule.

Find out the superficial content of the base, by the fifth Proposition foregoing, and multiply that by $\frac{1}{3}$ of its height, and it produceth the solid content of the Pyramid.

Example.

There is a Pyramid whose base is an Hexagon the side of which is 30, and its perpendicular height

height is 54 ; I demand the solid content of such a Pyramid ?

Here by the fifth Proposition I find the superficial content of the base to be 2340 ; then do I take $\frac{1}{3}$ of the perpendicular height of the Pyramid, which is 18, and thereby do I multiply 2340, (the plain of the base) and the product is 42120, which is the solid content of the given Pyramid.

Here note by the way, that a line drawn from the point at the top of the Pyramid, to the extremity of any part of the basis, is not the true height of any Pyramid, but a perpendicular let fall from the Cuspis (or top) to the centre of the base is the true height, and how to find out such perpendicular heights shall be shewn in the fourth Proposition of the 11 Chapter:

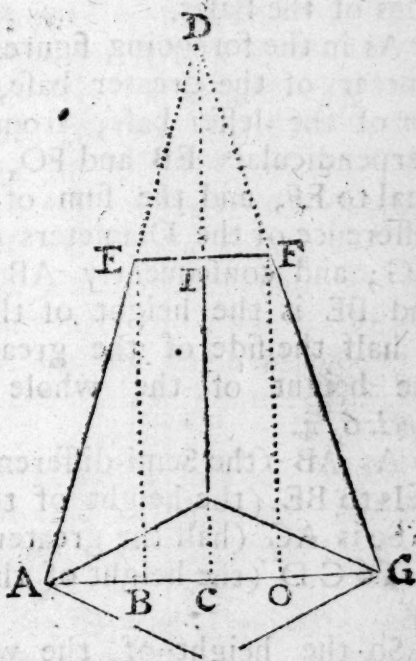
P R O R.

PROP. XIII.

*To measure the Frustum of a
Pyramid or Cone.*

THE Frustum
here given
to be measured is
AGEF, the side
of the greater base
at A being 24
Inches, and the
side of the lesser
base at E being 8
Inches, and the
length of it I, C,
20 foot equal to
EB, or FO.

It is evident that
if I find the solidi-
ty of the whole
Pyramid AGD, &
also the solidity of
the lesser Pyramid



EFD, and then subtract the content of EFD,
from the content of AGD, that there will re-
main the solidity of the Frustum AGEF; and
certainly this way of measuring the Frustum of a
Pyramid or Cone, is the most exact of any: And
it may be easily measured thus, first of all find

out the height of the whole Pyramid CD, which you may do by the following proportion, *viz.*

As the Semi-difference of the sides of the Bases,
Is to the height of the Frustrum,
So is the half side of the greater Base,
To the height of the whole Pyramid.

And this proportion will hold good if you work by the Semi-difference of the Diameters of the bases, as well as by the Semi-difference of the sides of the Bases.

As in the foregoing figure, let AG be the Diameter of the greater base, and EF the Diameter of the lesser base, from E and F let fall the perpendiculars EB and FO, then shall BO, be equal to EF, and the sum of AB and OG are the difference of the Diameters of the bases, EF and AG; and consequently AB is the Semi-difference, and BE is the height of the Frustrum, and AC is half the side of the greater base, and CD is the height of the whole Pyramid. Then by *Encl. 6. 4.*

As AB (the Semi-difference of Diameters)
Is to BE (the height of the Frustrum)
So is AC (half the greater Diameter)
To CD (the height of the whole Pyramid)

So the height of the whole Pyramid AGD, will be found to be 30 foot; for the greater Diameter A G, is 24 Inches, the lesser 8, the difference 16, the Semi-difference 8, therefore shall CD be 30 foot; for

$$8 : 20 :: 12 : 30$$

Now

Now having found the height of the whole Pyramid to be 30 foot, I thereby (according to the 12th. Proposition foregoing) find the content of the whole Pyramid to be 40 foot, then in the lesser Pyramid EFD there is given the side of its base EF, 8 Inches, and its height ID 10 Inches for CD 30—CI 20—ID 10, and by the said 12th. proposition I find the solid content of it to be 1.48 Feet, which being Subtracted from 40 (the content of the greater Pyramid) there will remain 38.52 feet for the true solid content of the given Frustum AGEF.

After the same manner is found the solidity of the Frustum of a Cone, the height of the whole Cone being found out by the difference of the Diameters of its bases; and by the 11th. proposition find the solidity of the whole Cone, and also the solidity of the lesser Cone, that is cut off from the Frustum, then Subtract the content of the lesser from the content of the greater, and the remainder will be the solid content of the Frustum.

This last proposition is useful in the measuring of tapering Timber Round or Squared, and for finding the liquid capacity of Brewers Conical, or Pyramidal Tuns.

Thus have I shewed the Use of Decimals in the Mensuration of the most useful Planes and Solids, I might proceed farther to shew their Application in the particular Mensuration of Board, Glass, Pavement, Plaistering, Painting, Wainscot, Tiling, Flooring, Tapistry, Brickwork, Timber and Stone; but it requireth (rather) a particular Treatise, than the narrow bounds here allowed for such a work.

CHAP. IX.

The Extraction of the Square Root.

IN the Solution of any Question, or in the working of any sum whatsoever belonging to any of the Rules of Vulgar or Decimal Arithmetick, there have been (at least) two things or numbers given, whereby the answer might be found; but in the extraction of the Square, Cube, and all other Roots, there is but one number given to find out the number sought, *viz.* there is a square Number given to find its Root, a Cube Number to find its Root, &c. And

I. A square Number is that which is produced by multiplying any number by (or into) it self, which Number given to be so multiplyed is called the Root; as if the Number 8 were given to be multiplyed by it self, it produceth 64, then is 8 called the Root, and 64 is its square, so the Root 12, hath for its square 144.

II. When a square Number is given, and its Root is required, the Operation it self is called the Extraction of the Square Root.

III. Square Numbers are of two kinds, *viz.* either Single or Compound.

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IV. A single square Number is that which is produced by the Multiplication of a Digit, or single Number into it self, and consequently such a square Number must be under 100, which is the square of 10, so 25 being given for a square number, it is a single square having for its Root 5. And 81 is a single square Number, having for its Root the Digit 9. All the single square numbers with their Roots, are contained in the following Tablet.

| Roots | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------|---|---|---|----|----|----|----|----|----|
| Squares | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |

V. When the Root of any square Number is required it being lesser than 100, and yet is not exactly a single square, expressed in the Tablet above, then you are to take the Root of that single square Number expressed in the said Tablet, which (being less) is nearest to the given square; as if it were 74 whose Root is required. I find that 81 (the square of 9) is too much, and 64 (the square of 8) is too little, but yet it is the nearest square number that is lesser than 74, and therefore I take 8 to be the square Root of 74, but yet it is plain that 8 is too little for the Root of 74. And to find out the Fractional part of this Root, you shall be plainly taught by and by.

VI. A compound square number is that which hath above 9 for its Root.

VII. The Root of a single square Number may be discovered at the first sight, but the Extradiction of a compound square Number is more tedious and difficult, its Root consisting of two places, at the least, and the square it self, of 100 at the least.

VIII. When a compound square number is given, and it is required to have its square Root extracted, before you can proceed to the Operation, your square number must be prepared, by pointing it at every second figure, beginning at the place of Units.

As, suppose you were to extract the square Root of 2304, first I put a point over 4 (it standing in the place of Units) and then passing over the second place (or place of Tens) which is 0, I put a point over the figure standing in the third place, (or place of Hundreds) which is 3, and 2304. the preparative work is done, as you may see in the Margent. Now if there had been more places in the given number, then I must have put a point over the figure standing in the fifth place, and another over that in the seventh &c. And here note, that as many points as you put over the given square number, so many Figures there will be in the Root, that is, the Root will consist of so many places.

So if there were given the number 33016516, to have its square Root extracted, after I

33016516

have

have pointed it according to the Direction before given, it will stand as in the Margent, and because the points that are put over it are in number 4, I conclude the Root it self will consist of 4 places, or figures.

IX. When you have thus prepared your number, then draw a crooked-line on the Right-hand of your number, behind which to place your Root, as you do for a Quotient in Division.

Note that when your number is prepared for Operation, as in the 8 Rule, the numbers containing between point and point, may not unfitly be termed Squares, and in the ensuing work, we shall so call them, as in the foresaid number 33016516, being pointed as before, I call 33 the first square, 01, the second, 65, the third, and 16 the fourth, and last square; every square (except sometimes the first consisting of two figures, or places, the last of which towards the Right-hand hath always a point over it, and if it so happen (as it often doth) that the last Figure (in any given square number) toward the left hand hath a point over it then that number alone shall be accounted the first square.

As if the number 676 were given, when it is pointed for the work according to Direction, as you see in the Margent I account 6 for the first square and 76 for the second.

These things being understood, we shall lay down those general Rules requisite for the management of the work it self.

X. When your number is prepared, find out the

the square Root of the first square, according to the 5 Rule foregoing, and place that Root behind the said crooked line. As

Let it be required to extract the square Root of the said number 2304, herethe first square number is 23, and (according to the said 5 Rule) its Root is 4, which I place behind the crooked line as you see in the Margent.

XI. Then square the said Root, and place its square which is 16 under the said first square 23, and having drawn a line underneath, subtract the said square 16, from 23, and place the Remainder, which is 7 underneath the said line as you may percieve by the work in the Margent.

$$\begin{array}{r} 2304 \quad (4 \\ 16 \\ \hline 7 \end{array}$$

XII. Then to the said Remainder bring down the Figures of the next square and annex them thereto on the Right-hand, so that they may make one intire number, which (for distinctions sake) we shall call the Resolvend.

As in this Example to the Remainder 7, I bring down the next square 04, and annex it thereto, and it maketh 704 for a Resolvend, as you may see in the Margent.

$$\begin{array}{r} 2304 \quad (4 \\ 16 \\ \hline 704 \text{ Resolv.} \end{array}$$

XIII. Always let the whole Resolvend (except the last Figure on the right hand) be esteemed a Dividend, on the left hand of which draw a crooked line before which to place a Divisor, as in Division.

So in this example, the Resol-
vend 704 is to be made a Divi-
dend, all but the last place which
is 4, so that the Dividend is 70,
before which I draw a crooked
line, as you see in the Margent.

$$\begin{array}{r} 2304(4 \\ 16 \end{array}$$

$$\begin{array}{r}) 704 \end{array}$$

XIV. Let the Quotient expressing the Root (or part of the Root sought) be doubled, or multiplied by 2, and that double or product shall be a Divisor, and must be placed on the left hand of the Resolvend, before the said crooked line.

So in our Example, the number 4 which was put for part of the Root being doubled makes 8, which I put before the Resolvend for a Divisor, as it appears in the Margent.

$$\begin{array}{r} 2304(4 \\ 16 \end{array}$$

$$\begin{array}{r} 8) 704 \end{array}$$

XV. Then (according to the Rule of Division in whole numbers) seek how often the said Divisor is contained in the said dividend, and put the answer down in the Quotient, and also on the Right hand of the Divisor.

As in our Example I seek how often the Divisor 8 is contained in the Dividend 70, which I find to be 8 times, therefore I put 8 in the Quotient for part of the Root, and also on the Right hand of the Divisor. See the work in the Margent.

$$\begin{array}{r} 2304(48 \\ 16 \end{array}$$

$$\begin{array}{r} 88) 704 \end{array}$$

XXVI. Then

XVI. Then by the Figure last put for part of the Root, multiply the said Divisor together with the Figure that you annexed to it (accounting them both as one intire number) and place the product underneath the said Resolvend, drawing a line under it, and then subtract it out of the said Resolvend, placing the Remainder beneath the line.

As in our Example, having placed 8 in the Quotient, and also on the Right-hand of the Divisor, then in the place of the Divisor, their stands 88, which I multiply by 8, the number last put in the Quotient, and the product is 704, which I place in order under the Resolvend 704, and having drawn a line underneath, I subtract the said product 704, from the Resolvend 704, and there remaineth 0, so is the work finished, and I find the square Root of 2304 to be 48. See the work in the Margent.

$$\begin{array}{r}
 2304 \text{ (48)} \\
 16 \\
 \hline
 88) 704 \\
 704 \\
 \hline
 (0)
 \end{array}$$

Here note that if at any time when you have multiplyed the number standing in the place of the Divisor, by the Figure last placed in the Quotient, or Root (as is directed in the last Rule) if the product be greater than the Resolvend, then conclude the work to be erroneous, to correct which put a lesser Figure in the Root, and proceed as is before directed.

Note also that the work of the 12, 13, 14, 15, and 16 Rules, must be repeated as often as there

there are points over the Figures, except for the first square, which is to be wrought according to the Directions given in the 10 and 11 Rules foregoing, and the work of those two Rules is to be observed, but once in the extraction of a square Root, tho' it consist of never so many squares or points.

These things will appear plain and easie in the working of one or two more Examples.

Example 2.

Let it be Required to extract the square Root of 33016516.

Here In order to the work. I first prepare my number by distinguishing it into squares, by pointing it according to the 8 Rule foregoing and thereby I find that 33, is the first square, and (according to the 10 Rule) I take the square Root of 33, which is 5, and place for the first Figure of the Root, then (according to the eleventh Rule) I square the Root (5) and it makes 25, which I place under the said first square number 33, and subtract it therefrom, and the remainder (8) I place below the line, as in the following work.

$$\begin{array}{r}
 \text{.} \\
 33016516 \quad (5 \\
 25 \\
 \hline
 8
 \end{array}$$

Then (according to the twelfth Rule) I annex to the said remainder (8) the next square (01) and

and it makes 801 for a Resolvend, then must 80 (according to the thirteenth Rule) be my dividend, and (according to the fourteenth Rule) I double the number (5) in the Root, and it makes 10 for a Divisor, and thereby I divide the said dividend (80) and I find that it Quotes 7, which (according to the fifteenth Rule) I put in the place of the Root after 5, and likewise before the Divisor, (10) so that in the place of the Divisor instead of 10, there is now 107.

Then (according to the sixteenth Rule) I multiply the said 107, by 7, (the figure last placed in the Root) and the product is 749, which I place orderly under the said Resolvend, and subtract it therefrom, and the remainder is 52 which I put below the line, as in the following work.

$$\begin{array}{r}
 33016516 \text{ (57)} \\
 25 \\
 \hline
 107) 801 \text{ Resolvend.} \\
 749 \\
 \hline
 52
 \end{array}$$

Then I repeat the same work over again, in finding the next Figure of the Root, as I did in finding the last, viz. to the remainder (52) (according to the twelfth Rule) I bring down and thereto annex the next (third) square (65) and it makes 5265 for a new resolvend, then (according to the thirteenth Rule) is 526 a new dividend, and (according to the fourteenth Rule) I take the Root (57) and double it for a new Divisor

visor and it makes 114, which place before the resolvend (5265.)

Then (according to the fifteenth Rule) I seek how often the divisor (114) is contained in the dividend, (526) and I find it will bear 4, which I place in the Root orderly, and also on the right hand of the Divisor, (114) and then there will be in the place of the Divisor, the number 1144, which (according to the sixteenth Rule) I multiply by the Figure (4) last put in the Root, and the product is 4576, which I place orderly under the resolvend (5265) and subtract it therefrom, and the remainder is 689 which I place under the line, as is before directed. See the whole work as followeth.

$$\begin{array}{r} \cdot \cdot \cdot \cdot \\ 33016516 \quad (574 \\ 25 \end{array}$$

$$\begin{array}{r} 107) \quad 801 \text{ Resolvend} \\ \quad 749 \text{ Product} \end{array}$$

$$\begin{array}{r} 1144) \quad 5265 \text{ Resolvend} \\ \quad 4576 \text{ Product} \end{array}$$

$$689$$

Then I again repeat the work of the 12, 13, 14, 15, and 16 Rules of this Chapter for finding the next Figure of the Root, viz. first I bring down (16) the next square number, and annex it to the remainder 689 (according to the twelfth Rule) and it makes 68916, for a new resolvend, of which (by the thirteenth Rule) 6891 is a new Dividend then (according to the fourteenth Rule)

Rule) I double the Root, and it makes 1148 for a divisor, which I place on the left side the resolvend, and then seek how often it is contained in the said dividend (6891,) and the answer is 6, which I place for part of the Root in order, and also on the right hand of the said Divisor, so that in the place of the Divisor 1148, will then stand the number 11486, which by the sixteenth Rule, I multiply by 6, (the Figure last placed in the Root) and the product is 11486, which I place in order under the resolvend, and subtract it therefrom, and the remainder is 0, and so the work is finished, whereby I find the square Root of 33016516, to be 5746, as by the whole operation appeareth.

$$\begin{array}{r} 33016516 \quad (5746 \\ 25 \end{array}$$

$$\begin{array}{r} 107) 801 \text{ Resolvend} \\ 749 \text{ Product} \end{array}$$

$$\begin{array}{r} 1144) 5265 \text{ Resolvend} \\ 4576 \text{ Product} \end{array}$$

$$\begin{array}{r} 11486) 68916 \text{ Resolvend} \\ 68916 \text{ Product} \end{array}$$

(0)

And if the Root had consisted of never so many places, yet for every Figure put therein (except the first, for which you are to observe the tenth and eleventh Rule) the work of the 12, 13, 14, 15, and 16 Rules must be repeated according

According to the second note after the sixteenth Rule foregoing.

Example 3.

A third Example may be this, let it be required to extract the square Root of 8328996.

In the working of this Example you will see the use of the first note upon the sixteenth Rule, for only the number 8 is the first square, as you may see by the pointing of the given number, and after the whole work of Extraction is finished, you will find the square Root of the given number, to be 2886, as in the following operation.

8328996 (2886 Root

4

48) 432 Resolvend
384 Product subtract

568) 4889 Resolvend
4544 Product subtract

5766) 34596 Resolvend
34596 Product subtract.

(0)

XVII. When there is given a number that is not a square number, that is, whose root cannot be exactly found, and you are desirous to find the Fractional part of the root as near as may be,

G

you

you are to observe the eighth rule in preparing your number for extraction, and then to annex thereto an even number of Cyphers at pleasure, and note, that as many pairs of Cyphers as you annex thereto, so many decimals will there be in the root expressed, (which though it come not to be the exact root, yet will it come so near the truth, that if the last Decimal Figure placed in the root, be increased by an unite, it will be too much) and as many points as there are over the given Integral square number, so many places will there always be in the Integral part of the Root, as in the following Example, where it is required to extract the square root of 129596.

First I proceed to the work of extraction according to the former rules as if it were an exact square Number, and find the integral root to be 359, as followeth.

$$\begin{array}{r} 129596 \quad (359 \\ 9 \end{array}$$

$$\begin{array}{r} 65) 395 \\ 325 \end{array}$$

$$\begin{array}{r} 709) 7096 \\ 6381 \end{array}$$

715 Remainder

But because (when the work is finished) there is a Remainder of 715, I annex a competent even number of Cyphers, to the given number, as of 4, or 6, or 8, and point them out in the same manner

manner as if there were significant figures in an Integer, then bring two of them down to the said remainder (715) and annex them thereto, so have you 71500 for a new resolvend ; Then find out a new divisor by doubling the root, as is before directed, and proceed as if the annexed Cyphers were significant figures, or whole Numbers, as far as you please, as in this example, where the work is carried on till there are 3 decimal figures in the Root ; and the work being finished, I find the root to be 359.994, and there is a remainder of 319964. See the work.

129596.000000 (359.694

9

65) 395

325

709) 7096

6381

7189) 71500

64701

71989) 679900

647901

719984) 3199900

2879936

319964 remains.

G 2

But

But if you proceed to put another decimal in the root you will find it to be 359.9944, and the remainder will be 3196864. Now you may perceive that the said root, is too little, because there is a remainder, but yet it is so near the truth that if the last figure thereof were increased by an unite, and so made 359.9945 it would then be too much, as you may prove at your leisure.

XVIII. The Square root of a vulgar Fraction that is commensurable to its root, is thus found, viz. extract the square root of the Numerator, for a new Numerator, and likewise the square root of the Denominator, for a new Denominator; so shall that new Fraction be the square root of the given Fraction; as for

Example.

Let it be required to extract the square root of $\frac{5}{36}$ first I take the square root of 25, which is 5, and place it for a new Numerator, then I take the square root of the Denominator 36, which is 6, and place it for a new Denominator, so is $\frac{5}{36}$ the square root of $\frac{25}{36}$, which was required; in like manner if $\frac{4}{9}$ were given to have its square root extracted, its root would be found to be $\frac{2}{3}$ and $\frac{1}{4}$ is the square root of $\frac{1}{16}$, the like is to be observed for any other.

Note. But here note diligently; before you proceed to extract the Square root of any Fraction, that you reduce it to its lowest Terms, for it may happen that in its given Terms, it may be incommensurable to its root, but being reduced to its lowest Terms it may be

be commensurable, and its root exactly found out, so $\frac{1}{2}$ is incommensurable to its root, but being reduced to $\frac{3}{6}$, its square root will be found to be $\frac{1}{2}$ as before.

XIX. The square root of a mixt number that is commensurable to its root is thus found out, viz. reduce the mixt number to an improper Fraction, and then extract the square root of the Numerator, and the square root of the Denominator, for a new Numerator, and a new denominator, as in the last Rule.

So if it were required to extract the square root of $1\frac{1}{2}$, first I reduce it to an improper Fraction, and so it is $\frac{3}{2}$ whose Square root is $\frac{\sqrt{3}}{\sqrt{2}} = 1\frac{1}{2}$, so if it were required to extract the square root of $3\frac{1}{2}$, first I reduce the given mixt number, to the improper Fraction $\frac{7}{2}$, and then extract the square root of the Numerator 49, and it I find to be 7, for a new Numerator, and likewise the square root of 2, the denominator, which I find to be $\sqrt{2}$, for a new denominator, so is $\frac{7}{\sqrt{2}} = 1\frac{1}{2}$ the square root of the given mixt number $3\frac{1}{2}$ which was required.

XX. When you are to extract the square root, of a Fraction that is incommensurable to its root, prefix before the given Fraction, this Character $\sqrt{}$, or \sqrt{q} . signifying the square root of that before which it is prefixed, so the square root of $\frac{2}{3}$ is thus expressed, $\sqrt{\frac{2}{3}}$ or $\sqrt{q. \frac{2}{3}}$, the like of any other. But if you would know as near as may be the square root of any such Fraction, reduce it to a decimal of the same value by the first Rule of the second Chapter, but let the decimal consist of an even number of places, viz. either of two, four, six, or eight, &c. places;

and the more places it consisteth of, so much the nearer the truth will the root be; Then extract the square root of that decimal (according to the Rules before delivered,) in every respect as if it were a whole number, so shall this root so found be very near the true root; and so near that if it consist of 3 places it shall not want $\frac{1}{1000}$ part of an unit of the true root, and if of 4 places, it shall not want $\frac{1}{10000}$ part of an unite of the truth.

So if I would extract the square root of $\frac{3}{4}$, first I reduce it to a decimal, which I find to be .75 and because I would have the root to consist of 4 places, I annex 6 Cyphers thereto and it makes .75000000, then extracting the square root thereof as if it were a whole number, I find it to be .8660, and there is a remainder of 4400, but if I would have it consist of 5 places, then I annex 2 more Cyphers to the said remainder, and make it 440000, and proceed, and then I find the root to be .86602, and the remainder to be 93596.

XXI. In like manner if it were required to extract the square root of a mixt number incommensurable to its root, as near as may be, first reduce the Fractional part to a decimal, but let it consist of an even number of places, viz. of 2, 4, 6, or 8, &c. places, then proceed to extract its square root, according to the Rules formerly delivered in this Chapter, in every respect, as if it were a whole number, so shall the root so found, be very near the truth, and the more places it consisteth of, so much the nearer will it be to the true root. And note that in the root there will be so many decimal places, as you placed

placed points over the decimal part of the square number.

So if it were required to extract the square root of $28\frac{1}{3}$ first I reduce the fractional part $\frac{1}{3}$ to a decimal and it makes .461538, so then the mixt number whose square root I am to extract is 28.461538 , which being pointed, and the work of extraction finished, according to the former Rules, I find its square root to be very near 5.334 and there is a remainder of 9982, But if I had proceeded yet farther, and made the decimal part to have consisted of 7 places, it would have had for its square root 5.3349, which doth not want $\frac{1}{10000}$ part of an unite of the true root.

But if you would not extract the square root of such a mixt number, then prefix before it this character, $\sqrt{\text{or } \sqrt{q.}}$ so if the said mixt number $28\frac{1}{3}$ were given I would express its square root thus, viz. $\sqrt{18\frac{1}{3}}$ or $\sqrt{q. 28\frac{1}{3}}$ the like is to be understood of any other.

XXII. When you are to extract the square root of a decimal Fraction, which hath 2 or 3 Cyphers possessing the two or three first places on the left hand of the given decimal, then cut off 2 of them with a dash of the Pen, and put a Cypher to possess the first place of the Root, and proceed to extract the square root of the remaining Figures, according to the former Rules as if there had been no such Cyphers before the given decimal; and if the given decimal have 4 Cyphers before it, cut them off with a dash of the Pen, and put 2 Cyphers in the root, and then proceed as before.

So if it were required to extract the square root of $\frac{5}{846}$, first I reduce to it to a decimal Fraction and it makes .005910, then I cut off the two first Cyphers, and place one Cypher in the root, then I proceed to extract the square root of the remaining Figures, viz. 5910, as if there had been no such Cyphers before them, and I find the root to be very near .077 as you may try at your leisure.

XXIII. The operation in the extraction of the square root is thus proved, viz. Multiply the root into it self, and (if there be no remainder after the work of extraction is finished) the product (if the work be truly done) will be equal to the number first given.

The proof of the Extraction of the Square Root.

As in the first Example, where it is required to extract the square root of 2304, which is there found to be 48. Now if I multiply 48 by it self, it produceth 2304, which is the given number, and therefore I conclude the operation to be true. But if after the work of extraction is finished, there is any remainder then, when you have multiplied the root by it self, to the product add the said remainder, and if the sum be equal to the given number, the operation is right, otherwise not. As in the Example of the seventeenth Rule, where it is required to extract the square root of 129596; and is there found to be 359.994, and the remainder is 319964. Now to prove the work, I multiply the root (359.994) by it self, and it produceth 129595.680036, which should be 129596, therefore to the said product I add the said remainder (319964,) and the sum is

129596

129596, and therefore I conclude the work to be truly wrought.

C H A P. X.

The Extraction of the Cube Root.

I. **A** Cube Number is that which is produced by multiplying any Number into it self and again into that product, which said given number is called the Cube Root.

As, suppose 5 were given to find its Cube, first I multiply 5 into its self, and it produceth 25, which is called the Square of 5, then I again multiply 25, (the said Square) by 5, and it produceth 125, which is called the Cube of 5, and here note that as 125, is called the Cube of 5, so is 5 called the Cube Root of 125.

II. The extraction of the Cube Root is nothing else then when by having a Cube Number given we find out its Cube root; which said Cube number is given always supposed to be a certain number of little Cubes, comprehended within one intire great Cube, which said Cube may very well be represented by a dyc, or any other

other solid body, having its length, breadth and depth equal; this being supposed, let there be laid 9 Dyes constituting a square, whose side shall be 3, and upon them let there be laid 9 more, Dyes, and upon them let there be laid 9 more, then will there be in all 27 Dyes, which will constitute one greater Cube, whose length, breadth and depth will be 3 Dyes, and this greater Cube comprehendeth 27 lesser Cubes. Now the extraction of the Cube root is by having the number of little Cubes (27) comprehended in the greater given Cube to find out how many of the lesser Cubes make up the side of the greater.

III. A Cube number is either Simple or Compound.

IV. A Simple Cube number is that which hath for its root or side, one of the 9 Digits, and it is therefore always lesser than 1000; so shall you find that 343 is a Simple Cube Number, whose side or root is 7, for $7 \times 7 \times 7 = 343$, all which said Simple Cubes, and Squares, as also their roots are expressed in the Tablet following.

| Roots | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------|---|---|----|----|-----|-----|-----|-----|-----|
| Squares | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |
| Cubes | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 |

V. A Compound Cube number is that which is produced by the multiplication of a Number consisting of two places (at the least 3 times into it

it self continually, and is therefore never less than 1000, so 1728 is a compound Cube number, produced by the multiplication of 12 into its self 3 times, for $12 \times 12 \times 12 = 1728$.

VI. When a compound Cube Number is given to have its Cube root extracted, before you can go about it, you must prepare it for the work by pointing it; which is thus done, *viz.* put a point over the first figure towards the right hand; *viz.* over the place of Units, then (passing the two next places) put a point over the fourth figure, or place of Thousands, and so proceed by putting a point over every third figure, as you did over every second figure in the extraction of the Square root, till you have finished your pointing; That being done, on the right hand of the said Cube number draw a crooked line, behind which to place its Cube root, as you do to place the Quotient in Division, as in the following Example.

Example I.

Let it be required to extract the Cube root of 262144.

In order to prepare this Cube Number, for the Extraction of its Cube root, I first put a point over the first figure (4) towards the right hand, and then overpassing the two next figures (14) I put another point over the fourth figure (2) and then is the given number distributed into several parts not unfitly called Cubes, *viz.* 262 (as far as the first

first point goeth) is the first Cube, and the 144 (from thence to the second point) is the second Cube, and then I draw a crooked line behind it as you see in the Margent.

VII. Having proceeded thus far, find out the Cube Root of the first Cube (262) but because it is not an exact Cube number, take the Cube root of that number in the foregoing Tablet, which being lesser than it is, yet is nearest to it, (which I here find to be 6,) and place it behind the crooked line for the first Figure in the Root, as you see in the following work.

262144 (6

VIII. This being done, Cube the said number which is placed in the root, and subscribe its Cube under the first Cube of the given number. So in this Example 216 being the Cube of 6, I place it under 262 the first Cube of the given number 262144, as followeth

262144 (6
216

IX. Draw a line under the Cube thus subscribed, and subtract it from the first Cube of the given number, placing the remainder orderly underneath the said line. So 216 (the Cube of 6) being subtracted from 262, the remainder is 46, which I place underneath the line as followeth

262144

$$\begin{array}{r} 262144 \text{ (6)} \\ 216 \end{array}$$

46

X. Bring down the next Cube number and annex it to the said remainder on the right hand therefore. So 144 being the next Cube, I bring it down and annex it to the remainder 46, and it makes 46144, which by Artists is usually called the Resolved.

$$\begin{array}{r} 262144 \text{ (6)} \\ 216 \end{array}$$

46144 Resolvend

XI. Draw a line underneath the Resolvend, then Triple the root, that is, multiply it by 3, and place its Triple under the Resolvend in such order, that the place of units in the said triple may stand under the place of tens in the Resolvend. So the triple of 6, is 18, which I place under the Resolvend 10, that 8 (the place of units in the said triple) may stand under 4 in the place of tens of the Resolvend; as you see following.

262144

$$\begin{array}{r} 262144 \end{array} (6$$

$$\begin{array}{r} 216 \\ \hline \end{array}$$

$$\begin{array}{r} 46144 \text{ Resolvend} \\ \hline \end{array}$$

$$\begin{array}{r} 18 \end{array}$$

XII. Square the said root, and then triple the said square of the root, and place the said triple square under the said triple root in such order that the place of unites in the triple square of the root, may stand underneath the place of Tens in the triple root, so in this Example, the square of the root 6, is 36, and the triple thereof is 108, which I place under 18, the triple root so, that 8 the place of unites in the said triple square of the root, may stand under 1, the place of Tens in 18, the said Triple root as followeth

$$\begin{array}{r} 262144 \end{array} (6$$

$$\begin{array}{r} 216 \\ \hline \end{array}$$

$$\begin{array}{r} 46144 \text{ Resolvend} \\ \hline \end{array}$$

$$\begin{array}{r} 18 \\ 108 \end{array}$$

XIII. Draw a line underneath the said triple root, and triple square of the root, as they are placed, and add them together in the same order

as

as they stand, so shall their sum be a Divisor. So in our Example, a line being drawn under 18 and 108, and they added together in the same order as they stand, their sum is 1098 for a Divisor, as in the following work.

$$\begin{array}{r} 262144 \end{array} \quad (6$$

$$216$$

46144 Resolvend

$$18$$

$$108$$

1098 Divisor

XIV. Draw a crooked line on the left hand of the resolvend, before which to place the said Divisor, and let the whole resolvend (except the place of unites therein) be esteemed a Dividend, then seek how often the said Divisor is contained in the dividend, and put the answer for the next Figure in the root. So in our Example seek how often 1098 the divisor is contained in 4614 the dividend, observing here the usual Rules of Division) and the answer I find to be 4 which I place for the next figure in the root, as in the Example.

262144 (64
216

1098) 46144 Resolvend

18 the Triple root.
108 the Triple Square of the root.

1098 Divisor

XV. Draw a line underneath the whole work, and then Cube the Figure last placed in the root, and place its Cube underneath the Resolvend in such sort that the place of units in the one may stand under the place of units in the other; so in our Example 64 being the Cube of 4 (the Figure last placed in the root) I place it under the Resolvend in such manner that the Figure 4 in the place of unites of the Cube 64, may stand under 4, the place of unites in the Resolvend, and then the work will stand as followeth.

262144

262144 (64
216

1098) 46144 Resolvend

18 the Triple Root
108 the Triple Square of the Root.

1098 Divisor

64 the Cube of 4

XVI. Square the figure last placed in the Root; and multiply its square by the triple Root subscribed underneath the Resolvend, (as is directed in the eleventh Rule of this Chapter) and subscribe the product under the Cube last put down, in such order; that the place of Units in the said product, may stand under the place of Tens, in the said Cube. So in our Example, the figure last placed in the Root is 4, which squared is 16, and 16 multiplied by 18 (the triple Root before set down) the product is 288, which I place under 64 (the cube of 4) in such sort that 8 (in the place of Units of the said product) may stand under 6 (the place of Tens) in the said cube of 4; view the work.

262144 (64
216 Cube of 6

1098) 46144 Resolvend

18 Triple Root.
108 Triple square of the Root.

1098 Divisor

64 Cube of 4
288 the square of 4 in the triple Root.

XVII. Multiply the triple square of the Root, (subscribed as is before directed in the twelfth Rule of this Chapter) by the figure last placed in the Root, and place the product under the number last subscribed, (which is the product of the square of the figure last placed in the Root multiplied by the said Triple Root) in such manner that the place of Units of this, may stand under the place of Tens in that; As in this Example, The Triple square of the Root is 108, which multiplied by 4 (the figure last placed in the Root) the product is 432, which I place under 288 (the number last subscribed) in such order that the figure 2 (in the place of Units of the said last product) may stand under 8, which is in the place of Tens, of the said number last subscribed; and then the work will stand as followeth.

262144 (64

216

46144 Resolvend

18

Triple root.

108

Triple square of the root.

1098 Divisor

64

Cube of 4

288

Square of 4 in Triple Root

432

Triple square of the Root in 4

XVIII. Draw another line under the work, and add the 3 numbers together; that were last placed under the Divisor, in the same order as they there stand, and let their sum be called the Subtrahend, which let be subtracted out of the Resolvend, nothing the Remainder; So in this Example I add 64, 288, 432 together in the same order as they stand, and their sum is 46144 the Subtrahend, which I subtract out of 46144 the Resolvend, and there is nothing remaineth, so the whole work is finished, and I find the Cube Root of 262144 to be 64, without any Remainder; See the whole work as followeth.

H 2

262144

$$\begin{array}{r}
 262144 \quad (64 \\
 216 \\
 \hline
 1098) \quad 46144 \quad \text{Resolvend} \\
 \hline
 18 \quad \text{Triple root.} \\
 108 \quad \text{Triple square of the root.} \\
 \hline
 1098 \quad \text{Divisor} \\
 \hline
 64 \quad \text{Cube of } 4 \\
 288 \quad \text{Square of } 4 \text{ in the trip. Root} \\
 432 \quad \text{Triple sq. of the Root in } 4. \\
 \hline
 46144 \quad \text{Subtrahend.} \\
 \hline
 (0)
 \end{array}$$

Now the Learner is to observe three things in general from the Rules before delivered, concerning the extraction of the Cube Root.

Observe 1. That the work contained in the 7. 8 and 9 Rules for finding out the first Figure of the Root, is not again to be repeated, throughout the whole work of Extraction, although the Root consist of never so many places, but the work of all the Rules following is to be repeated as often as a new figure is put in the root.

Observe 2. For every particular Cube in the number given, distinguished by the points, (except the first) there is to be found out a new resolvend, by annexing the next Cube to the remainder

mainder (according to the 10th Rule) and as often as there is a resolvend, so often must there be found a new Divisor (by the 11, 12 and 13 Rules) and as often as there is found a new Divisor so often must there be found a new Subtrahend (according to the 15, 16, 17, and 18 Rule before-going.)

Observe 3. When the Subtrahend chanceth to be greater than the resolvend, then you may conclude there is an error in your work, which must be corrected by putting a lesser figure in the Root.

Example 2.

Let it be required to extract the Cube Root of 48627125.

Having prepared the given number for the work of extraction, according to the 6th. Rule of this Chapter, I find it to be distributed into 3 Cubes, viz. 48, the first, 627, the second, and 125 the third. Then I proceed to the work; and first I find the Cube root of 48, (the first Cube) which is 3, then do I Cube 3, and place its Cube which is 27 under (48) the first cube, and subtract it therefrom and the remainder is 21, according to the 7, 8, and 9 Rules of this Chapter, and then will the work stand as you see in the Margent.

Then, to the said remainder 21, do I bring down, and thereto annex the next cube, which

is 627, and it makes 21627 for a resolvend according to the 10th. Rule foregoing. Then do I find out a Divisor according to the 11, 12 and 13 Rules of this Chapter, and first I triple the Root (3) and it makes 9, which I place under (2) the place of Tens in the Resolvend; 48627125 (3)
 Then do I square the said Root (3) and that makes 9, then do I triple its square (9) and that makes 27, which I place under the said Triple in such order as is directed in the 12th Rule, then drawing 279 Divisor
 a line underneath the work, I add the two said numbers together, (*viz.* the Triple Root, and the Triple Square of the Root.) in such order as they are there placed, and their sum is 279 for a Divisor, as per Margent.

Then according to the 14th Rule I seek how often the said Divisor 279 is contained in 2162 the Dividend; and I find the answer to be 6, which I place for the second figure in the Root; then do I in the next place go about to find out a subtrahend, and in order thereunto first (according to the fifteenth Rule of this Chapter) I cube the Figure (6) last placed in the root, and it maketh 216, which I place under the Resolvend in such order (as is directed in the said fifteenth Rule) that the place of Units of the one may stand under the place of Units of the other; Then (according to the 16th Rule of this Chapter) I square the figure (*viz.* 6) last placed in the

the root which makes 36, and then multiply it by (9) the said triple root, and the product is 324 which I place under (216) the said Cube of 6, in such order as is directed in the said 16th rule. Then do I multiply the said triple square (viz. 27) by the figure (6) last placed in the root, and it produceth 162, which I place under the last product (324) in such manner as is directed in the 17th rule. Then I add these 3 several numbers together in the same order as they stand, and their sum is 9656 for a Subtrahend, which subtracted out of (21627) the Resolvend, the remainder is 1971, as you may see by the following work.

Then do I look how often the said Divisor is contained in the Dividend (1971) and find it to be 2 times contained 636 (103) and proceed- ingly I place 2 in the root, and proceed- ing to the 17, 16, 15 and 14, and find it to be 2 times contained 162, and proceed- ingly I place 2 in the root, and proceed-

279) 21627 Resolvend

09 Triple root

277

279 Divisor

216 Cube of 6

324 Square of 6 by the triple root

162 Triple square of the root by 6

9656 Subtrahend

1971 Remainder

Then to the said remainder (1971) do I an-

nex the next cube number (125) according to the 10th Rule, and it makes 1971125 for a resolvend;

Then I proceed according to the 11, 12, and 13 Rules to find a Divisor, and therefore I first triple the whole Quotient (36) and it is 108, Then do I square the whole Quote (36) and it makes 1296, which being tripled is 3888, which being orderly placed under the triple Quote, and added thereto in that order, the sum is 38988 for a new Divisor: See the work in the Margent.

Then do I seek how often the said Divisor is contained in the Dividend (197112) and I find it to be 5 times contained therein, and accordingly I place 5 in the root, and proceed according to the 15, 16, 17 and 18 Rules to find out a Subtrahend, and therefore first, I cube the number (5) last placed in the root, and it makes 125, then do I square the said 5, and it makes 25, by which I multiply (108) the said triple root, and place the product (2700) under the said cube, as is before directed, then do I by the said 5, multiply (3888) the triple square of the root and the product (19440) do I place under the former product (2700) according to former directions, and add the 3 Numbers together, in the same order as they stand, and their sum is 1971125, (as appears per Margent) for a Subtrahend, which taken out of the said Resolvend there remainth (0) and so the work is finished

$$\begin{array}{r}
 125 \\
 2700 \\
 19440 \\
 \hline
 1971125
 \end{array}$$

finished, and I find the cube root of the given number 48627125 to be 365; view the whole work laid down as followeth.

48627125 (365
27

279) 21627 Resolvend

9 the Triple root

27 the triple square of the root

279 Divisor

216 the Cube of 6

324 the sq. of 6 in the tr. root

162 the triple square of the root in 6

19656 the Subtrahend

38988) 1971125 Resolvend

108 the triple root

3888 the triple sq. of the root

38988 the Divisor

125 the Cube of 5

2700 the sq. of 5 in the tr. root

19440 the tr. sq. of the root in 5

1971125 the Subtrahend

(0)

XIX. When it is required to extract the cube Root of a Number that is incommensurable to its

its Root and you are desirous to know the Fractional part of the root as near as may be, you are to annex to the given number a competent number of Cyphers, which number of Cyphers must be always a multiple of 3, viz. either 3, 6, 9, 12, &c. Cyphers, that is 000, 000000, or 000000000, &c. And having observed the 6 Rule for the punctuation of the given number, likewise point the annexed Cyphers, in the same manner as if they were significant figures, or integers; and observe that as many points as you put over the integral part, so many places will the integral part of the root consist of, and so many points as are put over the Cyphers, or Decimals, so many decimal places will there be in the root, this being observed the work it self in the extracting the Cube root of a decimal Fraction or of a mixt number of integers and decimals, is the same in every respect as if the number given were an integral Cube number, according to the rules before delivered in this Chapter. As in the following

Example.

Let it be required to extract the Cube root of 13798 which is a number incommensurable to its Cube root, and to find out its root as near as may be I annex to it 9 Cyphers (so by that means I shall have 3 Decimals in the root) and prepare it for Extraction by pointing it as is before directed, and as you see following.

13798.000000000

And

And having performed the work of extraction according to the former Rules, I find its Cube root to be 23.984, which (as by the remainder you may perceive) is somewhat too little, but yet so near the truth, that, if the decimal part were increased by an unite and so made 23.985 it would then be too much, and so consequently it cannot want (as it is) $\frac{1}{1000}$ part of an Unite of the truth, and if the root had had another figure placed in it, it would then have come so near the truth that it would not have wanted $\frac{1}{10000}$ part of an unite: for your farther satisfaction see the whole work performed as followeth.

[illegible]

| | | | | | |
|------------|------|-----|-----|-----|--------------|
| 13 | 798 | 000 | 000 | 000 | (23.984 Root |
| 8 | | | | | |
| 126) | 5798 | | | | Resolvend |
| | 6 | | | | |
| 12 | | | | | |
| 1 | 26 | | | | |
| | 27 | | | | |
| | 54 | | | | |
| 3 | 6 | | | | |
| 4 | 167 | | | | Subtrahend |
| 13939) | 1631 | 000 | | | Resolvend |
| | 69 | | | | |
| | 158 | 7 | | | |
| | 159 | 39 | | | Divisor |
| | | 729 | | | |
| | 55 | 89 | | | |
| 1 | 28 | 3 | | | |
| 1 | 484 | 919 | | | Subtrahend |
| 1714347) | 146 | 081 | 000 | | Resolvend |
| | | 7 | 17 | | |
| | 17 | 136 | 3 | | |
| | 17 | 143 | 47 | | Divisor |
| | | | 512 | | |
| | | 458 | 88 | | |
| | 137 | 090 | 4 | | |
| | 137 | 549 | 792 | | Subtrahend |
| 172519314) | 8 | 531 | 208 | 000 | Resolvend |
| | | | 71 | 94 | |
| | 1 | 725 | 121 | 2 | |
| | 1 | 725 | 193 | 14 | Divisor |
| | | | | 64 | |
| | | 1 | 151 | 04 | |
| | 6 | 900 | 484 | 8 | Subtrahend |
| | 6 | 001 | 635 | 004 | Remains |
| | | 629 | 572 | 096 | |

XX. If at any time it is required to extract the cube root of a vulgar Fraction, let such Fraction be first reduced to its lowest Terms; because it may not be commensurable to its root in the given Terms, but being reduced to its lowest Terms it may, and having so done to perform the work, this is

The Rule.

Extract the Cube Root of the Numerator, (by the former Rules) and place that for a new Numerator, then extract the Cube root of the Denominator, and place that root for a new Denominator, so shall this new Fraction be the Cube root of the given Fraction.

As for Example, Let it be required to extract the Cube root of $\frac{27}{64}$, first I take the Cube root of 27 (the Numerator) which is 3 and place it for a new Numerator. Then I take the Cube root of 64, (the Denominator) which is 4, and place it for a new Denominator, so shall this new Fraction $\frac{3}{4}$ be the Cube root of the given Fraction $\frac{27}{64}$.

In like manner if there were given $\frac{10\frac{1}{2}}{8}$ to have its Cube root extracted, I can easily discover that there cannot be found any Cube root exactly either for the Numerator or Denominator, in the Terms they are given in, but being reduced to their lowest Terms, they are $\frac{21}{4}$, whose cube root is $\frac{3}{4}$ as before.

In like manner the Cube root of $10\frac{1}{2}$ will be found to be $\frac{3}{4} = \sqrt[3]{9}$. $10\frac{1}{2}$.

XXI. But

XXI. But when there is given a vulgar Fraction to have its cube root extracted, it being incommensurable to its root, you may find its cube root very near if you reduce the given vulgar Fraction to a decimal, and then extract the Cube

root of that decimal (by the Rules before delivered) in every respect as if it were a whole Number, and then shall that be a decimal cube root, less than the truth, yet so near the truth that if you add an unite to the last decimal figure it will then be greater than the truth.

Here take notice by the way that your vulgar fraction being reduced to a decimal in Note order to have its cube root extracted, its equivalent decimal must consist of such a number of places as may be a multiple of 3, that is, it must consist of 3, 6, 9, 12, 15, &c. places, and the more places there is in the decimal, the nearer the truth will the root be.

Example.

Let it be required to extract the cube root of $\frac{1}{8}$: In order whereunto I reduce it to this decimal, viz. .625, which because it consisteth but of 3 places, (and so consequently can have but 1 figure in its root) I increase to 9 places by annexing 6 Cyphers thereto thus .625000000 and then the root will consist of 3 places, then do I proceed to extract its cube root, (according to the former Rules) and find it to be .854, &c.

and there will be a remainder of 2164136 as you may prove at your leisure.

XXII. When your given vulgar Fraction is reduced to a Decimal of the same value, and the 3, or 4 first places towards the left hand are possessed by Cyphers, then in this case you are to cut off 3 of them with a dash of the Pen, and for them place a Cypher to possess the first place in the root, and then proceed to extract the cube root of the remaining figures, according to the former Rules, as if there had been no such Cyphers at all.

As for Example.

Let there be given $\frac{8}{27}$ to have its cube root extracted; First reduce it to a decimal Fraction by the first Rule of the second Chapter of this Book, and it makes .000485613, &c. Now to extract the Cube root of this Fraction, I first prepare it, by pointing it in every respect as if it were a whole number, then with a dash of my Pen, I cut off the three first Cyphers and put a (0) to possess the first place in the root, then I proceed to extract the cube root of the remaining figures (485613) as if there had been no Cyphers at all before them; and having finished the work I find its cube root to be .078 as by the following work.

XXIII. When it is reduced to a decimal Fraction, to extract the Cube Root of a mixt number, reduce it to an improper Fraction, and if it hath a perfect Cube root, then extract the Cube root of the Numerator and

000 | 485613 (078

343

1491) 142613 Resolvend

21

147

1491 Divisor

512

1344

1176

131552 Subtrahend

11061 Remainder

In like manner if the decimal which is given to have its cube root extracted, have 6 Cyphers placed before the significant figures on the left hand, then cut off those 6 Cyphers with a dash of the Pen, and for them put two Cyphers to possess the two first places in the root, Then proceed to extract the Cube root of the remaining figures as if there had been no such Cyphers, &c.

XXIII. When it is required To extract the Cube Root of a mixt number. To extract the Cube Root of a mixt number, reduce it to an improper Fraction, and if it hath a perfect Cube root, then extract the Cube root of the Numerator, and

and place it for a new Numerator, and also extract the Cube Root of the Denominator, and place it for a new Denominator, so shall this new Fraction be the Cube Root of the given mixt Number.

Example.

Let it be required to extract the Cube Root of $5\frac{3}{43}$, having reduced it to an improper Fraction, I find it to be $\frac{1728}{343}$, and having extracted the Cube Root of the Numerator (1728) I find its root to be 12, for a Numerator, and the Cube root of 343 the Denominator is 7 for a Denominator, so that I conclude $\frac{12}{7}$ or $1\frac{5}{7}$ to be the Cube root of the given mixt Number $5\frac{3}{43}$, as you may prove at your leisure.

XXIV. But if the given mixt Number, whose Cube root is required, have not a perfect root then you are to reduce the Fractional part into a Decimal of the same value, (but let the number of decimal places be always a multiple of 3) and then proceed to extract the Cube Root of that mixt Number, as if it were a whole Number, always reserving so many decimal places in the Root as there are points over the decimal part of the mixt Number.

Example.

Let it be required to extract the Cube root of $28\frac{1}{4}$. First, reduce $\frac{1}{4}$ into its equivalent decimal, which is .75, but to make it consist of six places, I annex thereto four Cyphers, and then
I the

the said mixt number will be 28.750000, which being done, I proceed to the work as followeth.

28.750000 (3.06, &c.

27

279) 1750 Resolvend

9

27

279 Divisor

27090) 1750000 Resolvend

90

2700

27090 Divisor

216

3240

16200

1652616 Subtrahend

97384 Remains

So that I find by the work, the Cube root of 28.750000 to be 3.06, &c.

XXV. It is usual amongst Artists to express the Cube Root of a whole Number, mixt Number, or Fraction, either Vulgar, or Decimal, that

that is incommensurable to its Root, by prefixing this Character, (*viz.* $\sqrt[3]{c.}$) before the incommensurable number or quantity, so the Cube Root of 328 may be thus expressed $\sqrt[3]{c. 328}$, and the Cube Root of $24\frac{3}{4}$, thus $\sqrt[3]{c. 24\frac{3}{4}}$, or in a decimal mixt number thus $\sqrt[3]{c. 24.75}$ and of the fraction $\frac{3}{4}$ thus $\sqrt[3]{c. \frac{3}{4}}$, &c.

XVI. The operation in the extraction of the Cube Root is proved thus, *viz.* Cube the Root found out, that is, Multiply it three times into it self, and if any thing remain after the work is done, add it to the last product, and if that sum be equal to the given number, then the work is truly performed, otherwise not.

As in our first Example, where it is required to extract the Cube Root of 110592, and which is found to be 48; and to prove the work, multiply 48 by it self, whose product is 2304, which being again multiplied by 48, it produceth 110592, which is equal to the given number, and therefore I conclude the work to be right.

Likewise to prove the Example of the Nineteenth Rule, where it is required to extract the Cube Root of 13798 which is found to be the mixt number 23.984. Now to prove the work, I Cube the root, as before directed, and find it to be 13796.370427904 to which add the remainder, 1629572096 and their sum maketh the given number 13798 which proves the work to be right.

C H A P. XI.

The Use of the Square and
Cube Roots in solving some
Questions Arithmetical and
Geometrical.

P R O P. I.

*To find a mean proportional be-
tween two given Numbers.*

Multiply the given Numbers the one by the
other, and extract the square Root of the
product, so shall that square root be the mean pro-
portional sought.

Example.

Let the given numbers be 12 and 48, and let
it be required to find a mean proportional be-
tween them ; first multiply the given numbers
12 and 48 the one into the other, and their pro-
duct is 576, the Square Root of which is 24, so
that

that I conclude 24 to be a mean proportional between 12 and 48; for,

$$12 : 24 :: 24 : 48.$$

The square of the mean being equal to the product of the extremes.

This proposition is useful in finding the side of a square that shall be equal to any given parallelogram; for, (according to the first Proposition of the eighth Chapter of this Book,) if you multiply the contiguous sides of a Rectangular parallelogram the one by the other, that product will be its content, and if you extract the square root of that content, it will give you the side of a square, (in the same measure your parallelogram was) which will be equal to the given parallelogram.

P R O P. II.

To find the Side of a Square that shall be equal to the Content of any given superficies.

Find out the Content of the given superficies by the Rules laid down in the Eighth Chapter, and then extract the square root of the

Content, so will that Root be the side of a square equal to the given superficies.

Example.

There is a Rectangled Triangle whose base and perpendicular are 16 and 18, I demand the side of a square that will be equal to the given Triangle.

According to the second Proposition of the Eighth Chapter, I find the Content of this Triangle to be 144, the square Root of which is 12, and is the side of a square equal to the said Triangle.

In like manner if you extract the square root of the Content of a Circle, Pentagon, Hexagon, &c. or of any other figure regular or irregular, it will give the side of a square equal to that superficies.

P R O P. III.

Having any two of the sides of a Right-angled plain Triangle, given to find the third side.

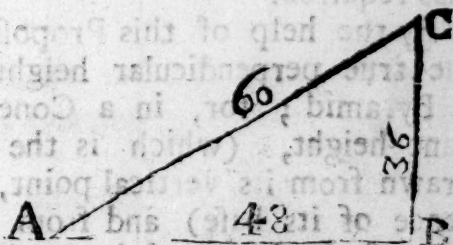
THIS most excellent and useful proposition is generally called *Pythagoras his Theoreme*, and in the 47. Pro. of *Euclides Elements* of Geom.

Geom. it is demonstrated, and proved that the Square made of the Hypotenuse, or slant side of a right angled plain Triangle is equal to the sum of the squares made of the base and perpendicular,

As for Example.

In the Triangle A B C, the Base A B is 48, and the perpendicular B C is 36, now I demand the length of the Hypotenuse A C.

To find out an answer to this, first I square the base A B, (48) which is 2304, then square the Perpendicular (36) and its square is 1296, the sum of which two squares is 3600, which is equal to the Square of the Hypotenuse A C, therefore the square root of 3600 will give the length of A C, which is 60.



P R O P. IV.

There is a Tower about which there is a Moat that is 48 foot wide, and a scaling Ladder that is 60 Foot long, will reach from the outside of the Moat, to the top of a Wall, that is within the said Moat, now I demand the height of the said Wall above the Water?

Let the Base $A B$ in the foregoing Triangle be the breadth of the Moat, and let the Hypothenuse $A C$ be the scaling Ladder, then is the perpendicular $B C$ the height of the Wall above the Water. Now it is plain that (because the square of $A C$ is equal to the sum of the squares of $A B$ and $B C$) if from the square of $A C$ which is 3600 you subtract the square of $A B$ which is 2304, there will remain 1296, which is the square of $B C$, therefore I extract the square Root of 1296, and find it to be 36, which is the height of the said Wall above the Water as was required.

By the help of this Proposition may be found the true perpendicular height of a Cone, or of a Pyramid; for, in a Cone, if you square the slant height, (which is the length of a line drawn from its vertical point, to the Circumference of its base) and from the square of that, subtract the square of the Semidiameter of its base, there will remain the square of the perpendicular height of that Cone.

Also, In a Pyramid, if from the square of the slant height of it, you subtract the square of that line which being drawn from the Centre of its base, should touch the end of the said slant line, (whether they meet at an angle or not) the remainder will be the square of the perpendicular height of that Pyramid, and its square Root will give the height it self.

P R O P. V.

*By the Content of a Circle to find
its Diameter.*

The proportion is

A ^{S 22.}
Is to 28.
So is the given Content
To the square of the Diameter,

Example.

There is a Circle whose superficial Content is
153.9385, I demand its Diameter?

$$22 : 28 :: 153.9385 : 195.9217.$$

The square Root of which is 13.99 (very near
14) for the Diameter required.

PROP.

PROP. VI.

*By the Content of a Circle to find
its Circumference.*

The Proportion is

A^{S 7} Is to 88
So is the given Content
To the square of the Circumference.

The square root of which is the Circumference
required.

Example.

There is a Circle whose superficial content is
153.9385, I demand the Circumference of that
Circle!

7 : 88 :: 153.9385 : 1935.2268701

The square root of which is 44 fere which is
the Circumference required.

II. The Cube Root is that by help of which
we resolve all questions Mathematical that con-
cern solidity, and by which we increase solid bo-
dies according to any given proportion. By it
we discover the solidity of a body that is capable
of length, breadth, and depth, (or thickness,) and

and by having the solidity given, we discover the side or diameter of such a body.

Some questions pertinent thereto may be such as follow.

PROP. VII.

THere is a Cube whose side is 4, I demand what shall be the side of a Cube whose solidity is double to the solidity of that Cube?

To answer this proposition, find out the Cube of 4 (the side of the given Cube) which is 64, and double it, which is 128, then extract the Cube root of 128, and it makes 5.0397 *ferè*, and that is the side of the Cube which is double to the Cube whose side is 4.

PROP. VIII.

THere is a Cube whose solidity is 128 foot, I demand the side of a Cube whose solidity is half as much?

Take $\frac{1}{2}$ of 128 = 64 the Cube root of which (*viz.* 4.) answers the question.

PROP. IX.

Having the solid Content of a Globe to find the side of a Cube whose solidity shall be equal to the given Globe.

Extract the Cube root of the given solid Content of the Globe, and it will give you the side of the Cube required.

Example.

There is a Globe whose solid Content is 1728 Inches, I demand the side of the Cube equal thereto?

Having extracted the Cube root of 1728, I find it to be 12, which is the side of the Cube required.

PROP. X.

HAVING the Diameter and Weight of a Bullet, to find the Weight of another Bullet whose Diameter is given.

As the Cube of the given Bullets Diameter,
Is to its Weight or Solidity.

So is the Cube of the Diameter of any other
Bullet,
To its Weight or Solidity.

Example.

There is a Bullet whose Diameter is 4 Inches, and its weight is 9 Pound, I demand the weight of another Bullet, whose Diameter is $6\frac{1}{4}$ or 6.25 Inches.

The Cube of 4 is 64.

The Cube of 6.25 is 244.140625

Then I say

64 : 9 : : 244.140625 : 34.33227

So

So that the weight required is 34.33227 pound^s and if you reduce the Decimal to the known parts of Averdupois weight, you will find the answer to be 34 lb—05 oz.—05 dr.

This kind of Proportion is by Artists Termed triplicate proportion.

In like manner, the Diameters of two Bullets, or Globes being given, and the solidity of one of them to find out the solidity of the other, it may be done by the same proportion, only changing the middlemost Term.

P R O P. XI.

TO find the side of a Cube equal to a given paralelepipedon.

Find out the solidity of the given paralelepipedon by the Eighth Prop. of the Eighth Chapter, then is the Cube Root thereof, the required side.

Example.

There is a paralelepipedon having the sides of its base 10 Foot 4 Inches, and 5 Foot 2 Inches, and its length is 20 Foot 8 Inches, I desire to know what is the side of a Cube whose content shall be equal to the given paralelepipedon?

The Superficial Content of the base is 7688 inches, which drawn into 248 the length in Inches, the product is 1906624 inches for its solid Content, the Cube root of which is 124 inches, for the side of a Cube equal to the given paralelepipedon.

In like manner if you would find at any time the side of the Cube equal to any solid Body whether Regular or Irregular : First, Find the solid

lid Content of that Body, and then extracting the Cube Root of its Iolid Content you have your desire.

PROP. XII.

Between two given Numbers to find two mean proportionals.

Divide the greater extream by the lesser, and extract the Cube Root of the Quotient, and by the said Cube Root multiply the lesser extream, then will the product give you the lesser mean propotional, then multiply the said lesser mean by the said Cubique Root, and that product will give you the greater mean propotional.

Example.

Let the two given extreames be 6 and 48 between which it is required to find 2 mean proportionals.

First, I divide 48 (the Greater Extream) by 6 (the Lesser Extream) and the Quotient is 8, the Cube Root of which is 2 then by (the Cube Root) 2 I multiply 6 (the lesser extream) and the product is 12 for the lesser mean propotional, and 12 being multiplied by 2 (the Cube Root) the product is 24, for the greater mean propotional sought. Thus have I found 12 and 24 to betwo mean proportionals between 6 and 48, for

$$6 : 12 :: 24 : 48$$

In like manner between 3 and 81 will be found 9 and 27, for two mean proportionals.

P R O P. XIII.

THE Concave Diameter of two Guns being known, and the quantity of Gun-powder that will charge one of them, to find out how much will be sufficient to charge the other.

The Capacities are one to another, as are the Cubes of their Diameters, and also the proportion is direct.

Example.

If 25 pound of Gun-powder be sufficient to charge a Gun, whose Concave Diameter is $1\frac{1}{4}$ Inches, or 1.5 Inch, how much powder will be sufficient to charge a Gun, whose Concave Diameter is 7 Inches? Answer, 25.47.

The Cube of 1.5 is 3.375 and the Cube of 7 is 343. wherefore the Proportion is as followeth.

$$3.375 : .25 :: 343 : 25.47$$

Or thus,

$$3.375 : 343 :: .25 : 25.47$$

P R O P. XIV.

THE Concave Diameters of two Guns being given, and the quantity of a weaker sort of Gun-powder sufficient to charge one of them,
to

to find out how much Gun-powder of a stronger sort (the proportion of the strength and weakness of the Gun-powder being also given) will be sufficient to charge the other Gun,

This is solved by two operations in the Rule of proportion, first to find out how much of the stronger sort of Gun-powder will be of equivalent strength with the given quantity of the weaker sort, and this proportion is reciprocal; The second is the same with that in the foregoing Proposition.

Example

There is a Gun whose Concave Diameter is $1\frac{1}{2}$ inches, and it requireth 25 pound of powder to Charge it, now there is another sort of Gun-powder which is much stronger than the former, and the proportion between their strength is as 5 to 2, now I demand how much of the strongest powder is sufficient to charge a Gun whose Concave Diameter is 7 inches.

To answer this, First, I find out how much of the strongest powder will charge that Gun, which is $1\frac{1}{2}$ inch in its Concave Diameter, which is done by the following proportion, viz.

$$5 : 2 :: .25 : .10$$

Thus have I found that $\frac{1}{10}$ of a pound of the strongest Powder will charge a Gun whose Concave diameter is $1\frac{1}{2}$ inch. And according to the last proportion, I find by a direct Proposition that 10.16 pounds of the same will be sufficient to charge a Gun whose Concave diameter is 7 inches, viz.

$$3.375 : 343 :: .10 : 10.16$$

C H A P.

C H A P. XII.

Concerning Simple Interest.

I. **W**Hen Money pertaining; or belonging to one person is in the hands, possession, or keeping; or is lent to another, and the Debtor payeth or alloweth to the Creditor, a certain sum in consideration of forbearance for a certain time, such consideration for forbearance is called Interest, Loane, or use Money; and the money so lent, and forborn is called the principal.

II. Interest is either Simple or Compound.

III. When for a sum of Money lent there is loane or interest allowed, and the same is not paid when it becomes due, and if such interest doth not then become a part of the Principal, it is called Simple Interest.

IV. In the taking of Interest for the continuance or forbearance of Money, respect must be had to the rate limited by Act of Parliament, which Act now in force, forbiddeth or restraineth all persons; whatsoever, from taking more than 6*l.* for the interest of an 100*l.* for a year, and according to the same proportion for a greater or a lesser sum, not confining the lender or borrower to the space of one year, no more than

it confineth him or them to the limitation of the sum to be lent, or borrowed, but that the sum may be either more or less than 100 *l.* and may continue in the hands of the Debtor, either a longer, or a shorter time than one year, according as the Lender and Borrower do agree, and oblige each other; now for any time greater than one year, the rate or proportion of Interest is by Act of Parliament limited, but the Act doth not say what part of 6 *l.* shall be the interest of an 100 *l.* for half a year, a quarter of a year, a month, a day, or for any time less than one year, and in this case several Artists do differ in their opinions, some would have the true proportional interest for any time less than a year to be discover'd by continual mean proportionals; as suppose it were required to know the interest of 100 *l.* for half a year at 6 *per Cent. per Annum*, they would have the Interest to be reckoned after the Rule of Compound interest, and so 30 *l.* is not the interest of a 100 *l.* for half a year, but is too much: But say they, to find out the true interest thereof: you are to find a mean proportional between 100, and 106, and that made less by 100, will give you the interest of 100 *l.* for half a year, and so by extracting of Roots they find out the interest for any time less than one year, but this is sufficiently laborious and painful if it be done without the help of Logarithms; but to perform this work to the 12 power for a Month, or to the 52 for a week, is very tedious, and to the 365 power for one day is scarcely possible to be effected by natural Numbers, but custom and daily practice tell us that the interest of Money for any time less than one year ought

to be computed according to the Rules of Simple Interest, and so 3 l. is the undoubted interest of 100 l. for 6 months, and 30 shillings is the interest of 100 l. for a quarter of a year; but here note by the way that by 6 months is not meant 6 times 4 weeks, or 6 times 28 days, but by six months, or half a year is to be understood the half of 365 days, and a quarter of a year is $\frac{1}{4}$ of 365 days, and by 1 month is understood $\frac{1}{12}$ of 365 days, so that a month consisteth of $30\frac{1}{12}$ days.

Upon the aforesaid custom of computing the interest of Money for time less than one year, this following Analogy seems to be assumed for a safe exposition of the statute (and which is indeed the ground, and reason it self of Simple Interest)

Vide Sect. 6 of the
5 chap. of Mr. Ker-
lies Appendix to
Wing. Arith.

viz. That such proportion as 365 days (or one year) hath to the interest of any sum for a year, such proportion hath any part of one year, or any number of days propounded to the interest of the same sum, for that time propounded. And this (as was said before) is the whole ground work, and very foundation of the manner of computing of Simple Interest.

V. Rebate, or Discount, is, when there is an allowance of so much *per Cent.* for Money paid before it be due, and Of Rebate, as the increase of Money at interest is found out by continual proportionals Arithmetical or Geometrical increasing, so is the Rebate or discount of Money found out by continual proportionals decreasing Arithmetically or Geometrically, that is according as the al-

lowance,

lowance is, either after Simple or Compound Interest; Now the nature of Rebate or Discount is thus; when there is a sum of Money, (suppose 100 l.) to become due at the end of a certain time to come, (*viz.* at the end of 12 Months;) and it is agreed upon by the Debtor and Creditor that there shall be made present payment of the whole Debt, and it is likewise agreed that in consideration of this present payment that the Creditor shall allow the Debtor after the rate of 6 per Cent. per Annum: Now upon this agreement the Creditor ought to receive so much money as being put out to interest for the same time it was paid before 'twas due, and at the same rate of interest, that the discount was reckoned at, then would it amount or be increased to the sum that was first due.

The manner of working Questions in Rebate at Simple Interest shall be shewn in the ninth Rule of this Chapter, and of working Questions in Rebate at Compound Interest shall be shewn in the Fourth Rule of the next Chapter.

VI. When the interest of a 100 £. for a year is known, the interest of any other sum, for the same time, is also found out, by one single rule of direct proportion, viz. The interest of a 100 £. for a year by the statute is 6 £. I demand what is the interest of 75 £. for the same time, and at the same rate of Interest? The proportion is as followeth.

[Faint mirrored bleed-through from reverse side]

$$100 : 6 :: 75 : 4.5 = 4 - 10$$

Or if you would have the Answer to produce both principal and interest, then make the second

cond. number to be the sum of the given principal and interest, and the fourth proportional will answer your desire. Thus

$$100 : 106 :: 75 : 79.5 = 79 - 10$$

VII. When the interest of 100 for a year is given, and the interest of any other sum of pounds, shillings and pence is required for a year, the answer may be easily found after the practical method delivered in the following Example.

Let it be required to find the interest of 148 l.—13 s.—04 d. for one year after the rate of 6 per Cent. per Annum, Simple Interest?

First, I place the given numbers according to the direction given for the Rule of 3, which will stand thus, viz.

$$\begin{array}{ccccccc} l. & & l. & & l. & s. & d. \\ 100 & : & 106 & : & 148 & -13 & -04 \end{array}$$

Now it is evident that if I multiply 148 l.—13 s.—04 d. (which is the third number) by 6 (which is the second number) and divide the product by 100 (which is the first number) the Quotient will be the answer; Therefore I proceed thus, viz. first I multiply the pence by 6, which makes 24 pence, or two shillings, therefore I set down 0 under the pence, and carry 2 to the next, then I go to 13 s. saying 6 times 13 is 78, and 2 that I carried is 80 s. which is 4 l. therefore I set down 0 under the shillings, and carry 4 to the pounds, then I proceed, saying 6 times 8 is 48, and 4 that I

carry

carry is 52, then I set down 2, and carry 5, &c. proceeding thus till the work be finished, and then will the product be 890 *l.*—00 *s.*—00 *d.* which product should be divided by 100 (the first number) but it being an unite with two Cyphers, I cut off two figures from the right hand of the pounds, with a dash of the pen, and the figures on the left hand of the said dash, are so many pounds, and those on the right hand of it, are the Decimal parts of a pound, whose value may be found out by the 3 Rule of the 2 Chap. But remember, that if there be any shillings or pence, in the product you are to add them to their respective products in your Reduction.

The work of the foregoing Example is as followeth.

$$\begin{array}{rclcl} \textit{l.} & & \textit{s.} & & \textit{l.} & \textit{s.} & \textit{d.} \\ 100 & : & 6 & : : & 148-13-04 & & 6 \end{array}$$

| | |
|----|----------|
| 8 | 92—00—00 |
| — | 20 |
| — | — |
| 18 | 40 |
| — | 12 |
| — | — |
| 4 | 80 |
| — | 4 |
| — | — |
| 3 | 20 |
| — | — |

So that by the work I find the interest of 148 *l.*—13 *s.*—4 *d.* for one year after the rate of 6 per Cent. per *An.* to be 8 *l.*—18 *s.*—04 *d.*—3.2 *qu.*

An-

Another Example may be this, viz. I demand the interest of 368 l.—15 s.—3 d. for one year, at 6 per Cent. per An. Answer 22 l.—02 s.—6 d. as by the work following.

100 : 6 :: 368—15—03

| | l. | s. | d. |
|-----------|-----|-------|----|
| 368—15—03 | 368 | 15 | 03 |
| 6 | | | 6 |
| <hr/> | | | |
| 22 | 12 | 11 | 06 |
| | | 20 | |
| | | <hr/> | |
| | | 25 | 1 |
| | | 12 | |
| | | <hr/> | |
| | 6 | 18 | |

VIII. The Interest of 100 l. being known for a year, or 365 dayes, the interest of any other sum may be known for any other time, or number of dayes, more or less than a year, by two single Rules of 3 Direct, viz. First, find out what is the interest of the given sum, for one year, or 365 dayes, according to the last Rule, then having found out that, you may (by another single Rule of 3 Direct) find out its interest for any other time more or less.

Example.

What is the interest of 322 l. for 6 years after the rate of 6 per Cent. per Annum, Simple Interest?

K 4

First,

First I find what is the Interest of 322 l. for a year by the following proportion,

$$\begin{array}{ccccccc} \text{l.} & & \text{l.} & & \text{l.} & & \text{l.} \\ 100 & : & 6 & :: & 322 & : & 19.32 \\ & & & & 6 & & \end{array}$$

$$100) 19.32 \text{ (19.32)}$$

Thus having found the interest of 322 l. for a year to be 19.32 l. at 6 per Cent. by the following proportion I find out its interest for 6 years, to be 115 l.—18 s.—04 $\frac{3}{4}$ d, and that added to the principal, makes 437 l.—18 s.—04 $\frac{3}{4}$ d. for the sum due to the Creditor at the end of the said time.

| year | l. | year | l. | l. | d. |
|------|--------|------|-----|----|------------------|
| 1 | 19.32 | 6 | 115 | 18 | 04 $\frac{3}{4}$ |
| | 6 | | 322 | | |
| | 115.92 | | 437 | 18 | 04 $\frac{3}{4}$ |
| | 20 | | | | |
| | 18.40 | | | | |
| | 2 | | | | |
| | 4.80 | | | | |
| | 4 | | | | |
| | 3.20 | | | | |

And here take notice that the second number in this last proportion, must always be only the interest of the sum proposed, and not the sum of

of the principal and interest, as in the second proportion under the sixth Rule.

After the same manner is the interest of 1 *l.* (at the rate of 6 per Cent. per *Annum*, or any other rate of interest,) discovered for a day, by the help of which the interest of any sum whatsoever may be discovered for any number of dayes as shall be shown by and by.

$$\begin{array}{ccccccc} & \textit{l.} & & \textit{l.} & & \textit{l.} & & \textit{l.} \\ \text{First} & 100 & : & 6 & :: & 1 & : & .06 \end{array}$$

$$\begin{array}{ccccccc} & \textit{day} & & \textit{l.} & & \textit{day} & \\ \text{Secondly} & 365 & : & .06 & : & 1 & : & .0001643835 \end{array}$$

So that by the foregoing proportions I have found that the interest of 1 *l.* at 6 per Cent. per *Annum* for a day is .0001643835 *l.*

Now if you would know the Interest of any other sum for any number of days more or less than 365, you may do it by help of the said number after this manner, *viz.*

Multiply the sum whose interest is required by the said number, and that product will give you the interest of the said sum for one day, then multiply that Product by the number of days given, and the last Product will give you the interest of the said sum for the number of dayes in the question. Take the following question for an example, *viz.*

What is the interest of 568 *l.* for 213 days after the rate of 6 per Cent. per *Annum*?

0001643835

568

13150680

9863010

8219175

0933698280

213

2801094840

933698280

1807390560

19.8877733640

Facit 19 ^{l.} — 17 ^{s.} — 09 ^{d.}

Having finished the work as you see, I find the answer to be 19.8877, &c. which upon sight I discover to be 19 ^{l.} — 17 ^{s.} — 09 ^{d.} by the brief way of valuing a decimal Fraction of Coyne laid down in the 4 Rule of the second Chapter before-going.

But when the interest of any sum of Money is required for any number of days as aforesaid, at any other rate of interest than at 6 ^{per Cent.} ^{per Annum}, the aforesaid number will not then serve for the work, but you are to find out particular multipliers for the several rates of interest as is before directed. All which I have expressed from 4 to 10 ^{per Cent.} in the following Table.

When

| | | | |
|-------------------------|----|--|-------------|
| When you would find | 4 | } per Cent. per An. the Multiplier is | .0001095890 |
| the Interest of any sum | 5 | | .0001369863 |
| for any number of dayes | 6 | | .0001643835 |
| at the rate of | 7 | | .0001917808 |
| | 8 | | .0002191780 |
| | 9 | | .0002465753 |
| | 10 | | .0002739726 |

So that when you would find out the interest of any sum of Money for any number of dayes according to the direction before given, at any Rate from 4 to 10 per Cent. per Annum, Simple Interest, you may perform the work by the multiplier in the foregoing Table which is placed against each respective rate of Interest.

IX. When the present worth of a sum of Money due at the end of any time to come is required, Rebate being allowed at any rate of Simple Interest, it may be found out by the following method; viz. First, Find out the interest of 100 l. for the time that the Rebate is to be allowed for, and at the same rate of interest propounded, then make the sum of an 100 pound, and its interest for the proposed time, to be the first number in the Rule of 3, and 100 l. the second number, and the given sum whose present worth is required, let be the third number, and the fourth number in a direct proportion shall answer the question, as in the following Example, viz.

What present Money will satisfie a debt of 100 l. that is due at the end of a year yet to come, Discount or Rebate being allowed at the Rate of 6 per Cent. per Annum.

According to the foregoing Directions, I state the numbers as followeth, and the fourth proportional

proportional number or answer to the question is
 $94\ 33962\ l. = 94\ l. - 06\ s. - 09\ \frac{1}{2}\ d. \text{ fere.}$

$$106 : 100 :: 100 : 94\ 33962$$

The reason of the said Analogy will appear if you consider, that there ought to be so much ready money paid, that if it were put out to interest at the same rate of Int. that Rebate was allowed for, and for the same time, the same would then be augmented to the sum that was at first due, as in the last question, there is given 100 *l.* which is due at the end of 12 Months, now I say, that there ought to be so much money paid down to satisfy this debt, as being put out to interest at 6 per Cent. for 12 Months, would then be increased to 100 *l.* which is the first sum due, and again it is as evident that, if there were 106 *l.* due at the end of 12 Months, or a year, and present payment is agreed upon, allowing Rebate at 6 per Cent. per Annum, that then there ought to be paid the sum of 100 *l.* in full discharge of the said debt of 106 *l.* for if when I have received the said sum of 100 *l.* I put it out to interest for one year at the rate of 6 per Cent. it will then be increased to 06 *l.*

Therefore to solve the said question, the proportion here used is no more than if I should say, If 106 *l.* be decreased to 100 *l.* what will 100 *l.* be decreased to? The answer is, to 94 *l.* - 6s. - 9d. $\frac{1}{2}$ and for proof, if you will seek what that sum will be increased to at the end of 12 Months, at the rate of 6 per Cent. you will find it to be 100.

Example 2.

How much present Money will satisfy a debt
 of

of 82 *l.*—15 *s.* due at the end of 126 days, yet to come allowing Rebate after the rate of 6 per Cent. per Annum?

First I find the interest of 100 *l.* at the same rate of interest for 126 days by the following proportion.

$$\begin{array}{ccccccc} \text{day} & & \text{l.} & & \text{day} & & \text{l.} \\ 365 & : & 6 & : : & 126 & : & 2.0712 \end{array}$$

Then do I add 2.0712 *l.* (the interest of 100 *l.*) to 100 *l.* and the sum is 102.0712 which I make the first number in the Rule of 3, and 100 *l.* the second, and 82.75 *l.* (the sum given to be Rebated) the third number, and the fourth number in a direct proportion is the answer to the question, see the work as followeth.

$$\begin{array}{ccccccc} \text{l.} & & \text{l.} & & \text{l.} & & \text{l.} \\ 102.0712 & : & 100 & : : & 82.75 & : & 81.0708 \\ & & & & 100 & & \end{array}$$

$$102.0712) 8275.00 \quad (81.0708$$

So that by the work it appears that 82 *l.*—15 *s.* due at the end of 126 days yet to come, will be satisfied with the present payment of 81 *l.*—01 *s.*—04 $\frac{3}{4}$ *d.* Rebate be allowed after the rate of 6 per Cent. per Annum.

The proof of the Rule.

Find out (by the eighth Rule foregoing) how much the present money that is paid upon Rebate, will amount to being put out to interest for the same time, and at the same rate of interest that Rebate was allowed for, and if the amount be equal to the sum that was due at the end

of

of that time then you may conclude the work to be rightly performed, otherwise not:

As for *Example.*

In the foregoing question it was found that 81. 0708 *l.* being paid presently would satisfy a debt of 82. 75 due at the end of a 126 dayes to come, and to prove it, let us see whether 81.0708 being put out to interest for 126 days at the rate of 6 per Cent. per Annum, will be increased to 82.75 *l.* (the sum which was said to be due at the end of 126 days to come) which I do by these two proportions following according to the eighth Rule.

| | | | | | |
|--------|-----|-----------|---|-----|-----------|
| | day | <i>l.</i> | | day | <i>l.</i> |
| First, | 365 | : | 6 | : | 126 |
| | | : | : | : | 2.0712 |

| | | | | | |
|-----------|-----------|---|-----------|---|-------------|
| | <i>l.</i> | | <i>l.</i> | | <i>l.</i> |
| Secondly, | 100 | : | 2.0712 | : | 81.0707 |
| | | : | : | : | 1.6791, &c. |

So you see that I have found the interest of 81.0708 for 126 days to be 1.6791, &c. which added to the principal 81. 0708 the sum is 12.7499 which by the brief way of valuing the Decimal of a pound sterling is 82 *l.*—15 *s.* and indeed it doth not want $\frac{1}{10}$ part of a farthing of the exact sum, which is occasioned by the defective Decimal wherefore I conclude the work to be rightly performed.

Upon the foregoing ninth Rule is grounded the manner of calculating the ensuing Table of Multipliers, which sheweth in decimal parts of a pound, the present worth of a pound Sterling due at the end of any number of years to come,
not

not exceeding 30, Simple Interest being computed at 6 per Cent. per Annum.

The first number in the Table being found out by this following proportion, viz.

As 106 *l.* is to 100 *l.* so is 1 *l.* to .943396, and the second number in the Table being the present worth of 1 *l.* due at the end of two years to come, is thus found out, viz. First I consider that 12 *l.* is the simple interest of 100 *l.* for 2 years, which added to 100 *l.* makes 112 *l.* wherefore I say as 112 *l.* is to 100 *l.* so is 1 *l.* to .892857 *l.* which is the present worth of 1 *l.* due at the end of two years to come.

The several proportions and operations for the whole Calculation being as followeth, viz.

| | | | | | |
|---|-----|-------|------|---|---------|
| 1 | 106 | : 100 | :: 1 | : | .943396 |
| 2 | 112 | : 100 | :: 1 | : | .892857 |
| 3 | 118 | : 100 | :: 1 | : | .847457 |
| 4 | 124 | : 100 | :: 1 | : | .806451 |
| 5 | 130 | : 100 | :: 1 | : | .769230 |
| 6 | 136 | : 100 | :: 1 | : | .734294 |

And after the same manner are all the numbers in the following Table Calculated; which being well understood, the way of calculating most of the ensuing Tables will easily be obtained; and its use you will find immediately after the Table it self.

level, as those are in the next Chapter concerning Compound Interest, but Simple Interest being not so generally in practice, I shall therefore

The use of the preceding Table

Table evident (by the ninth Rule foregoing) that

| years | TABLE I. | |
|-------|---|-----------|
| | Which sheweth in Decimal parts of a pound the present worth of 1 l. due at the end of any number of years to come under 31, at the rate of 6. per Cent. per Ann. Simple Interest. | |
| | | 11.602409 |
| | | 12.581395 |
| | | 13.561797 |
| | | 14.543478 |
| | | 15.526315 |
| | | 16.510204 |
| | | 17.495049 |
| | | 18.480769 |
| | | 19.467289 |
| | | 20.454545 |
| 1. | 943396 | 21.442477 |
| 2. | 892857 | 22.431034 |
| 3. | 847457 | 23.420168 |
| 4. | 806451 | 24.409836 |
| 5. | 769230 | 25.400000 |
| 6. | 735294 | 26.390625 |
| 7. | 704225 | 27.381679 |
| 8. | 675075 | 28.373134 |
| 9. | 649350 | 29.364963 |
| 10. | 625000 | 30.357143 |

After the same method might this Table be continued to any number of years at pleasure, I might also have calculated for other rates of interest, as those are in the next Chapter concerning Compound Interest, but Simple Interest being not so generally in practice, I shall therefore forbear.

The use of the preceding TABLE.

It is evident (by the ninth Rule foregoing) that

that if any sum be paid with an allowance of Rebate, you are to make 100 *l.* with its interest (for the same time you Rebate for) both in one sum, to be the first number in the Rule of 3 100 the second, and the sum to be rebated the third, then will the fourth proportional be the answer; and the same may be wrought by any other number and its interest, as well as by 100 *l.* and its interest *mutatis mutandis*: Now in the Table beforegoing there is expressed in Decimal parts of a pound, the present worth of 1 *l.* due at the end of any number of years to come under 31, &c. that is to say, if you take the money signified by those Decimals, and put it out to interest at 6 per Cent. per Annum, Simple interest for so many years as are expressed in the Collum of years against the said Decimal, then will that sum at the end of the said Term, be augmented to 1 *l.* wherefore if you have any sum whatsoever to be rebated for any number of years within the limits of the Table, make 1 *l.* the first number in the Rule of 3, and the Decimal in the Table against the number of years to be rebated for, make that the second, and the sum whose present worth is required the third number, so will the fourth proportional be the answer. But (because the first number (being Unity) neither multiplieth nor divideth if you take the number in the Table, correspondent to the number of years for which you would reckon Rebate, and thereby multiply the sum whose present worth is required, the product will give you the Answer.

Example.

There is a sum of Money, viz. 560 l. due at the end of 8 years to come, but the Debtor and Creditor agree that present payment shall be made, and the Debtor to be allowed Rebate after the rate of 6 per Cent. per Annum, Simple Interest. Now I demand how much present money will satisfy the said Debt? Answer, 378.378 l. = 378 l. — 07 s. — 06 $\frac{3}{4}$ d. see the following work.

$$\begin{array}{r}
 \text{l.} \qquad \text{l.} \qquad \text{l.} \qquad \text{l.} \\
 1 : .675675 :: 560 : 378.378 \\
 \hline
 560 \\
 40540500 \\
 3378375 \\
 \hline
 378.378000
 \end{array}$$

First (the Rebate being to be reckoned for 8 years) I look for 8 in the Column of years, and just against it on the right hand, I find .675675 which I multiply by 560 (the sum whose present worth is required,) and the product is 378.378, which by the brief way of valuing the fraction of a pound sterling) I find at first sight to be 378 l. = 07 s. — 06 $\frac{3}{4}$ d.

This Question if it had been wrought by the foregoing ninth Rule would have produced the same answer, for, the Int. of 100 l. for 8 Mon. is 48 l. and 100 \div 48 = 148 wherefore by the Rule of 3 I say

l. : l. : l. : l.
148 : 100 : : 560 : 378.378

X. When an Annuity or yearly Income in arrears for any number of years, and you would know the increase, or amount of it, allowing Simple Interest at a certain rate *per Cent. per Annum*, for each yearly payment from the time it first became due, the operation will be somewhat more tedious than to find the amount of one single sum, according to the eighth Rule of this Chap. which will clearly appear by solving the following question; *viz.*

There is an Annuity, or an income of 100 *l. per Annum*. forborne to the end of 6 years, I demand how much is due at the end of the said Term, allowing interest at the rate of 6 *per Cent. per Annum* Simple Interest? Answer 690 *l.*

In order to the solution of this Question, I consider, First, that

It is evident that for the last year, *viz.* the sixth years payment, there must be no interest at all Reckoned, because it becomes not due till the end of the sixth year; Secondly there must be reckoned the interest of 100 *l.* for one year, *viz.* that which is due at the end of the fifth year; Thirdly, there must be reckoned the interest of 100 *l.* for two years, *viz.* that which is due at the end of the fourth year. Fourthly, There must be reckoned the interest of 100 *l.* for three years, *viz.* that which is due at the end of the third year, Fifthly, the interest of 100 *l.* for 4 years, *viz.* that which is due at the end of the second year: And Sixthly, The interest of 100 *l.* for 5 years, *viz.* that which is due at the end

of the first year, and is forborne the second, third, fourth, fifth and sixth years; all which interests being added together, and their sum added to the sum of each years income, the sum will exhibit the total sum, due at the end of the said six years, which you may perceive by the following work to be 690 *l.* which is the answer to the foregoing Question.

| | | |
|-------------------------------|---|-----------|
| | | <i>l.</i> |
| The Interest of 100 <i>l.</i> | 1 | 06 |
| at 6 per Cent. per An. | 2 | 12 |
| Simple Interest, for | 3 | 18 |
| | 4 | 24 |
| | 5 | 30 |

The sum of the interest is 90

The sum of the annuities is 600

The Total amount is 690

The Construction of Table II.

UPon the foregoing reason is grounded the Calculation of the following Table, which sheweth the amount of 1 *l.* annuity, being forborn to the end of any number of years under 31, Interest being allowed for each yearly payment after the rate of 6 per Cent. per Annum, Simple Interest.

The first number in the Table being 1 *l.* which is that due at the end of the first year, no interest being due for that; the second number in the

the Table is 2.06, which is the first and second years payment, and the interest of 1 *l.* for one year, being that which was due at the end of the first year; The third number in the Table is 3.18 *l.* being the increase of 1 *l.* for 2 years added to the second number in that Table which is 2.06, for the amount of 1 *l.* at the end of 3 years is 1.12 which added to 2.06 the second number it makes 3.18 for the third number; The fourth number is the amount of 1 *l.* for 3 years which is 1.18 added to the number before it, *viz.* the third number, proceeding in the same method, till you have composed the Table at your pleasure, each number in the Table being 1 *l.* and the amount of 1 *l.* (for so many years as it standeth against in the Table made less by one,) added to the number immediately preceeding it.

| | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1.00 | 2.06 | 3.18 | 4.34 | 5.52 | 6.71 | 7.91 | 9.12 | 10.34 | 11.57 |
| 2.00 | 4.12 | 6.36 | 8.60 | 10.84 | 13.08 | 15.32 | 17.56 | 19.80 | 22.04 |
| 3.00 | 6.18 | 9.54 | 12.90 | 16.26 | 19.62 | 22.98 | 26.34 | 29.70 | 33.06 |
| 4.00 | 8.24 | 12.72 | 17.44 | 22.16 | 26.88 | 31.60 | 36.32 | 41.04 | 45.76 |
| 5.00 | 10.30 | 16.98 | 22.30 | 28.60 | 34.90 | 41.20 | 47.50 | 53.80 | 60.10 |
| 6.00 | 12.36 | 21.24 | 28.56 | 36.84 | 45.12 | 53.40 | 61.68 | 69.96 | 78.24 |
| 7.00 | 14.42 | 25.50 | 34.82 | 45.10 | 55.40 | 65.70 | 76.00 | 86.30 | 96.60 |
| 8.00 | 16.48 | 29.76 | 41.08 | 53.36 | 65.64 | 78.92 | 92.20 | 105.48 | 118.76 |
| 9.00 | 18.54 | 34.02 | 47.34 | 61.62 | 76.90 | 93.18 | 109.46 | 125.74 | 142.02 |
| 10.00 | 20.60 | 38.28 | 53.60 | 69.88 | 87.16 | 105.44 | 123.72 | 141.98 | 160.24 |

The use of Table II.

In the preceding Table in the Column under the word Years, are set down every Year from 1 to 30, and the number in the Table placed against each year, is the amount of 1 *l.* annually, in pounds and decimal parts of a pound, being forborne so many years as it is placed

| T A B L E II. | | |
|---------------|---|----------|
| Years | Which sheweth in pounds and Decimal parts of a pound the amount of 1 l. annuity being forborne to the end of any number of years under 31, Simple Interest being computed after the Rate of 6 per Cent. per An. | Years |
| | | 11 14.30 |
| | | 12 15.96 |
| | | 13 17.68 |
| | | 14 19.46 |
| | | 15 21.30 |
| | | 16 23.20 |
| | | 17 25.16 |
| | | 18 27.18 |
| | | 19 29.26 |
| | | 20 31.40 |
| 1 | 1.00 | 21 33.60 |
| 2 | 2.06 | 22 35.86 |
| 3 | 3.18 | 23 38.18 |
| 4 | 4.36 | 24 40.56 |
| 5 | 5.60 | 25 43.00 |
| 6 | 6.90 | 26 45.50 |
| 7 | 8.26 | 27 48.06 |
| 8 | 9.68 | 28 50.68 |
| 9 | 11.16 | 29 53.36 |
| 10 | 12.70 | 30 56.10 |

The use of Table II.

In the preceeding Table in the Collum under the word Years, are set down every Year successively from 1 to 30, and the number in the Table placed against each year, is the amount of 1 l. annuity, in pounds and decimal parts of a pound, being forborne so many years as it is placed

placed against. The use of it will plainly appear by the solving of one, or two Questions, viz.

There is an Annuity of 134 l.—10 s.—6 d. all forborn to the end of 4 years; I demand how much is due to the Creditor at the end of the said Term, Simple Interest being allowed after the rate of 6 per Cent. per Annum?

Facit 586 l.—10 s.—07 d.

To answer this Question, first, I look for 4 years, in the Collum of years, and the number against it is 4.36 which is the amount of 1 l. Annuity for 4 years; therefore having turned the 1 s.—6 d. (in the given annuity) into a Decimal (which is .525) I say by the Rule of 3 thus,

$$\begin{array}{rclcl}
 l. & l. & l. & l. & \\
 1 & : & 4.36 & :: & 1 \quad 4.525 & : & 586.529 \\
 & & & & 4.36 & & \\
 & & & & \hline
 & & & & 807150 & & \\
 & & & & 403595 & & \\
 & & & & 538.00 & & \\
 & & & & \hline
 & & & & 586.52900 & &
 \end{array}$$

Thus by the work I find the answer to be 586.529 l. the value of which Decimal by the brief way of valuing a Decimal laid down in the 4th. Rule of the 2d. Chapter, I find to be 586 l.—10 s.—7 d.

And it is plain that in solving Questions by this Table, that (the first number in the Rule of 3 being unite) if you multiply the given Annuity

by the proper Tabular Number, that then the product will be the answer.

Example 2.

What is the amount of an Annuity of 150 l. 10 s. being forborn to the end of the 7 years, allowing Simple Interest after the rate of 6 per Cent. per Annum? Answer, 1243 l.—02 s.—07 $\frac{1}{2}$ d. fere.

The given Annuity is

150.5

The Tabular number for 7 years is

8.26

9030

3010

12040

Facit 1243.130

XI. When an Annuity or yearly Income, for a certain number of years to come, is to be sold for ready Money, and the seller for ready Money, and the seller is to allow the Buyer Rebate at Simple Interest for his present payment, then in this case the buyer ought to pay so much present Money for each yearly payment, as being put out at Simple Interest for so many years as it is Rebated for, it would then amount to one yearly payment, and the sum of all those present worths will be the present worth of the Annuity required, the Rule will appear very plain by the following Example.

There

There is an Annuity or Lease of 100 *l.* per Annum to continue 6 years yet to come to be sold for ready Money, the Seller being to allow the Buyer Rebate at 6 per Cent. per Annum, Simple Interest now I desire to know how much present Money will buy out the said Lease?

Facit 499 *l.*—09 *s.*—04½ *d.* fere.

It is evident that if we find out the present worth of 100 *l.* due at the end of the first year, and also the present worth of 100 *l.* due at the end of the second year, and the present worth of 100 *l.* due at the end of the third year, and likewise the present worth of 100 *l.* due at the end of the fourth, fifth, and sixth years, and add all these present worths together, their sum will be the present worth of the given Annuity; which several present worths are found out according to the ninth Rule, by the several proportions following, viz.

| years | <i>l.</i> | <i>l.</i> | <i>l.</i> | <i>l.</i> |
|-------|-----------|-----------|-----------|-------------|
| 1 | 106 | : 100 | :: 100 | : 94.339622 |
| 2 | 112 | : 100 | :: 100 | : 89.285714 |
| 3 | 118 | : 100 | :: 100 | : 84.745762 |
| 4 | 124 | : 100 | :: 100 | : 80.645169 |
| 5 | 130 | : 100 | :: 100 | : 76.923076 |
| 6 | 136 | : 100 | :: 100 | : 73.529411 |

The present worth of
the said Annuity is } 499.468754

So that you see by the foregoing proportions, the present worth of 100 *l.* per Annum to continue six years, allowing Rebate at 6 per Cent. per An-

Annum, Simple Interest, is 499.468754 *l.* = 499 *l.*
9 *s.* — 04 $\frac{1}{4}$ *d.*

Upon the foregoing eleventh Rule is grounded the construction and calculation of the following Table which sheweth the present worth of 1 pound annuity to continue any number of years under 31 Simple Interest being computed after the rate of 6 per Cent. per *Annum*; the first number in the Table is .943396 which is the present worth of 1 pound due at the end of a year to come. The second number in the Table is 1.836253, which is the sum of the present worths of 1 *l.* due at the end of two years to come, and if 1 *l.* due at the end of one year to come added together; And the third number in the Table is 2.683710 which is the sum of the present worths of 1 *l.* due at the end of 3, 2, and 1 years to come, after the same method is the whole Table calculated.

But the numbers in the said Table may more easily be found out thus, *viz.* Look in the first Table, and let the first number of that be the first number of this third Table, and let the sum of the first number in this, and the second number in that be the second number in this Table, and for the third number in this Table take the sum of the second in this, and the third in that Table, and in this manner you may proceed till you have composed the whole Table.

TABLE

| Years | TABLE III. | |
|-------|--|--------------|
| | Which sheweth the present worth of 1 l. annuity to continue any num- ber of years under 31, Simple Interest being computed at 6 per Cent. per An. | |
| | | 11 8.251334 |
| | | 12 8.832729 |
| | | 13 9.394526 |
| | | 14 9.938004 |
| | | 15 10.464319 |
| | | 16 10.974523 |
| | | 17 11.469572 |
| | | 18 11.950341 |
| | | 19 12.437630 |
| | | 20 12.892175 |
| 1 | .943396 | 21 13.334652 |
| 2 | 1.836253 | 22 13.765686 |
| 3 | 2.683710 | 23 14.175524 |
| 4 | 3.490161 | 24 14.585360 |
| 5 | 4.259391 | 25 14.985360 |
| 6 | 4.994685 | 26 15.375985 |
| 7 | 5.698900 | 27 15.757664 |
| 8 | 6.374575 | 28 16.120798 |
| 9 | 7.023925 | 29 16.485761 |
| 10 | 7.648925 | 30 16.842904 |

The Use of the foregoing Table III.

In the foregoing third Table, in the left hand Column under the Title of years, are expressed all the integral numbers, from one to 30, which signifie so many years, and the numbers in the Right hand Column which are placed against the number of years are pounds, and decimal parts

parts of a pound sterling, and every one of them are the present worth of 1 pound Annuity to continue so many years to come as are placed against them in the Collum of years, Rebate being allowed at Simple Interest 6 per Cent. per An.

As, suppose there were a Lease of 20 shillings per annum to continue 6 years, to be sold for present Money, allowing the buyer Rebate at 6 per Cent. per Annum Simple Interest. I desire to know how much is its present worth? To answer this, I look in the Collum of years for 6, and in the next Collum on the Right hand just against 6 you have 4.994685 l. = 4 l. — 19 s. — 10 $\frac{1}{4}$ d. which is the answer to the Question. And by the help of this Table may the present worth of any Annuity to continue any number of years under 31 be found out, allowing Rebate at 6 per Cent. per Annum, Simple Interest, by one single Rule of 3 Direct, according to the manner of solving the following question, viz.

Quest. 1.

There is a Lease of 18 years yet to come, of the yearly value of 130 l. to be sold for ready Money, and the purchaser is to be allowed Rebate after the rate of 6 per Cent. per Annum, Simple Interest, now I demand how much is the present worth of this Lease?

Facit 1553 l. — 10 s. — 10 $\frac{1}{4}$ d.

First, I look in the Table for 18 years, and over against it on the right hand I find 11.950341 which is the present worth of 1 pound annuity to continue 18 years, &c. Therefore by the Rule of 3 Direct, I say

This Question is but the converse of the former, and may be thus solved, viz. The Tabular number for 18 years at 6 per Cent. per Annum is 11.950341 *l.* which is multiplied by the given annuity 130 *l.* and the Quotient will give you the sum which it will purchase, viz.

$$\begin{array}{r} 11.950341 \\ \times 130 \\ \hline 1553.544330 \end{array}$$

l. s. d.
1553.544330 = 1553 — 10 — 10 $\frac{3}{4}$.

So that by the work you find the answer to be 1553.544 *l. &c.* or 1553 *l.* — 10 *s.* — 10 $\frac{3}{4}$ *d.* very near, which said answer is nothing else but the product of the Tabular number, (11.950341 *l.*) multiplied by the given annuity (130 *l.*) For it is evident, that if the present worth of 1 pound annuity to continue 18 years be 11.950341 *l.* then the present worth of 130 *l.* per annum to continue the same number of years (and Rebate being allowed at the same rate per Cent. per An. for the one as for the other) must be 130 times as much. But when rebate is to be allowed after any other rate then 6 per Cent. per Annum, then the foregoing Table will not at all be useful, but you must have recourse to a Table calculated for the same rate of interest, which you may easily perform at leisure by the foregoing rules.

Quest. 2.

What Annuity to continue 18 years will 1553.5443 30 purchase, allowing the Buyer Simple interest at 6 per Cent. per Annum?

Facit 130 *l.*

This

This Question is but the converse of the former, and may be thus resolved, *viz.* Take the Tabular number corresponding to 18 years, which is 11.950341 by which divide the given purchase Money, and the Quotient will give you the annuity that it will purchase, *viz.*

$$\begin{array}{ccccccc} & l. & & l. & & l. & \\ 11.950341 &) & 1553.54433 & (& 130 \end{array}$$

So that by the work I find it will purchase an Annuity of 130 *l.* to continue 18 years.

The reason of the work is plain, for if the Tabular number correspondent to 18 years be the present worth of 1 *l.* Annuity to continue 18 years to come, then it is certain that so much Money as is expressed by that Tabular number, will purchase an Annuity of 1 *l.* to continue 18 years: And consequently we may find by help of the said Table what annuity any other sum of Money will purchase to continue any number of years not exceeding 30, by a single Rule of 3 Direct, as in the last Question, the proportion is as followeth, *viz.*

$$\begin{array}{ccccccc} & l. & & l. & & l. & & l. \\ 11.950341 & : & 1 & : : & 1553.54433 & : & 130 \end{array}$$

And it is no more in effect than a sum in Division, for the second number (being 1) neither multiplyeth nor divideth, &c.

By what hath been said concerning the use of the foregoing Table, you may perceive that the present worth of an Annuity is found out by multiplication, and to know what annuity any sum will purchase is performed by Division.

I might have made Tables for other Rates of Interest, but Simple Interest being seldom allowed in the purchasing or valuing of Leases and Annuities, they being generally purchased at Compound Interest, or Interest upon Interest, makes me forbear, and indeed at Simple Interest a Lease is over-valued.

CHAP.

CHAP. XIII. Of Compound Interest.

I. **W**Hat hath been said in the last Chapter, I judge sufficient for the understanding of the Nature and Use of Simple Interest, and that being well understood, the nature of Compound Interest will not seem difficult to the studious Learner, and the better he is acquainted with the nature of Simple Interest, so much the easier will he come to the knowledge of the nature, and use of Compound Interest.

II. Compound Interest is, when a sum of Money is put out to Interest, and the Interest thereof becoming due is still continued in the hands of the Debtor, so as to become part of the principal, interest being reckoned for it from the time it becometh due, for which reason it is called interest upon interest: And as Simple Interest increaseth by a series of Arithmetical proportionals continued; so doth Compound Interest increase by a rank or series of continual Geometrical proportionals. For when a sum of Money is put out to interest at any rate *per Cent. per Annum*, (as suppose a 100 *l.* to be put out to receive at the end of one year 6 *l.* for its interest) it is evident that if the interest (being 6 *l.*) be continued in the hands of the Debtor, there will be at the end of the second year the increase
of

of 106 *l.* which is 112.36 *l.* and at the third years end there will be the increase of 112.36 *l.* so that every number proceedeth from that going before it, after the same rate or reason as 100 proceedeth from 100, as you see following.

| <i>l.</i> | <i>l.</i> | <i>l.</i> | <i>l.</i> |
|-----------|-----------|-------------|--------------|
| 100 | : 106 | :: 106 | : 112.36 |
| 100 | : 106 | :: 112.36 | : 119.1016 |
| 100 | : 106 | :: 119.1016 | : 126.247696 |

So that by the Augmentation of 100 *l.* in 4 years you have this rank of Geometrical proportionals continued, viz. 100, 106, 112.36, 119.1016 and 126.247696 which is in number 5, viz. more by one than is the number of years the last of which is the amount of 100 *l.* at 6 per Cent. for 4 years reckoning Compound Interest, or Interest upon Interest, and each of these proportionals proceedeth from that going before it as 106 proceedeth from 100, that is to say, every of the said proportionals, is in such proportion to that which goeth before it as 106 is to 100, or as 100 is to 106, so is any one of them, to that which followeth it, or if you take any 3 of them which are placed together, there is this proportion between them, viz. As the first of those three is to the second, so is the second to the third, and the third to the fourth, and the fourth to the fifth, and the fifth to the sixth, &c. whence it is evident that they have amongst themselves this following Qualification, viz. that the square of

any one of them is equal to the Rectangle, or Product, made by that which is placed immediately before it, and that immediately after it, and the same would it be if there were never so many Terms, and is a peculiar property of all numbers that are Geometrical proportionals continued

III. The Interest of 100 *l.* for a year being known, the Compound Interest of any other sum for any number of years may be likewise found out by so many single Rules of 3, as there are given years, for,

As 100 *l.* is to its increase for one year, so is any other sum to its increase for the same time, and so is the first years increase to the second, and the second years increase to the third, and so is the third years increase to the fourth, &c.

Example.

Let it be required to find how much 350 *l.* will be increased to, being put out to Interest at 6 per Cent. per Annum, Compound Interest for 5 years? Answer, 468 *l.*—7 *s.*—4 $\frac{1}{2}$ *d.* fere, See the following work.

$$\begin{array}{lcl}
 \text{350} & : & 371 \\
 \text{371} & : & 393.26 \\
 100 : 106 & : & 393.26 : 416.8556 \\
 & & 416.8556 : 441.866936 \\
 & & 441.866936 : 468.87895216
 \end{array}$$

Where-

Whereby you see that 350 l. being put out to interest after the rate of 6 per Cent. will at the first years end be increased to 371 l. And 371 l. being put out for the second year, will be increased to 393.26 l. and 393.26 l. being made a principal, and put out at the same rate for the third year, will at the end thereof be increased to 416.8556 l. and at the end of 5 years it will be increased to 468.37895216 l.

And upon the aforefaid Grounds is calculated the following Table 1, whose Construction and use immediately followeth the same.

| | | | | |
|----|----------|--------|--------|--------|
| 12 | 501605 | 330022 | 522003 | 311510 |
| 11 | 422003 | 250000 | 522223 | 502110 |
| 10 | 382004 | 513505 | 540004 | 511005 |
| 9 | 342282 | 501510 | 532310 | 521811 |
| 8 | 31141032 | 180833 | 510482 | 533003 |
| 7 | 282220 | 110000 | 120012 | 512805 |
| 6 | 222233 | 100000 | 118012 | 120000 |
| 5 | 171142 | 12030 | 111818 | 182203 |
| 4 | 140010 | 12030 | 100248 | 141385 |
| 3 | 131000 | 14182 | 120013 | 128081 |
| 2 | 151018 | 13323 | 140522 | 140035 |
| 1 | 131120 | 1305 | 121010 | 130018 |
| | 114203 | 11010 | 137204 | 132111 |
| | 110320 | 11330 | 111400 | 110010 |
| | 110000 | 100000 | 110000 | 100000 |
| | | 8. | 7. | 6. |

TABLE I.

Which sheweth what one Pound will amount to, being forborn to the end of any number of years to come, not exceeding 30 Compound Interest being computed at 5, 6, 7, 8, 9, or 10 per Cent. per Annum.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|---------|---------|---------|---------|---------|---------|
| 1 | 1.05000 | 1.06000 | 1.07000 | 1.08000 | 1.09000 | 1.10000 |
| 2 | 1.10250 | 1.12360 | 1.14490 | 1.16640 | 1.18810 | 1.21000 |
| 3 | 1.15762 | 1.19101 | 1.22504 | 1.25971 | 1.29502 | 1.33100 |
| 4 | 1.21550 | 1.26247 | 1.31079 | 1.36048 | 1.41158 | 1.46410 |
| 5 | 1.27628 | 1.33822 | 1.40255 | 1.46932 | 1.53862 | 1.61051 |
| 6 | 1.34009 | 1.41851 | 1.50073 | 1.58687 | 1.67710 | 1.77156 |
| 7 | 1.40710 | 1.50363 | 1.60578 | 1.71382 | 1.82803 | 1.94871 |
| 8 | 1.47745 | 1.59384 | 1.71818 | 1.85093 | 1.99256 | 2.14358 |
| 9 | 1.55532 | 1.68947 | 1.83845 | 1.99900 | 2.17189 | 2.35794 |
| 10 | 1.62289 | 1.79084 | 1.96715 | 2.15892 | 2.36736 | 2.59374 |
| 11 | 1.71033 | 1.89829 | 2.10485 | 2.33163 | 2.58042 | 2.85311 |
| 12 | 1.79585 | 2.01219 | 2.25219 | 2.51817 | 2.81266 | 3.13842 |
| 13 | 1.88564 | 2.13292 | 2.40984 | 2.71962 | 3.06580 | 3.45217 |
| 14 | 1.97993 | 2.26090 | 2.57853 | 2.93719 | 3.34172 | 3.79749 |
| 15 | 2.07892 | 3.39655 | 2.75903 | 3.17216 | 3.64248 | 4.17724 |

A Continuation of the preceding TABLE I.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|---------|---------|---------|----------|----------|----------|
| 16 | 2.18287 | 2.54035 | 2.95216 | 3.42594 | 3.97030 | 4.59497 |
| 17 | 2.29201 | 2.69277 | 3.15881 | 3.70001 | 4.32763 | 5.05447 |
| 18 | 2.40661 | 2.85433 | 3.37993 | 3.99601 | 4.71712 | 5.55991 |
| 19 | 2.52695 | 3.02559 | 3.61652 | 4.31570 | 5.14166 | 6.11590 |
| 20 | 2.65329 | 3.20713 | 3.86968 | 4.66095 | 5.60441 | 6.72749 |
| 21 | 2.78596 | 3.39956 | 4.14056 | 5.03383 | 6.10880 | 7.40024 |
| 22 | 2.92526 | 3.60353 | 4.43040 | 5.43654 | 6.65860 | 8.14027 |
| 23 | 3.07152 | 3.81975 | 4.74053 | 5.87146 | 7.25787 | 8.95430 |
| 24 | 3.22509 | 4.04893 | 5.07236 | 6.34118 | 7.91108 | 9.84973 |
| 25 | 3.38635 | 4.29187 | 5.42743 | 6.84847 | 8.62308 | 10.83470 |
| 26 | 3.55567 | 4.54938 | 5.80735 | 7.30935 | 9.39915 | 11.91818 |
| 27 | 3.73345 | 4.82234 | 6.21386 | 7.98806 | 10.24508 | 13.10999 |
| 28 | 3.92012 | 5.11178 | 6.64883 | 8.62710 | 11.16713 | 14.42099 |
| 29 | 4.11613 | 5.41838 | 7.11425 | 9.31727 | 12.17218 | 15.86309 |
| 30 | 4.32194 | 5.74349 | 7.61225 | 10.06265 | 13.26767 | 17.44940 |

The

The Construction of the foregoing TABLE I.

By the third Rule foregoing it is evident that the Interest of 100 *l.* for a year being known, the Compound Interest for any other sum may be found out for any number of years; According to which Rule all the numbers in the said Table are found out, being the amount of 1 *l.* at Compound Interest for any number of years, not exceeding 30, being put out at any of these Rates, *viz.* 5, 6, 7, 8, 9, or 10 *per Cent. per Annum*, which numbers are found out by the Rule of Proportion thus,

$$100 : 105 :: \left\{ \begin{array}{l} 1 : 1.05 \\ 1.05 : 1.1025 \\ 1.1025 : 1.157625 \\ 1.157625 : 1.21550625 \end{array} \right.$$

By which means the four first numbers in the second Column of the Table (being placed under the number 5) are found, and by a continuation of the same operation are all the rest of the numbers in that Column found out; which is indeed nothing else but a continual multiplication of the first number, (*viz.* 1.05,) into it self 29 times, and so the last number in that Column is the thirtieth power of 1.05, and the same Column may be continued to any other number of years at pleasure above 30; the numbers in this Column being the interest of 1 *l.* at 5 *per*

5 per Cent. per Annum Compound Interest for 30 years.

The numbers in the third Colum under the Figure 6, are the increase of 1 l. at 6 per Cent. per Annum. Comp. Int. for 30 years, and are found out by multiplying 1.06 into it self 29 times according to the Rule of Continual Multiplication. The like is to be understood of all the rest.

The use of the foregoing TABLE.

In the first Colum of the Table under the Title years, are expressed the number of years from 1 to 30, and in the second Colum under the figure 5, and against every respective year are expressed the increase of 1 l. being put out at 5 per Cent. per Annum, Compound Interest.

In the third Collum under the number 6 is expressed the yearly increase of 1 l. being put out at 6 per Cent. per Annum, Compound Int. And so in the 4, 5, 6, and 7 Collums, are the yearly amounts of 1 l. at 7, 8, 9, and 10 per Cent. per Annum, Compound Interest.

All which numbers in the said Table are multipliers, for the producing of the amount or Increase of any other sum being put out at Compound Interest, at any rate of Interest, and for any number of years therein expressed, as will appear by the following Examples.

Example 1.

I demand the full amount of 365 l. being put

to Interest for 9 years, Interest being Computed after the rate of 6 per Cent. per Annum, Compound Interest? Facit 616 l.—13 s.—01½ d.

Here because the sum proposed is put out at 6 per Cent. and for 9 years, I look in the Collum of 6 per Cent. which is the third Collum of the Table under the figure 6, and just against 9 in the Collum of years, I find 1.68947 which is the increase of 1 l. being forborn the same time, and at the same rate of Interest, wherefore by the Rule of 3 I say

$$\begin{array}{r}
 \text{l.} \qquad \text{l.} \qquad \text{l.} \qquad \text{l.} \\
 1 : 1.68947 :: 365 : 616.65655 \\
 \hline
 365 \\
 844735 \\
 1013682 \\
 506841 \\
 \hline
 616.65655
 \end{array}$$

So that by the work I find that if the sum of 365 l. be all forborn to the end of 9 years, and interest be computed for the same at 6 per Cent. per Annum, Compound Interest, it will then be increased to 616.65655 which is 616 l.—13 s. 1½ d.

Example 2.

What will 128 l.—16 s.—08 d. be increased to? The utmost improvement thereof being made for 15 years at 7 per Cent. per Annum, Compound Interest?

Facit 355 l.—09 s.—01 d.

First,

First, turn the 16 s.—8 d. into the Decimal of a pound by the 2d Rule of the 2d Chapter foregoing, and you will find it to be .8333, so that the given sum is 128.8333, &c.

Now to answer this question, I look into the foregoing Table, and in the Collum of 7 per Cent. and just against 15 years I find 2.75903 which is the uttermost increase of 1 l. for 15 years at 7 per Cent. Compound Interest, by which if you multiply the given sum, the product will be the answer to the question, as by the following work will plainly appear.

$$\begin{array}{r}
 1 : 2.75903 :: 128.8333 : \\
 \hline
 2.75903 \\
 \hline
 3864999 \\
 115949970 \\
 6441665 \\
 9018331 \\
 2576666 \\
 \hline
 355.454939699
 \end{array}$$

By the foregoing work the answer is found to be 355.4549, &c. = 355 l.—09 s.—01 d.

But if any sum be put out at Compound Int. for months, or days over and above the given number of years, then the work will be somewhat different from the former; for first you must find out the amount of the given sum, for the given number of years, and then by the 8th Rule of the foregoing Chapter find out the Interest of that amount for the odd time, being either months or dayes under a year, and that Int. being added to the aforesaid amount, that sum will

will be the answer to the question: this is so obvious that it needeth no Example.

IV. When a sum of Money due at the end of any number of years to come is to be satisfied with present money, allowing rebate at Compound Interest, there must be found a Rank or Series of continual proportionals, more in number by one than the number of years for which the discount is proposed, of which rank or series of proportionals, the sum to be satisfied by present payment must be the first, and the second must decrease from that after the same rate or proportion as 100 decreaseth from the sum of 100 added to its interest for one year, after the rate of Interest propounded; that is to say, as 100 proceedeth from 106, or 108 if the interest be 6 or 8 *per Cent.* and after the same rate or reason must the third decrease from the second, and the fourth from third, &c.

When a question is stated for the rebate of Money at Compound Interest, it is solvable by as many single Rules of 3, as the number of years for which the sum proposed is to be Rebated, and it is nothing else but the inverse of the third Rule of this Chapter, as may be proved by the working of the following question, taken out of the said Rule, where it is proved that 350 *l.* being forborn in the Debtors hands for 5 years at 6 *per Cent.* Compound Interest, it will then be increased to 468.38001216; now let the said Question be inverted thus, *viz.*

There is a sum of Money, *viz.* 468.38001216 due at the end of 5 years to come, now I demand how

how much present Money will satisfy the said Debt, rebate being allowed after the rate of 6 per Cent. per Annum, Compound Interest?

First, I say, as 106 is to 100, so is the sum due at the end of 5 years, viz. 468.38001216, to 441.867936, which is the sum due at the fourth years end, and so is the sum due at the fourth years end, to the sum due at the third years end, &c. as by the work appeareth.

| | | | |
|-----|--------------|---|------------|
| | l. | | l. |
| | 468.38001216 | : | 441.867936 |
| | 441.867936 | : | 416.8556 |
| 106 | : 100 | : | 416.8556 |
| | | : | 393.26 |
| | | : | 371 |
| | | : | 350 |

So that by the foregoing work you see that if 468.38001216 l. be due at the end of 5 years to come, and is to be satisfied by the payment of present money, rebate being allowed at 6 per Cent. per Annum, Compound Interest, 350 l. is the sum required.

And upon this Rule is grounded the Calculation of the following Table, which sheweth what 1 l. due at the end of any number of years to come, not exceeding 30 is worth in present Money, Rebate being reckoned at any of these rates, viz. 5, 6, 7, 8, 9, or 10 per Cent. per An. Compound Interest.

TABLE II.

Which sheweth the present worth of one pound payable at the end of any number of years to come, not exceeding 30, Rebate being yearly allowed at 5, 6, 7, 8, 9, or 10 per Cent. per Annum, Compound Interest.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|---------|---------|---------|---------|---------|---------|
| 1 | .952381 | .943396 | .934579 | .925925 | .917431 | .909090 |
| 2 | .907029 | .889996 | .873438 | .857338 | .841680 | .826446 |
| 3 | .863837 | .839619 | .816297 | .793832 | .772183 | .751314 |
| 4 | .822702 | .792093 | .762895 | .735029 | .708425 | .683013 |
| 5 | .783526 | .747258 | .712986 | .680583 | .649331 | .620921 |
| 6 | .746215 | .704960 | .666342 | .630169 | .596267 | .564474 |
| 7 | .710681 | .665057 | .622749 | .583490 | .547034 | .513158 |
| 8 | .676839 | .627412 | .582009 | .540268 | .501866 | .466507 |
| 9 | .644608 | .591898 | .543933 | .500248 | .460427 | .424097 |
| 10 | .613913 | .558391 | .508349 | .463193 | .422410 | .385543 |
| 11 | .584679 | .526787 | .475092 | .428882 | .387532 | .350494 |
| 12 | .556837 | .496989 | .444012 | .397113 | .355534 | .318630 |
| 13 | .530321 | .468839 | .414964 | .367697 | .326178 | .289664 |
| 14 | .505067 | .442300 | .387817 | .340461 | .299246 | .263331 |
| 15 | .481017 | .417265 | .362446 | .315241 | .274538 | .239392 |

A Continuation of the preceding TABLE II.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|---------|---------|---------|---------|---------|---------|
| 16 | .458111 | .393646 | .338734 | .291890 | .251869 | .217629 |
| 17 | .436296 | .371364 | .316574 | .270269 | .231073 | .197844 |
| 18 | .415520 | .350343 | .295864 | .250249 | .211993 | .179858 |
| 19 | .395733 | .330512 | .276508 | .231712 | .194489 | .163508 |
| 20 | .376889 | .311804 | .258419 | .214548 | .178430 | .148643 |
| 21 | .358942 | .294155 | .241513 | .198655 | .163698 | .135130 |
| 22 | .341849 | .277505 | .225713 | .183940 | .150181 | .122846 |
| 23 | .325571 | .261797 | .210947 | .170315 | .137781 | .111678 |
| 24 | .310067 | .246978 | .197146 | .157699 | .126405 | .101525 |
| 25 | .295302 | .232998 | .184249 | .146018 | .115967 | .092296 |
| 26 | .281240 | .219810 | .172195 | .135201 | .106392 | .083905 |
| 27 | .267848 | .207367 | .160930 | .125186 | .097607 | .076277 |
| 28 | .255093 | .195630 | .150402 | .115913 | .089548 | .069343 |
| 29 | .242946 | .184556 | .140562 | .107327 | .082154 | .063039 |
| 30 | .231377 | .174110 | .131367 | .099377 | .075371 | .057308 |

The

The Construction of the foregoing T A B L E.

By the Fourth Rule of this Chapter is plainly shewn the manner of finding the present worth of any sum of Money due at the end of any number of years to come, Rebate being computed at Compound Interest, and after the same manner are all the numbers in the foregoing Table found as you may see by the following Example, where the four first numbers in the third Column are methodically found out by the Rule, that being the Column of Rebate at 6 per Cent. per Annum,

| | | | | | | | | | | |
|-----|--|-----|---|-----|---|---|---|------------|---|-------------------|
| I | | 106 | : | 100 | : | : | : | 100 | : | 943396226415, &c. |
| II | | 106 | : | 100 | : | : | : | 9433962264 | : | .88999644, &c. |
| III | | 106 | : | 100 | : | : | : | 88999644 | : | .83961928, &c. |
| IV | | 106 | : | 100 | : | : | : | 83961928 | : | .79209460, &c. |

So that by the foregoing proportions, I say first, if 106 *l.* be decreased to 100 *l.* what will 1 *l.* be decreased to? Answer, to .94339 *l.* &c. = 18 *s.* 10 $\frac{1}{2}$ *d.* The five first figures thereof being the first number in the third Column of the foregoing Table, and it sheweth that the present worth of 1 *l.* due at the end of one year to come, Rebate being allowed at 6 per Cent. is 94339 *l.* = 18 *s.* — 10 $\frac{1}{2}$ *d.*

Secondly, I say, by the Rule of 3, If 106 *l.* be decreased to 100 *l.* what will .94339, &c. be decreased to? The answer is .88999 *l.* &c. = 17 *s.* 09 $\frac{1}{2}$ *fere.* And this is the second number in the third

third Column of the said Table, and is placed against 2 years in the first Column, and sheweth the present worth of 1 *l.* due at the end of two years to come, Rebate being allowed after the Rate of 6 per Cent. per Annum, Compound Interest.

And after the same manner are all the rest of the numbers in the said Column of 6 per Cent. found out, and also all the other Decimal Fractions in the second, fourth, fifth, sixth, &c. Columns, shewing the Rebate of one pound for any number of years not exceeding 30, at 5, 7, 8, 9, and 10 per Cent. (*mutatis mutandis.*)

The use of the foregoing TABLE II.

The first Column is the number of years for the Rebate of 1 *l.* and the numbers in the rest of the Columns, are Decimal Fractions shewing the present worth of 1 *l.* due at the end of so many years to come, as they are placed against in the Column of years, Rebate being allowed at the same rate of interest under which they are placed, the figures 5, 6, 7, 8, 9, and 10 placed at the top, denoting the same. An Example or two will make its use more plain.

Example 1.

I demand how much present Money will satisfy a Debt of 684 *l.* due at the end of 6 years to come, allowing rebate after the rate of 8 per Cent. per Annum, Compound Interest? To answer this Question, look in the Column of 8 per Cent. and against 6 years I find this number, viz. 63017 which sheweth that if 1 *l.* be due at the end of 6 years-

6 years to come, its present worth is .63017 *l.* Rebate being allowed after the rate of 8 per Cent. per Annum. Compound Interest.

Therefore I say by the Rule of 3, If 1 *l.* be decreased to .63017 *l.* what will 684 *l.* be decreased to at that rate? *Facit* 431.03628 *l.* = 431 *l.* 0 *s.* 8 $\frac{1}{2}$ *d.* as by the work appeareth.

$$\begin{array}{ccccccc} \textit{l.} & & \textit{l.} & & \textit{l.} & & \textit{l.} \\ 1 & : & .63017 & : & 684 & : & 431.03628 \\ & & 684 & & & & \end{array}$$

$$\begin{array}{r} 252068 \\ 504136 \\ 378102 \end{array}$$

$$431.03628$$

$$\textit{Facit} \quad \begin{array}{ccc} \textit{l.} & \textit{s.} & \textit{d.} \\ 431 & -00 & -08 \frac{1}{2} \textit{ fere} \end{array}$$

So that you see the sum proposed being multiplied by the proper Tabular number, produceth the answer to the question, for the number 1, which is here the first number in the Rule of 3 doth not either multiply or divide, and therefore the answer is found out by the multiplication only. Observe the work of the next Example.

Example.

What is the present worth of 164 *l.*—15 *s.* due at the end of 9 years to come, allowing Rebate after the rate of 6 per Cent. per Annum, Compound Interest? *Facit* 97 *l.*—10 *s.*—03 $\frac{1}{4}$.

Look in the Table aforesaid in the Collum of 6 per Cent. and against 9 in the Collum of years you will find this number, viz. .59189 which is the present worth of 1 *l.* due at the end of 9 years

years to come, and is the proper multipliar for finding the answer to this Question, as by the work.

$$\begin{array}{r}
 l. \quad 1. \quad 1. \quad 1. \\
 1 : .59189 :: 164.75 : 97.5138775 \\
 \hline
 295945 \\
 414323 \\
 236756 \\
 355134 \\
 59189 \\
 \hline
 97.5138775
 \end{array}$$

The answer found by the foregoing operation is $97.5138775 = 97 \text{ l.} - 10 \text{ s.} - 03 \frac{1}{4}$.

But if the given time for the Rebate of any sum consisteth of odd Months or Days, besides years, then in such case, the Rebate (at the given Rate of Interest) for the odd time must be found by the 9th. Rule of the 12 Chap. foregoing) for the given sum, and then the present worth of the given sum thus decreased, must be found for the number of years as in the two last Examples.

Example 3.

There is $640 \text{ l.} - 10 \text{ s.}$ due at the end of six years, and 3 months to come, what is its present worth, Rebate being allowed at the Rate of 7 per Cent. per Annum, Compound Interest?

First, I find the decrease of 640.5 l. for Months thus, viz.

$$\begin{array}{cccc} \text{mon.} & \text{l.} & \text{mon.} & \text{l.} \\ 12 & : 7. & : 3 & : 1.75 \end{array}$$

So that I find the Int. of 100 l. for 3 Months at 7 per Cent. to be 1.75 l. which added to 100 l. makes 101.75, then to find the decrease of 640.5 l. for 3 Months: I say,

$$\begin{array}{ccccccc} \text{l.} & & \text{l.} & & \text{l.} & & \text{l.} \\ 101.75 & : & 100 & :: & 640.5 & : & 629.484 \end{array}$$

So that I find by the last proportion, that if at the end of 6 years 3 months: There was due 640.5 l. yet at the end of 6 years there will be due but 629.484 l. whose present worth by the foregoing directions will be found to be 419 l. - 9 s. for,

$$\begin{array}{ccccccc} \text{l.} & \text{l.} & \text{l.} & \text{l.} & \text{l.} & \text{s.} \\ 1 & : .66634 & :: & 629.484 & : & 419.450 \text{ \&c.} = 419 - 9 \end{array}$$

Read the 9th. Rule of the Twelfth Chapter foregoing, and you will easily understand the method here used for solving Questions of this nature.

V. Questions in Rebate at Compound Interest may be resolved by the First Table of this Chapter which sheweth the increase of 1 l. at Compound Interest, &c. But as in the second Table you make the Tabular numbers multipliers, to find out the present worth of a sum; so if

if you would find out the present worth of a sum by the first Table, you must then make those Tabular Numbers Divisors; the Reason whereof is plain, for the first Table sheweth the increase of 1 *l.* for 30 years, &c. But they may likewise serve to shew what sum of Money due at the end of any number of years to come under 31 (allowing Rebate according to the rates of Interest therein mentioned) 1 *l.* present Money will satisfy. Now to Resolve Questions in Rebate by this Table, look in the Collum of the proposed Interest or Rebate, and against the proposed number of years is the Tabular number for your work, which must be according to the following proportion, viz.

As the Tabular Number so found,

Is to 1,

So is the sum proposed to be Rebated,

To its present worth.

To make this a little more plain, I shall Answer the first Question in the Use of the second Table, by the help of the first Table only, which is as followeth, viz.

I demand how much present Money will satisfy a Debt of 684 *l.* due at the end of 6 years to come, allowing Rebate after the rate of 8 per Cent. per Annum, Compound Interest?

Look in Table 1. in the Collum of 8 per Cent. and against 6 years you will find this number, viz. 1.58687, therefore the proportion is as followeth.

$$\begin{array}{ccccccc} \text{l.} & & \text{l.} & & \text{l.} & & \text{l.} \\ 1.58687 & : & 1 & :: & 684 & : & 431.03719 \end{array}$$

So that by this proportion the answer is 431.03719 *l.* = 431 *l.* — 00 — 08 $\frac{3}{4}$ *d.* very near to the answer before found by the second Table of Rebate.

VI. When an annuity is in arrear, and it is required to know its utmost improvement, accounting Interest upon Interest for each particular sum from the time it becomes due, to the end of the given Term of years. The manner how to work such Questions will be apparent by the working of the following Question, *viz.*

There is an Annuity of 150 *l.* to continue to the end of five years, and the utmost improvement thereof to be made after the rate of 6 *per Cent. per Annum*, Compound Interest; now I demand how much will then be due to the Creditor?

It is evident that there must be found out, first the amount of 150 *l.* for one year, *viz.* that which is due at the end of the fourth year, it lying in the Debtors hands all the fifth year.

Secondly, There must be accounted the improvement of 150 *l.* for 2 years, *viz.* that which is due at the end of the third year, it lying in the Debtors hands the fourth, and fifth years.

Thirdly,

Thirdly, There must be accounted the improvement of 150 *l.* for 3 years, *viz.* that which is due at the end of the second year, it lying in the hands of the Debtor the third, fourth, and fifth years.

And in the fourth place there must be accounted the utmost improvement of 150 *l.* for 4 years, *viz.* that which is due at the end of the first year, it lying in the Debtors hands the second, third, fourth, and fifth years.

And besides there must be accounted 150 *l.* due at the end of the fifth year, no Interest being reckoned for that, because it becometh not due till the expiration of the last year, and then the sum of all these is the utmost amount of that annuity.

The solving of Questions concerning Annuities at Compound Interest, will not be any thing different in their operation, from the manner of solving a Question concerning a single sum of money put out for years at Compound Interest, by the third Rule before-going. As suppose that instead of an Annuity of 150 *l.* there was a single sum of 150 *l.* put out for 4 years at Compound Interest, at 6 per Cent. what would be its utmost improvement at the end of the said Term?

Here you will easily perceive that in solving the one, the other is also solved.

See the work according to the foregoing third Rule.

| | | | | | |
|------------|-----------|-----|----------|---|------------|
| | | | 150 | : | 159 |
| <i>l.</i> | <i>l.</i> | | 159 | : | 168.54 |
| 100 | : | 106 | : | : | 178.6524 |
| | | | 178.6524 | : | 189.361544 |
| | | | | : | 150 |
| <hr/> | | | | | |
| 845.553944 | | | | | |

Now if the foregoing proportions be well considered, you will find that

The sum due at the end of } *l.*
the fifth year, being that years } 150
Rent is —————

And 150 *l.* due at the end of the } *l.*
fourth year, will at the fifth years } 159
end be encreased to —————

And 150 *l.* due at the end of the } *l.*
third year, will at the end of the } 168.54
fifth year be increased to —————

And 150 *l.* due at the second } *l.*
years end, will at the fifth years } 178.6524
end be increased to —————

And 150 *l.* due at the first } *l.*
years end, will at the fifth years } 189.371544
end be increased to —————

The sum of all these being } *l.*
due at the five years end is — } 845.553944

So

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So that if an Annuity of 150 *l.* be all forborn to the end of five years, and it be improved to the utmost after the rate of 6 per Cent. per Annum, Compound Interest, it will then be increased to the sum of $845.553944 = 845 \text{ l. } 11 \text{ s. } 0 \frac{1}{2} \text{ d.}$

Now if the particular numbers in finding out the augmentation of the said Annuity according to the manner before prescribed, be well viewed, and the method in finding them out be well considered, it will appear, that if an Annuity, payable by yearly payments, be all forborn to the end of any number of years, and the utmost improvement thereof be made at Compound Interest, the total then due at the end of the said time, or term of years, will be the sum of a series, or Rank of continual proportionals as many in number as the years of the Annuities forbearance, the first being the Annuity, or yearly payment it self, and the second proceeding from the first after the same Rate or proportion as 100 *l.* and its Interest for a year added together, proceedeth from 100 *l.* and after the same Rate doth the third proceed from the second, and the fourth from the third, &c.

N 4 The

The manner of Calculating the following Third TABLE.

And upon this Rule is grounded the Calculation of the following Table, which sheweth what 1 *l.* Annuity (being forborn to the end of any number of years to come, not exceeding 30) will be increased to Compound Interest, being computed after any of the Rates mentioned at the head of the Table.

But considering that as an Annuity increaseth yearly at Compound Interest, the sum due at each years end, is the sum of a series of continual proportionals equal in number to the yearly payments, and that the first number is the annual payment its self, therefore may a Table to shew the Annual increase of 1 *l.* Annuity with great ease be made from the first Table, shewing the yearly increase of 1 *l.* at Compound Interest, as will plainly appear by what followeth.

Let us pitch upon making the Collum of 6 *per Cent. per Annum*, in the third Table? Look in the first Table, and you will find the Collum of 6 *per Cent.* to have for its first number 1.06, and the second number 1.12360 &c. And to make the Collum of 6 *per Cent.* in the third Table proceed thus, for the first number in the said third Table put 1, or 1.00000, and for the second number in the third Table, take the sum of the first number in the third Table (which is 1.00000,) and the first number in the first Table (which is 1.06) and that makes 2.06 for the

| Years | TABLE III. Which sheweth in Pounds and Decimal parts of a Pound the increase or Amount of 1 l. Annually, to continue any number of Years not exceeding 30; Compound Interest being computed at 5, 6, 7, 8, 9, and 10 per Cent. per Annum. | | | | | |
|-------|--|----------|----------|----------|----------|----------|
| | 5. | 6. | 7. | 8. | 9. | 10. |
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 2.05000 | 2.06000 | 2.07000 | 2.08000 | 2.09000 | 2.10000 |
| 3 | 3.15250 | 3.18360 | 3.21490 | 3.24640 | 3.27810 | 3.31000 |
| 4 | 4.31012 | 4.37461 | 4.43994 | 4.50611 | 4.57312 | 4.64100 |
| 5 | 5.52563 | 5.63709 | 5.75073 | 5.86660 | 5.98471 | 6.10510 |
| 6 | 6.80191 | 6.97531 | 7.15329 | 7.33592 | 7.52333 | 7.71561 |
| 7 | 8.14200 | 8.39383 | 8.65402 | 8.92280 | 9.20043 | 9.48717 |
| 8 | 9.64910 | 9.89746 | 10.25980 | 10.63662 | 11.02847 | 11.43588 |
| 9 | 11.02656 | 11.49131 | 11.97798 | 12.48755 | 13.02103 | 13.57947 |
| 10 | 12.57789 | 13.18079 | 13.81644 | 14.48656 | 15.19292 | 15.93742 |
| 11 | 14.20678 | 14.97164 | 15.78359 | 16.64548 | 17.56029 | 18.53116 |
| 12 | 15.91712 | 16.86994 | 17.88845 | 18.97712 | 20.14071 | 21.38428 |
| 13 | 17.71298 | 18.88213 | 20.14064 | 21.49529 | 22.95338 | 24.52271 |
| 14 | 19.59863 | 21.01506 | 22.55048 | 24.21492 | 26.01918 | 27.97498 |
| 15 | 21.57856 | 23.27596 | 25.12902 | 27.15211 | 29.36091 | 31.77248 |

A Continuation of the preceding TABLE III.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|----------|----------|----------|-----------|-----------|-----------|
| 16 | 23.65749 | 25.67252 | 27.88805 | 30.32428 | 33.00339 | 35.94972 |
| 17 | 25.84036 | 28.21287 | 30.84021 | 33.75022 | 36.97370 | 40.54470 |
| 18 | 28.13238 | 30.90565 | 33.45024 | 37.45024 | 41.30133 | 45.59917 |
| 19 | 30.53900 | 33.75999 | 37.44625 | 41.44626 | 46.01845 | 51.15909 |
| 20 | 33.06595 | 36.78559 | 40.76196 | 45.76196 | 51.16011 | 57.27499 |
| 21 | 35.71925 | 39.99272 | 44.86517 | 50.42292 | 56.76453 | 64.00249 |
| 22 | 38.50521 | 43.39228 | 49.00573 | 55.45675 | 62.87333 | 71.40274 |
| 23 | 41.43047 | 46.99582 | 53.43614 | 60.89329 | 69.53193 | 79.54302 |
| 24 | 44.50199 | 50.81557 | 58.17667 | 66.76475 | 76.78981 | 88.49732 |
| 25 | 47.72709 | 54.86451 | 63.24903 | 73.10593 | 84.70089 | 98.34705 |
| 26 | 51.11345 | 59.15638 | 68.67646 | 79.95441 | 93.32397 | 109.18176 |
| 27 | 54.66912 | 63.70576 | 74.48382 | 87.35076 | 102.72313 | 121.09994 |
| 28 | 58.40258 | 68.52810 | 80.69769 | 95.33882 | 112.96821 | 134.20993 |
| 29 | 62.32271 | 73.63979 | 87.34652 | 103.96593 | 124.13535 | 148.63092 |
| 30 | 66.43884 | 79.05818 | 94.46078 | 113.28321 | 136.30753 | 164.49402 |

The

The Use of the third TABLE:

The numbers 5, 6, 7, 8, 9, 10 at the head of the Table are the several Rates of Interest, of 100 *l.* for a year, and the numbers placed in the several Collums under those numbers, shew the yearly increase of 1 pound Annuity, at the same Rate of Interest as it is placed under, and for so many years as it is placed against in the Collum of years on the left hand of the Table; and the use of these numbers will be manifest by the method used in solving the following Question, *viz.*

There is an Annuity of 34 *l.*—8 *s.* payable by yearly payment, forborn unto the end of Twelve years; Now, I demand how much is due at the end of the said Term, Compound Interest being allowed at 6 *per Cent. per Annum*?
Facit 580 *l.*—6 *s.*—6 *d.* and somewhat more as will appear by the following operation.

The increase of the said Annuity being proposed at 6 *per Cent.* I look in the Collum which hath the number 6 placed at the head of it, and against the number 12 in the Collum of years I find the number 16.86994 which sheweth that if 1 *l.* Annuity be forborn to the end of 12 years, and there be allowed Compound interest at 6 *per Cent.* it will then be increased to 16.86994 = 16 *l.*—17 *s.*—04 $\frac{1}{2}$ *d.* therefore I say by the Rule of Proportion.

1 : 16.86994 : : 344 : 580.324936

344

6747976

6747976

5060982

580325936

Whereby it is apparent that those tabular numbers are only Multipliers for the producing of the amount of any given annuity for any number of years not exceeding 30, any Rate of Compound Interest, being allowed from 5 to 10 per Cent. Inclusive, &c.

VII. Questions concerning the increase of Annuities at Compound Interest may be likewise solved by the first Table in this Chapter, according to the following method, viz.

When an Annuity is in arrear, and it is required to know to what sum it is augmented, Compound Interest being computed, &c. Find out what principal will in one year gain the Annual Rent proposed, allowing the proposed Rate of Interest. Then (as is taught in the use of the said first Table) find the increase of the said principal for the number of years, and at the rate of interest proposed, and from the amount thereof subtract the said principal, then will that Remainder be the amount of the given Annuity for the given time, as will appear by solving the first Question of the sixth Rule foregoing

foregoing, which is this, *viz.* there is an Annuity of 150 *l.* forborn to the end of 5 years what is its amount at 6 per Cent. per Annum, Compound Interest?

Now to answer this, I find out a principal that at 6 per Cent. will gain 150 *l.* in one year, which I do by the following proportion, *viz.*

$$\begin{array}{cccc} l. & l. & l. & l. \\ 6 & : 100 & :: 150 & : 2500 \end{array}$$

So that I find 2500 *l.* to be the answer, then supposing the said principal 2500 *l.* to be put out to interest at 6 per Cent. Compound Interest for 5 years, look in the first Table in the Column of 6 per Cent. and against 5 years you will find 1.338225, &c. which being multiplied by 2500, produceth 3345.563944 from which if you subtract the said principal 2500 *l.* there will remain 845.563944 for the answer which is the same with that found before.

VIII. When an Annuity to continue any number of years is to be bought with ready money, there ought to be paid so much money, as being put out at Compound Interest, at any Rate, and for the time of the Leases continuance, its total amount may be equal to the utmost improvement of the said Annuity, being all forborn to the time of the Leases expiration, Compound Interest being Com-

puted at the same rate. And the manner of finding out such a present worth, is as in the following Example,

viz.

The manner of finding the present worth of annuities, Rebate being allowed at Comp. Int.

viz. There is an Annuity of 468.38001216 l. to continue 5 years what is its present worth, allowing Rebate after the rate of 6 per Cent. per Annum, Compound Interest.

Here it is plain that there must first be computed the present worth of the said Annuity, due at the end of the first, second, third, fourth and fifth years, and the sum of all these present worths, will be the present worth of the said annuity, as will appear by the following work which is wrought by the fourth Rule of this Chapter.

The present worth of } l.
468.38001216 l. due at the end } 441.867936
of the first year is-----}

The same sum due at the }
end of two years, is in ready } 416.8556
money worth. -----}

The same sum due at the }
end of three years is worth-----} 393.26

The same sum due at the end }
of four years is worth-----} 371

The same due at the end }
of five years is worth in ready } 350
money. -----}

The sum of the said present }
worths is-----} 1972.983536

Which is the present worth of an Annuity of 468.3001216 to continue 5 years Rebate being al-

allowed at the Rate of 6 per Cent. per Annum, Compound Interest.

The Construction of the following TABLE IV.

And upon the same grounds with the solution of the last Question is calculated, the following fourth Table, which sheweth the present worth of 1 *l.* Annuity to continue any number of years, not exceeding 30, and payable by yearly payments, Rebate being allowed after the rate of 5, 6, 7, 8, 9, and 10 per Cent. per An. Compound Interest.

But the nature of the following Table being rightly considered, you will find the making of it to be easily performed by help of the numbers in the second Table of this Chapter.

As for Example.

Let us pitch upon the making of the Collum of 6 per Cent. First, I turn to the second Table, and by the numbers in the Collum of 6 per Cent. I do the work ; The first number in the second Table, I make to be the first number in the fourth, and to that fourth I add the second number in the second Table, and their sum is the second number in the fourth Table ; then to this second number do I add the third number in the second Table, and their sum is the third number

ber in the fourth Table and after the same manner are all the rest of the numbers in that Column made, and also those in the rest of the Columns, *mutatis mutandis*.

But remember when ever you Calculate one Table by the help of another, to continue the Table you make use of, to more places than you intend the numbers in your Table to consist of for fear of errors through the Addition of Defective Decimals.

O T T A B L E

TABLE IV.

Which sheweth the present worth of one pound Annuity payable by yearly payment, and to continue any number of years not exceeding 30, Rebate being allowed at 5, 6, 7, 8, 9, or 10 per Cent. per Annum, Compound Interest.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|----------|---------|---------|---------|---------|---------|
| 1 | .95238 | .9433 | .93457 | .92952 | .91743 | .90909 |
| 2 | 1.85941 | 1.83339 | 1.80801 | 1.80801 | 1.75911 | 1.73553 |
| 3 | 2.72324 | 2.67301 | 2.62431 | 2.62431 | 2.53129 | 2.48685 |
| 4 | 3.54595 | 3.46510 | 3.38721 | 3.38721 | 3.23971 | 3.16986 |
| 5 | 4.32947 | 4.21236 | 4.10019 | 3.10019 | 3.88965 | 3.79078 |
| 6 | 5.07569 | 4.91732 | 4.76653 | 4.76653 | 4.48591 | 4.35526 |
| 7 | 5.78637 | 5.58233 | 5.38928 | 5.38928 | 5.03295 | 4.86841 |
| 8 | 6.46321 | 6.20979 | 5.97129 | 5.97129 | 5.53481 | 5.33492 |
| 9 | 7.10782 | 6.80169 | 5.51529 | 6.51523 | 5.99524 | 5.75901 |
| 10 | 7.72173 | 7.36008 | 7.02358 | 6.02358 | 6.41765 | 6.14456 |
| 11 | 8.30641 | 7.88687 | 7.49867 | 7.49867 | 6.80519 | 6.49506 |
| 12 | 8.86325 | 8.38384 | 7.94268 | 7.94268 | 7.16072 | 6.81369 |
| 13 | 9.39357 | 8.85268 | 8.85765 | 7.35765 | 7.48690 | 7.10335 |
| 14 | 9.89864 | 9.29498 | 8.74546 | 8.74546 | 7.78614 | 7.36668 |
| 15 | 10.37965 | 9.71224 | 9.10791 | 9.10791 | 8.06068 | 7.60608 |

A Continuation of the preceding TABLE IV.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|----------|----------|----------|----------|----------|---------|
| 16 | 10.83776 | 10.15089 | 9.44664 | 8.85136 | 8.31255 | 7.82371 |
| 17 | 11.27406 | 10.47725 | 9.76322 | 9.12163 | 8.54363 | 8.02155 |
| 18 | 11.68958 | 10.82760 | 10.05908 | 9.37188 | 8.75562 | 8.20141 |
| 19 | 12.08531 | 11.15811 | 10.33559 | 9.60359 | 8.95011 | 8.36492 |
| 20 | 12.46220 | 11.46992 | 10.59401 | 9.81814 | 9.12854 | 8.51356 |
| 21 | 12.82115 | 11.76407 | 10.83557 | 10.01681 | 9.29224 | 8.64869 |
| 22 | 13.16300 | 12.04158 | 11.06124 | 10.20074 | 9.44242 | 8.77154 |
| 23 | 13.48857 | 12.30337 | 11.27218 | 10.37105 | 9.58020 | 8.88329 |
| 24 | 13.79864 | 12.55035 | 11.46933 | 10.52875 | 9.70661 | 8.98474 |
| 25 | 14.09394 | 12.78335 | 11.65358 | 10.67477 | 9.82257 | 9.07704 |
| 26 | 14.37518 | 13.00316 | 11.82577 | 10.80997 | 9.92897 | 9.16094 |
| 27 | 14.64303 | 13.21053 | 11.98671 | 10.93516 | 10.02657 | 9.23722 |
| 28 | 14.89812 | 13.40616 | 12.13711 | 11.05107 | 10.11612 | 9.30656 |
| 29 | 15.14071 | 13.59071 | 12.27767 | 11.15840 | 10.19828 | 9.36960 |
| 30 | 15.39244 | 13.76482 | 12.40904 | 11.25778 | 10.27365 | 9.42691 |

The Use of the foregoing TABLE.

The first Collum is the number of years from 1 to 30, and the number 5, 6, 7, 8, 9 10, at the head of the Table are the Rates of Interest of 100 l. for a year, and the numbers in each of these Collums under the said rates of interest are the present worths of 1 l. Annuity to continue for the number of years which is placed against them, allowing Rebate after the rate of Interest at the head of each Collum, and are multipliers serving to find the present worth of any other Annuity, as will appear by the following

Example.

There is an Annuity of 48 l. to continue 12 years, and payable by yearly payments, to be sold for present money, I demand what it is worth, allowing Rebate at 6 per Cent. per Annum, Compound Interest? Facit $402.424 = 40 \text{ l. } 8 \text{ s. } 10 \text{ s. } 3 \text{ d.}$ which is thus found out by the foregoing Table, viz. look in the said Table, in the Collum of 6 per Cent. and against 12 in the Collum of years, you have this number, viz. 8.38384, which is the present worth of 1 l. Annuity to continue twelve years, Rebate being allowed, &c. therefore by the Rule of proportion, I say

Then by the second Table I find the present worth of 402.42432 which is 402.42432 : 48 :: 8.38384 : 6707072
 402.42432
 48
 6707072
 3353536

Moreover by the same Table I find the present worth of 402.42432 which is 402.42432 : 48 :: 8.38384 : 6707072
 402.42432
 48
 6707072
 3353536

So that I find the answer to be 402.42432, which is found by multiplying the said Tabular number by 48; as you see by the work.

Otherwise find a principal which may bear such proportion to the given Annuity that is to be Rebated) as 100 beareth to the Rate of Interest allowed in the Rebate. Then find the present worth of this principal so found, by the Directions given in the use of the second Table of this Chapter, then subtract the said present worth from the principal found as before, and the remainder will be the present worth of the given Annuity, Rebate being allowed as proposed.

Example.

What is the present worth of an Annuity of 50 l. to continue 3 years, allowing Rebate at 8 per Cent. per Annum, Compound Interest?

First, I find a principal that shall be to the given number 50 as 100 is to 8 which I find to be 625 l. by the following proportion, viz.

l. l. l. l.
 8 : 100 :: 50 : 625

O 3

Then

Then by the second Table I find the present worth of 625 *l.* which is 496.145 *l.* which I subtract from the said principal 625 *l.* and there remaineth 128.855 *l.* = 128 *l.* — 17 *s.* — 1 $\frac{1}{4}$ *d.* *vere* which is the present worth of 50 *l.* *per Annum*, to continue 3 years, Rebate being allowed at 8 *per Cent. per Annum*, Compound Interest.

Moreover by the numbers in the foregoing fourth Table, you may at first sight discover how many years purchase any Lease to continue any number of years, not exceeding 30 is worth in ready money, Compound Int. being Computed on both sides at any of the rates mentioned at the head of the Table.

Example:

Suppose there were a Lease issuing out of Lands to continue 16 years to be sold for ready money, allowing Rebate at 8 *per Cent. per Annum*, Compound Interest, I demand how many years purchase the said Lease is worth?

Look in the Table 4, in the Collum of 8 *per Cent.* and against 16 years you will find 8.85136 which sheweth that it is worth 8.85136 years purchase which is somewhat above 8 years, and 3 quarters; But if the said Lease had been of Houses, and 10 *per Cent.* were thought a convenient allowance for the same, then you will find it to be worth 7.82371 years purchase which is 7 years, and above 3 quarters purchase.

IX. When there is a sum of money propounded, and it is required to know what annuity to con-

continue any given number of years, it will purchase according to any given Rate of Interest, you may suppose any annuity at pleasure, then by the directions *Of the purchase of Annuities at Comp. Interest.* given in the use of the fourth Table; or else by the eighth Rule of this Chapter, find the present worth of the supposed annuity for the number of years, and at the rate of Interest propounded, which being done, you may find what annuity to continue the said number of years, the sum propounded will purchase by the following proportion, viz.

As the present worth of the supposed annuity.

Is to the said annuity.

So is the sum propounded,

To the annuity required.

As for Example.

Let it be required to find out what annuity to continue 4 years, 800 l. present money will purchase, Compound Interest being computed at 6 per Cent. per Annum? Facit 230.873 l.

First, suppose an annuity at pleasure to continue 4 years, as suppose 150 l. then do I find the eighth Rule of this Chapter the present worth of the said annuity to be 519.76584 l. therefore by the Rule of proportion I say

$$\begin{array}{ccccccc} \text{l.} & & \text{l.} & & \text{l.} & & \text{l.} \\ 519.76584, & \text{&c.} & : & 150 & :: & 800 & : 230.873 \end{array}$$

The Construction of the following TABLE V.

Upon the reason of the foregoing Rule is grounded the Calculation of the following Table for the purchasing of Annuities; and it may somewhat more readily be Calculated thus, *viz.*

It is evident by the construction of the first Table of this Chapter, that 1 *l.* present is equivalent to 1.06 due at the end of a year to come; therefore is 1.06 the first number in the Collum of 6 *per Cent.* of the following Table; because 1 *l.* will purchase 1.06 *l.* Then it is also evident by the fourth Table that the present worth of 1 *l.* Annuity to continue two years at the same rate is 1.83339, &c. that is 1.83339, &c. will purchase a Lease of 1 *l.* *per Annum* to continue two years, Compound Interest, being allowed at 6 *per Cent.* therefore by the Rule of 3 Direct, I say,

$$\begin{array}{ccccccc} l. & & & & l. \\ 1.13339, \text{ \&c.} & : & 1 & :: & 1 & : & .54543, \text{ \&c.} \end{array}$$

By which I find that 1 *l.* ready money will buy a Lease of .54543 *l.* *per Annum* to continue 2 years therefore it is the second number in the following Table. Likewise by the fourth Table I find that 2.67301 is the present worth of 1 *l.* Annuity to continue 3 years at the same rate of Interest, wherefore by the Rule of proportion I say,

$$\begin{array}{ccccccc} l. & & l. & & l. & & l. \\ 2.67301, \text{ \&c.} & : & 1 & :: & 1 & : & .37411, \text{ \&c.} \end{array}$$

Where-

Whereby I find that 1 *l.* will purchase an Annuity of .37411, to continue 3 years. Compound Interest being allowed at 6 *per Cent.* wherefore .37411, is the third number in the said Table; whereby it is evident that if you divide 1, or Unite by the several numbers in the said Collum of 6 *per Cent.* in the fourth Table, successively, the several Quotients will give you the numbers successively, for the Collum of 6 *per Cent.* in the fifth Table; And after the same manner are all the numbers in the other Collums of the said fifth Table found out (except the first number of each Collum, which must be the same with the first numbers in each Collum of the first Table) *mutatis mutandis.*

But it is absolutely necessary that the numbers in the said fourth Table, be continued to more places than there are expressed, to prevent the errors that otherwise will arise, by dividing by defective Decimals.

T A B L E

| T A B L E V. | | | | | | |
|---|---------|---------|---------|---------|---------|---------|
| Which sheweth what Annuity payable by yearly payments to continue any number of years not exceeding 30, one pound will purchase, Compound Interest, being computed at 5, 6, 7, 8, 9, or 10 per Cent. per Annum. | | | | | | |
| Years | 5. | 6. | 7. | 8. | 9. | 10. |
| 1 | 1.05000 | 1.06000 | 1.07000 | 1.08000 | 1.09000 | 1.10000 |
| 2 | .53780 | .54543 | .55309 | .56076 | .56846 | .57619 |
| 3 | .36720 | .37411 | .38105 | .38803 | .39505 | .40211 |
| 4 | .28209 | .28859 | .29519 | .30192 | .30866 | .31947 |
| 5 | .23097 | .23739 | .24389 | .25045 | .25709 | .26379 |
| 6 | .19701 | .20336 | .20979 | .21631 | .22291 | .22960 |
| 7 | .17281 | .17913 | .18555 | .19207 | .19869 | .20545 |
| 8 | .15472 | .16103 | .16746 | .17401 | .18067 | .18744 |
| 9 | .14069 | .14702 | .15348 | .16007 | .16679 | .17364 |
| 10 | .12950 | .13586 | .14237 | .14902 | .15582 | .16274 |
| 11 | .12038 | .12679 | .13335 | .14007 | .14694 | .15396 |
| 12 | .11284 | .11927 | .12590 | .13269 | .13965 | .14576 |
| 13 | .10645 | .10296 | .11965 | .12652 | .13356 | .14077 |
| 14 | .10101 | .10758 | .11434 | .12129 | .12843 | .13574 |
| 15 | .09634 | .10296 | .10979 | .11682 | .12405 | .13147 |

A Continuation of the preceding TABLE V.

| Years | 5. | 6. | 7. | 8. | 9. | 10. |
|-------|--------|--------|--------|--------|--------|--------|
| 16 | .09226 | .09895 | .10585 | .11298 | .12029 | .12781 |
| 17 | .08869 | .09544 | .10242 | .10962 | .11704 | .12466 |
| 18 | .08534 | .09235 | .09941 | .10670 | .11421 | .12192 |
| 19 | .08274 | .08962 | .09675 | .10412 | .11173 | .11954 |
| 20 | .08024 | .08718 | .09439 | .10184 | .10954 | .11745 |
| 21 | .07799 | .08500 | .09228 | .09983 | .10761 | .11562 |
| 22 | .07597 | .08304 | .09040 | .09803 | .10590 | .11400 |
| 23 | .07413 | .08127 | .08871 | .09642 | .10438 | .11257 |
| 24 | .07247 | .07967 | .08718 | .09497 | .10302 | .11129 |
| 25 | .07095 | .07822 | .08581 | .09367 | .10180 | .11016 |
| 26 | .06956 | .07690 | .08456 | .09250 | .10071 | .10915 |
| 27 | .06829 | .07569 | .08342 | .09144 | .09973 | .10825 |
| 28 | .06712 | .07459 | .08239 | .09048 | .09885 | .10745 |
| 29 | .06604 | .07357 | .08144 | .08961 | .09805 | .10672 |
| 30 | .06496 | .07260 | .08058 | .08882 | .09733 | .10607 |

The

The use of the foregoing Table V.

The use of the foregoing Table will appear in the solution of the following Question, *viz.*

A Merchant hath 1500 *l.* by him, which he is willing to lay out upon an Annuity, issuing out of Lands to continue 20 years, beginning presently Compound Interest being Computed on both sides at 6 *per Cent. per Annum.* Now I demand what Annuity the said sum will buy? Facit 130.77 *l.* = 130 *l.* — 15 *s.* — 05 *d.* very near.

To answer this question, I look in the Collum of 6 *per Cent.* of the foregoing fifth Table. and against 20 in the Collum of years I find .08718, which is the annuity that 1 *l.* present money will purchase to continue 20 years, wherefore by the Rule of three Direct, I say.

$$\begin{array}{cccc} l. & l. & l. & l. \\ 1 & : & 08718 & :: 1500 : 130.77 \end{array}$$

X. Questions concerning the purchasing of Leases and Annuities may be solved very well by the numbers in the fourth Table, if you make them Divisors instead of Multipliers.

Let the last Question be proposed, and solved by the fourth Table, *viz.*

What Annuity to continue 20 years will 1500 *l.* ready Money purchase, Compound Interest being allowed at 6 *per Cent.*

To answer this I look in the fourth Table, in the Collum of 6 *per Cent.* against 20 years and there I find this number, *viz.* 1146992, which is the present worth of 1 *l.* Annuity to continue 20 years, Compound Interest being allowed at 6 *per Cent.* And if it be the present worth of 1 *l.* Annuity, I conclude it will purchase 1 *l.*

An-

Annuity to continue the same number of years wherefore I say by the Rule of 3 Direct,

$$\begin{array}{ccccccc} l. & & l. & & l. & & l. \\ 11.46992 & : & 1 & :: & 1500 & : & 130.77 \end{array}$$

So that the answer is the same with the former which was found by help of the Fifth Table.

All the foregoing Tables might have been continued to any greater number of years at pleasure ; But although these Tables are calculated but for 30 years ; yet they may be made serviceable for years above 30, as shall be shewed by and by.

Arithmetical Questions to exercise the Learner in the Precedent Tables.

Quest. 1. There is a Lease of 20 years to begin presently, which in ready Money is worth 1200 *l.* But suppose the said Lease were not to begin till the expiration of 8 years, I demand what would be the present worth of the said Lease Rebate, being allowed at 8 per Cent. per Annum, Compound Interest?

The main intent of this Question is to shew the use of the second Table, for if you find the present worth of 1200 *l.* due at the end of 8 years, at 8 per Cent. the Question is answered, which according to the directions given after the said Table, will be found to be 648.3216 *l.* = 648 *l.*—06 *s.*—05 *d.*

Quest. 2. A oweth to B 600 *l.* to be paid in 6 years, viz. 100 *l.* every year, but being weakened in his Estate, is not able to perform ; but
an

an Estate being to come into his hands at the end of 10 years; *B* is willing to forbear it all till then, and to be allowed Compound Interest at 8 *per Cent.* for his forbearance; I demand how much will be due to *B* at the 10 years end?

This Question is solved by help of the third and first Tables; for first 100 *l.* is to be paid in the nature of Annuity for 6 years, therefore by the third Table I find the amount of an annuity of 100 *l.* to continue 6 years at 8 *per Cent.* which is 733.592 *l.* and will be due at the expiration of 6 years, and then is that sum to be forborn to the end of 10 years, which is 4 years after the 6 years; which being a single sum, its amount is found by the first Table to be 998.037 *l.* &c. which is the answer to the question.

Quest. 3. There is a Lease to continue 21 years to be sold for 1000 *l.* but the Lessee desireth rather to pay an annual Rent: Now the question is what that annual Rent ought to be Compound Interest being computed at 10 *per Cent. per Annum*?

The intent of this Question is to find what annuity to continue 21 years 1000 will purchase at 10 *per Cent.* which is to be done by the fifth Table thus,

Because the time is for 21 years, look in the Collum of years for 21 and just against it in the Collum of 10 *per Cent.* you will find .11562, by which multiply 1000, and the product is 115.62 *l.* and so much will 1000 *l.* purchase for 21 years at 10 *per Cent.* Compound Interest.

Quest. 4. *A* and *B* have each of them a Lease to continue 20 years; *A* hath 80 *l. per Annum*, and *B* 120 *l. per Annum*, and they agree to make

an exchange, upon this condition, that *A* shall pay in ready money the excess of his Estate, allowing him Compound Interest at 8 per Cent. Now I demand how much ready money *A* ought to give *B* upon this exchange, according to that condition?

Subtract 80 *l.* from 120 *l.* and the Remainder is 40 *l.* and so much per Annum is the Lease of *B* worth more than that of *A*, therefore *A* must pay *B* so much money as will purchase 40 *l.* per Annum to continue 20 years at 8 per Cent. which by the third Table will be found to be 392.7256 *l.*

Quest. 5. There is a House to be let by Lease for 21 years, for which the Lessor will have 50 *l.* fine, and 70 *l.* per Annum, but the Lessee is willing to pay the greater fine, that he may have the Rent but 40 *l.* per Annum, now I demand what fine he ought to pay upon that condition Compound Interest being allowed at 8 per Cent. per Annum?

Take the difference between 40 and 70, which is 30 for the abatement in the yearly Rent for 21 years; Then by the fourth Table find the present worth of 30 *l.* per Annum for 21 years at 8 per Cent. which is 300.5043 *l.* = 300 *l.* — 10 *s.* — 01 *d.* which added to the said 50 *l.* fine makes 350 *l.* — 10 *s.* — 01 *d.* for the fine to be paid upon the said condition.

Quest. 6. There is a Lease to be let of 20 *l.* per Annum, and 250 *l.* fine for 24 years, and the Lessee is willing to pay the greater Rent, that he may pay but 50 *l.* fine, now I demand what Rent he ought to pay upon that condition, Compound Interest being computed at 7 per Cent. per Annum?

It is manifest, that if the Lessor taketh 50 *l.* ne, he abateth 200 *l.* therefore find by the fifth Table what Annuity to continue 24 years, 200 *l.* will purchase at 7 *per Cent.* The Tabular number is .08718, which multiplyed by 200 produceth 17.436 = 17 *l.*—8 *s.*—9 *d.* and so much must the Lessee raise his rent if he will have 200 *l.* abated of his fine, to which if you add 20 *l.* the proposed Rent, the sum is 37 *l.*—8 *s.*—09 *d.* for the yearly Rent to be paid to satisfie the said condition.

Quest. 7. What Annuity to continue 20 years, may I grant presently, for 900 *l.* to be paid 6 years hence, accompting 6 *per Cent. per Annum*, Compound Interest.

First find by the second Table the present worth of 900 *l.* due 6 years hence, at 6 *per Cent.* which is 634.464 *l.* = 634 *l.*—09 *s.* 03 $\frac{1}{2}$ *d.*

Then by the Fifth Table find what Annuity to continue 20 years 634.464 will purchase at 6 *per Cent.* And you will find the answer to be 55.31257152 *l.* = 55 *l.*—06 *s.*—03 *d.* and so much I ought to grant yearly for 20 years for 900 *l.* to be paid me at the end of 6 years.

Quest. 8. I have 6 years of an old Lease, yet to come, and would take a new Lease in reversion for 21 years, after the expiration of the old Lease, the annual Rent whereof is 40 *l.* But I would pay such a sum of Money present as a fine, that for my Lease in Reversion for the said 21 years, I may pay but 15 *l. per Annum*, Now I demand how much present money I ought to pay the Lessor, to satisfie these conditions, Compound Interest being computed at 8 *per Cent.*

The difference between 40 and 15 is 25, and so much the Lessee desireth to have abated in his Rent,

Rent, wherefore by the fourth Table find the present worth of 25 *per Annum* for 21 years at 8 *per Cent.* which is 250.42025 *l.* = 250 *l.* — 08 *s.* — 05 *d.* Then by the second Table find the present worth of 250.42025 *l.* due at the end of 6 years to come, at 8 *per Cent.* which is 157.807 *l.* &c. = 157 *l.* — 16 *s.* — 01 $\frac{3}{4}$ And so much ought I to give to satisfy the said conditions.

Quest. 9. There is a Lease to be let for 12 years, for 20 *l.* *per Annum*, and 20 *l.* fine, but the Lessee desireth to take a Lease of the same for 21 years, and to pay the same Rent, the Question is, what fine ought to be paid for the Lease of 21 years, accounting Compound Interest at 6 *per Cent*?
Facit 280 *l.* — 12 *s.* — 05 *d.*

By the fifth Table seek what Annuity to continue 12 years, 200 *l.* will purchase at 6 *per Cent.* which you will find to be 23.854 *l.* Then by the Fourth Table find the present worth of 23.854 *l.* Annuity to continue 21 years at 6 *per Cent.* which is 280.620 *l.* &c. = 280 *l.* — 12 *s.* — 05 *d.* and so much ought the Lessee to pay for a fine, to have his Lease for 21 years.

Quest. 10. A Gentleman hath 1000 *l.* which he would lay out to purchase an Annuity of 100 *l.* to be paid by yearly payments; Now the Question is, how many years must the said Annuity continue, Compound Interest being allowed on both sides at 8 *per Cent.* *per Annum*?

First, Divide 1000 by 100, and the Quotient will be 10, which sheweth that the Buyer giveth 10 years purchase for the said Annuity.

Then in the Fourth Table, and in the Collum of 8 *per Cent.* look for the number 10, which cannot be exactly found, but the nearest to it and less than it, is 9.81814 which is placed against

It is manifest, that if the Lessor taketh 50 *l.* hence, he abateth 200 *l.* therefore find by the fifth Table what Annuity to continue 24 years, 200 *l.* will purchase at 7 *per Cent.* The Tabular number is .08718, which multiplied by 200 produceth 17.436 = 17 *l.* — 8 *s.* — 9 *d.* and so much must the Lessee raise his rent if he will have 200 *l.* abated of his fine, to which if you add 20 *l.* the proposed Rent, the sum is 37 *l.* — 8 *s.* — 9 *d.* for the yearly Rent to be paid to satisfy the said condition.

Quest. 7. What Annuity to continue 20 years, may I grant presently, for 900 *l.* to be paid 6 years hence, accompting 6 *per Cent. per Annum*, Compound Interest.

First find by the second Table the present worth of 900 *l.* due 6 years hence, at 6 *per Cent.* which is 634.464 *l.* = 634 *l.* — 09 *s.* 03 $\frac{1}{2}$ *d.*

Then by the Fifth Table find what Annuity to continue 20 years 634.464 will purchase at 6 *per Cent.* And you will find the answer to be 55.31257152 *l.* = 55 *l.* — 06 *s.* — 03 *d.* and so much I ought to grant yearly for 20 years for 900 *l.* to be paid me at the end of 6 years.

Quest. 8. I have 6 years of an old Lease, yet to come, and would take a new Lease in reversion for 21 years, after the expiration of the old Lease, the annual Rent whereof is 40 *l.* But I would pay such a sum of Money present as a fine, that for my Lease in Reversion for the said 21 years, I may pay but 15 *l.* *per Annum*, Now I demand how much present money I ought to pay the Lessor, to satisfy these conditions, Compound Interest being computed at 8 *per Cent.*

The difference between 40 and 15 is 25, and so much the Lessee desireth to have abated in his Rent,

Rent, wherefore by the fourth Table find the present worth of 25 *per Annum* for 21 years at 8 *per Cent.* which is 250.42025 *l.* = 250 *l.* — 08 *s.* — 05 *d.* Then by the second Table find the present worth of 250.42025 *l.* due at the end of 6 years to come, at 8 *per Cent.* which is 157.807 *l.* &c. = 157 *l.* — 16 *s.* — 01 $\frac{3}{4}$ And so much ought I to give to satisfy the said conditions.

Quest. 9. There is a Lease to be let for 12 years, for 20 *l.* *per Annum*, and 20 *l.* fine, but the Lessee desireth to take a Lease of the same for 21 years, and to pay the same Rent, the Question is, what fine ought to be paid for the Lease of 21 years, accounting Compound Interest at 6 *per Cent* ?
Facit 280 *l.* — 12 *s.* — 05 *d.*

By the fifth Table seek what Annuity to continue 12 years, 200 *l.* will purchase at 6 *per Cent.* which you will find to be 23.854 *l.* Then by the Fourth Table find the present worth of 23.854 *l.* Annuity to continue 21 years at 6 *per Cent.* which is 280.620 *l.* &c. = 280 *l.* — 12 *s.* — 05 *d.* and so much ought the Lessee to pay for a fine, to have his Lease for 21 years.

Quest. 10. A Gentleman hath 1000 *l.* which he would lay out to purchase an Annuity of 100 *l.* to be paid by yearly payments; Now the Question is, how many years must the said Annuity continue, Compound Interest being allowed on both sides at 8 *per Cent.* *per Annum* ?

First, Divide 1000 by 100, and the Quotient will be 10, which sheweth that the Buyer giveth 10 years purchase for the said Annuity.

Then in the Fourth Table, and in the Collum of 8 *per Cent.* look for the number 10, which cannot be exactly found, but the nearest to it and less than it, is 9.81814 which is placed against

20 years, and the nearest to it greater than it is, is 10.01681, therefore I conclude that the Annuity must continue above 20 years, but not 21 years, and to find out how much it must continue more than 20 years, I work thus, *viz.* First, I find the difference between the said Tabular numbers 10.1681 and 9.81814, which is .19867. Then I find the difference between the lesser of the said Tabular Numbers, *viz.* 9.81814 and 10, the Number that I would find in the Table, which is .18186, then by the Rule of proportion, I say.

| | | | | | | |
|-----------|---|-------------|---|-----------|---|-------------|
| <i>l.</i> | | <i>year</i> | | <i>l.</i> | | <i>year</i> |
| .19867 | : | 1 | : | .18186 | : | .9153 |

which is as much as to say, as the greater difference .19867 is to one year, so is the lesser difference to .9153 parts of a year, which is 47 Weeks, and 5 Days, therefore the number of Years sought in the Question is 20 Years, 47 Weeks and 5 Days.

Quest. 11. A Gentleman bought a Lease of 100 *l.* per Annum to continue 18 years, for 960 *l.* now I demand what Rate of Compound Interest was their implied in such a bargain?

To Answer this, First, I divide 960 by 100, and the Quotient is 9.6 which sheweth how many years purchase it was worth; then because the Lease was to continue 18 years, I look in the fourth Table in the Collum of years for 18, and carry my Eye exactly in the line against it, looking for the said Quotient 9.6 which I cannot find exactly, but the next (lesser) number to it, is, 9.37188 in the Collum of 8 per Cent. and the next (bigger) number to it is 10.05908, in the Collum of 7 per Cent. wherefore I conclude that the Rate of interest implied is between 7 and 8
per

per Cent. and to know how much it is more than 7, I do thus, take the difference between the two said Tabular numbers which you will find to be 68720 also subtract (9.6) the said Quotient, from 10.05908 (the greater Tabular number) and the remainder is .45908, then by the Rule of Proportion, I say,

$$\begin{array}{ccccccc} l. & l. & l. & l. \\ 68720 & : & .45908 & :: & 1 & : & .6680 \end{array}$$

That is to say, as the difference between the two Tabular numbers is to the lesser Remainder so is 1 *l.* the difference between 7 and 8 *per Cent.* to .668 the proportional part to be added to 7 *l.* which is 13 *s.* — 04 $\frac{1}{2}$ *d.* so that 7 *l.* 13 *s.* 04 $\frac{1}{2}$ *d.* is very near the Interest required.

How to find out Tabular Numbers for years exceeding 30.

It may many times fall out, that the number of years proposed in a Question, may exceed the number of years limited in the foregoing first, second, third, fourth and fifth Tables, and in such cases that defect may be supplied by the method used in the solution of the following Questions.

Quest. 12. Suppose 80 *l.* were put out to Interest at 5 *l. per Cent.* Compound Interest for 40 years, I demand how much it will then be amounted to?

This Question is to be solved by the first Table, thus, *viz.* Take any two Numbers in the Collum of years, which together will make up 40, and then take the Tabular numbers in the

Collum of 5 *per Cent.* which stand against those two numbers, and multiply them together, and then multiply that product by 80 *l.* the given sum, and the last product will be the Answer.

As suppose you take 30 and 10, or 21 and 19, or 31 and 9, or 25 and 15, &c.

But we will pitch upon 30 and 10, and the Tabular number against 30 in the Collum of 5 *per Cent.* is 4.32194, and against 10 is 1.62889 which two numbers being multiplyed, produce 7.03996, &c. which is the amount of 1 *l.* for 40 years at 5 *per Cent.* then I multiply 7.03996, &c. by 80 *l.* and the product is 563.197 *l.* &c. = 563 *l.* — 03 *s.* — 11 $\frac{1}{2}$ *d.* fere.

The Answer would have been the same, if we had pitched upon any other two numbers to have made up 40. And for Tryal hereof, let us pitch upon 25 and 15, the Tabular number against 25 is 3.38635, and the Tabular number against 15 years is 2.07892, and the product of these two Tabular numbers is 7.0399, &c. which multiplyed by 80, produceth 563.197 *l.* as before, and so much will 80 *l.* be increased to in 40 years at 5 *per Cent per An.* Compound Interest. The like is to be understood for any other number of years.

Quest. 13. Suppose 420 *l.* to be payable at the end of 50 years to come, What is its present worth, Rebate being allowed at 5 *per Cent. per Annum,* Compound Interest?

This Question is of the same nature with those belonging to the second Table, and is answered thereby, according to the method used in solving the last Question by the first Table, *viz.* the given time being 50 years; I pitch upon 30 and 20, and the Tabular number against 30 is .231377 and

and that against 20 is .376889 and the product of these two is .087203, &c. which is the present worth of 1*l*. due 50 years hence at 5 per Cent. per An. wherefore I multiply .087203, &c. by 420, and the product is 36.62544, &c. = 36 *l*. 12 *s*. 6 *d*. and so much is the present worth of 420*l*. due 50 years hence at 5 per Cent. per Annum. Compound Interest.

Quest. 14. An Heir being beyond the Sea, did not return till 36 years after an Estate of 30*l*. per annum was fallen to him by the Death of the Proprietor; the Question is, what was then due to him, Compound Interest being computed at 6 per Cent. per Annum?

This Question is of the nature of those belonging to the third Table, and the manner of solving it is thus, viz.

Find out (by the seventh Rule of this Chapter) what principal will in one year gain 30*l*. at 6 per Cent. by the following proportion.

$$\begin{array}{ccccccc} \textit{l.} & \textit{l.} & & \textit{l.} & & \textit{l.} & \\ 6 & : & 100 & : : & 30 & : & 500 \end{array}$$

Having found 500 *l*. to be the principal, seek (after the manner of the 12 Question) by the first Table the amount or Increase of 500 *l*. for 36 years at 6 per Cent. which you will find to be 4073.5998 *l*. &c. from which if you subtract the said principal 500*l*. the remainder is 3573.5998 *l*. &c. = 3573 *l*.—12 *s*.—00 *d*. fere. An so much was due to the Heir at his return.

Quest. 15. There is an Annuity of 30*l*. to continue 37 years, the Question is what it is worth in ready money, Compound Interest being computed at 6 per Cent. per Annum?

By the second way of solving Questions under the

the fourth Table, for a principal which will gain 30 *l.* in one year, at 6 *per Cent.* which is here 500 *l.* then according to the method used in solving the thirteenth Question foregoing, find the present worth of 500 *l.* for 37 years at 6. *per Cent.* which will be found to be 57.896537 which subtracted from 500 *l.* leaves 442.1034, &c. = 442 *l.* 2 *s.* 0 $\frac{3}{4}$ *d.* And so much is the present worth of the foresaid Annuity.

Quest. 16. What Annuity to continue 40 years will 500 *l.* purchase Compound Interest being computed at 6 *per Cent.* *per Annum*?

It is evident by the tenth Rule of this Chapter, that if you find out the present worth of 1 *l.* Annuity for any number of years, and at any rate of Interest, it may easily be found what Annuity to continue the same number of years any other sum will purchase at the same rate of interest by one single Rule of 3 Direct: Therefore,

Find out the present worth of 1 *l.* Annuity to continue 40 years at 6 *per Cent.* by the method used in solving the last Question, which will be found to be 15.04632 *l.* = 15 *l.* — 00 — 11 *d.* which sum of Money will purchase an Annuity of 1 *l.* to continue 40 years at 6 *per Cent.* therefore to know what Annuity of 500 *l.* will purchase for the same time, say by the Rule of Proportion.

15.04632 *l.* : 1 *l.* :: 500 *l.* : 33.230, &c.

which will be found to be 33.230 &c. = 33 *l.* — 04 — 07 $\frac{1}{4}$ *d.* *ferè*, and such an Annuity to continue 40 years will 500 *l.* purchase Interest being allowed at 6 *per Cent.*

Cockers

ARTIFICIAL

ARITHMETICK,

SHEWING

The Genesis or Fabrick of the Logarithmes and their use in the extraction of Roots, solving of Questions in Anatocisme, or Compound Interest, and in the other Rules of Arithmetick, in a Method not usually Practised.

Composed by *EDWARD COCKER*,
late Practitioner in the Arts of Writing A-
rithmetick and Engraving.

Perused, Corrected and Published
By *JOHN HAWKINS*, School-
Master at St. Georges's Church in Southwark.

Nil tam difficile est quod non solertia vincat.

L O N D O N,
Printed by *James Orme*, in the Year 1703.

The meaning of such Characters as are used in the ensuing Treatise.

÷ IS the sign of Addition, and is as much as to say *plus*, signifying that the Numbers or Quantities between which it is placed, are to be added together as $4÷7$ signifieth that 4 and 7 are to be added together.

—Is the sign of Subtraction, and as much as to say *minus*, signifying that the Number which followeth it is to be subtracted out of the Number which produceth it, as $8-5$ signifieth that 5 is to be subtracted from 8.

× Is the sign of Multiplication, and signifieth that the Numbers between which it is placed, are to be multiplyed together, as $6×8$ signifieth that 6 and 8 are to be multiplyed together.

=Is a sign of Equality, and signifieth that the Numbers or Magnitudes between which it is placed, are equal as $3÷6=7÷2$ signifyeth that 3 and 6 are equal to 7 and 2 : Likewise $18-6=4÷8=12$ and $4×7=28$ &c. If this be not a sufficient Explanation, read the 13, 14, 15, and 19 Sections of the first Chapter of my *Algebraical Arithmetick*.

This forms pp. 261, 62.

Cocker's

ALGEBRAICAL

ARITHMETICK

CONTAINING

The Doctrine of Composing,

and Resolving an

EQUATION,

With all other Rules requisite for the understanding of that Mysterious Art, according to the Method used by Mr. JOHN KERSET, in his incomparable Treatise of ALGEBRA.

Composed by EDWARD COCKER,
late Practitioner in the Arts of Writing
Arithmetick, and Engraving.

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By JOHN HAWKINS, School-Master
at St. George's Church in Southwark.

Plato foribus Academiae inscribi iussit ; Nemo
Arithmetices Ignarus hic Ingrediatur.

L O N D O N ,
Printed in the Year 1702.



THE SECOND BOOK OF ARTIFICIAL ARITHMETICK,

I: **A** *Artificial Arithmetick* is performed by Artificial Numbers, very fitly called Logarithmes.

II. Logarithmes are borrowed Numbers which differ among themselves by Arithmetical proportion, as the numbers which they signified differ by Geometrical proportion.

III. Logarithmetical Arithmetick is an Artificial use of Numbers, invented for ease in Calculation, wherein each natural Number is so fitted with an Artificial, that what is usually produced by Multiplication of natural Numbers, is here effected by the Addition of their Artificial Numbers: And what natural Numbers perform by Division, is here effected by the Subtraction of their artificial Numbers, and what natural

Num-

Numbers do perform by long and tedious operations in the extraction of Square, Cube, Biquadrate, &c. Roots is here easily effected by Bipartition, Tripartition, Quadrupartion, &c. of their artificial Numbers, and so the hardest parts of Calculation is avoided by an easie posthaphæresis, as our Trigonometrical Calculators of late have sufficiently experienced, by avoiding very tedious Multiplications and Divisions in the use of the Tables of Natural Sines, Tangents,

* *The Lord Nepair
Baron of Merchiston
in Scotland.*

Secants to the Everlasting
Credit of the honourable
* Author of this late and incomparable invention.

IV. The parts of *Artificial Arithmetick* are the same with Natural Arithmetick, but we shall treat them in this order, viz. First, of the Nature of Logarithmes; Secondly, of their Genesis, or the Invention of the Table of Logarithmes And Thirdly, of the use of the Logarithmes in Multiplication, Division, the Extraction of Roots, &c.

CHAP.

C H A P. II.

Of the nature of Logarithmes.

1. **L**ogarithmes are Numbers so fitted to proportional numbers, that themselves retain equal differences.

Let there be assigned a series or rank of numbers in Geometrical proportion, as those in the Collum A viz. 1, 2, 4, 8, 16, 32, &c. And let there be as many other numbers placed over against them in Arithmetical progression, that is having equal differences as those in the Collums B. C. D. E. or any other numbers whatsoever of the like Nature. Then,

| A | B | C | D | E |
|------|----|----|----|----|
| 1 | 0 | 2 | 5 | 0 |
| 2 | 1 | 4 | 8 | 3 |
| 4 | 2 | 6 | 11 | 6 |
| 8 | 3 | 8 | 14 | 9 |
| 16 | 4 | 10 | 17 | 12 |
| 32 | 5 | 12 | 20 | 15 |
| 64 | 6 | 14 | 23 | 18 |
| 128 | 7 | 16 | 26 | 21 |
| 256 | 8 | 18 | 29 | 24 |
| 512 | 9 | 20 | 32 | 27 |
| 1024 | 10 | 22 | 35 | 30 |
| 2048 | 11 | 24 | 38 | 33 |
| 4096 | 12 | 26 | 41 | 36 |

Forasmuch as these numbers in the Collums B. C. D. E. are of equal-difference among themselves, therefore shall they be the Logarithmes of the numbers in the Collum A, each of the

the respective number against which it is placed. So in the Collum B. the number 4, is the Logarithme of 16 in the Collum A, and in the Collum C the number 10 is the Logarithme of 16 in the Collum A, and in the Collum D, 29 is the Logarithme of 256 in the Collum A, &c.

And as the numbers in the said Collums, B, C, D, E, are Logarithmes of the respective numbers in the Collum A, so they may be Logarithmes of any other rank, or series of numbers in Geometrical proportion.

II. If four numbers are Arithmetical proportionals, either Continued, or Discontinued, the sum of the means is equal to the sum of the extremes.

Let us choose 8, 10, 12, 14, in the Collum C. I say that the sum of the Extreams, 8 and 14, are equal to the sum of the two means, 10, and 12. For, $8 + 14 = 10 + 12 = 22$. Or if they are discontinued as, 10, 12, 22, 24. in the Collum C; for $10 + 24 = 12 + 22 = 34$. The like of any other; this being a peculiar property of all Numbers that are Arithmetically proportional.

III. If four Numbers are in Geometrical proportion, either continued, or discontinued, the product arising from the Multiplication of the two extremes, is equal to the product of the two means.

So 4, 8, 16, 32, in the Collum of A are Geometrical proportionals continued, and the product of the Extreams 4 and 32, is equal to the product of the means, 8, and 16, for, $4 \times 32 = 8 \times 16 = 128$.

Also

Also, 4, 8, 64, 128 are Geometrical proportionals discontinued, and the product of 4 and 128, the extremes, is equal to the product of 8 and 64 the two means, for $4 \times 128 = 8 \times 64 = 512$.

Hence it follows, that what Geometrical proportionals perform by Multiplication, the same will the Logarithmes (being Arithmetical proportionals) perform by Addition.

Let there be given four Geometrical proportionals in the Collum A, viz. 8, 16, 128, and 256, and let their Logarithmes be 8, 10, 16, and 18 in the Collum C; I say that as 8×256 the product of the extremes is equal to 16×128 the product of the means, so is $8 + 18$ the sum of the Logarithmes of the extremes is equal to $10 + 16$ the sum of the Logarithmes of the means. Therefore.

If 3 Numbers are given to find the fourth proportional, it may be found by Addition and Subtraction of their Logarithmes, (for, as in Natural Numbers if you multiply the second and third together, and divide their product by the first, the Quote will be the fourth proportional number so if you add the Logarithmes of the second and third together; and from their sum subtract the Logarithmes of the first, the remainder will be the Logarithme of the fourth proportional number.

Example.

Let there be given 2, 16, and 64, and let it be required to find a fourth proportional number there-to, which is 512.

The

The Logarithmes of the given numbers are 3, 12, and 18, Now if you add 12 and 18 together (which are the Logarithmes of the second and third) their sum is 30, (which is the Logarithme of 1024, the product of the second and third) and if from 30 the said sum of the Logarithmes, you subtract 3, (the Logarithme of the first) there will remain 27, which is the Logarithme of the 512 the fourth proportional number sought for.

$$12 + 18 = 30 - 3 = 27$$

And

$$2 : 16 :: 64 : 512$$

IV. By what hath been said, you may perceive that to natural numbers there may be fitted divers kinds of Logarithmes, but we shall pitch only upon that kind which were framed by Mr. *Briggs* at the request of the Baron of *Merchiston*, who hath chosen these Geometrical proportionals, viz. 1. 10. 100. 1000. 10000. 100000, &c. To which numbers he hath assumed the Logarithmes following, viz. for the number 1, the Logarithme 0.000000, for 10 the logar. 1.000000, for 100, the logar. 2.000000 for 1000 the logar. 3.000000, for 10000, the logar. 4.000000, &c. as in the following Table.

| A | B |
|-------------|-----------|
| 1 | 0.000000 |
| 10 | 1.000000 |
| 100 | 2.000000 |
| 1000 | 3.000000 |
| 10000 | 4.000000 |
| 100000 | 5.000000 |
| 1000000 | 6.000000 |
| 10000000 | 7.000000 |
| 100000000 | 8.000000 |
| 1000000000 | 9.000000 |
| 10000000000 | 10.000000 |

The numbers in the Collum A are the series of Geometrical proportionals, and the numbers in the Collum B, are the respective Logarithmes of each of those Geometrical proportionals, themselves being Arithmetical proportionals, where note that the Figures 1, 2, 3, 4, &c. which are separated from the rest by a point or prick, are called the Indices, or Characteristicks of the logarithme, because they declare how many places the numbers by them signified do consist of; the Characteristick of any Logarithme being always an unite less than the number of places, which the number by it signified doth consist of: As in the foregoing Table you may perceive that the logarithme of 1, is 0.000000, and the logarithme of 10 is 1.000000, and the logarithme of 100 is 2.000000, &c. so that the Index, or Characteristick of 1, and of all numbers from 1 to 10 is 0. and the Characteristick of 10 and of all numbers from 10 to 100 is 1: And the Characteristick of 100 and of all numbers from 100 to 1000 is 2, and the Characteristick of each number being an unite

unite less than the number of places of which the number by it signified doth consist, as was said before.

The Logarithmes of this kind ought all to consist of an equal number of places, that is to say they ought not to be, one Log. of 10 places, another of 8, &c. but all of them to be of 6, of 7, of 8, &c. places,

C H A P. III.

Of the Genesis or Fabrick of the Logarithmes.

I. **T**HE Logarithme of 1 being assumed to be 0.000000, and the Logarithme of 10 to be 1.000000. the Logarithme of 100 to be 2.000000, &c. In the next place it will be requisite to shew the way and manner of Calculating the Logarithmes of the intermediate numbers, viz. of the numbers between 1 and 10, which are 2, 3, 4, 5, &c. and between 10 and 100, which are 11, 12, 13, 14, 15, 16, &c. and between 100, and 1000, which are 101, 102, 103, 104, &c. which to do, observe the following Rules.

II. Find so many continual means between 1 and 10, till that continual mean which cometh nearest 1, may be a mixt number less than 2,
and

and so near 1, that it may have as many Cyphers placed before the significant Figures of the Numerator, as you intend your logarithmes to consist of places; But our Directions here shall be for the making a Table of logarithmes to consist of 7 places; wherefore find so many continual means between 1 and 10, till the last may have 7 Cyphers placed before the significant Figures of its Numerator, in order whereunto, annex to the number 10 a competent number of Cyphers, (*viz.* 28, because the work may be the more exact) and extract the Square Root of that number so enlarged, which being done, you will find its Square Root to be 3.16227766016837, This being done, annex to the said Root 14 Cyphers more, and extract the Square Root thereof, which you will find to be 1.77827941003892.

Again annex to the Root last found 14 Cyphers more, and extract the Square Root thereof, which you will find to be 1.33352143216332, and thus proceeding successively by annexing of Cyphers, and a continual extraction of the Square Root, until you have found a Square Root, or Continual mean, having 7 Cyphers placed before the significant Figures of its Numerator, which will be found after 27 several Extractions to be 1.00000001715559.

So the 3 last continual means between 10 and 1 will be found to be

| | |
|---|------------------|
| { | 1.00000006862238 |
| | 1.00000003431119 |
| | 1.00000001715559 |

All which 3 continual means are less than 2, and so near 1, that there are 7 Cyphers placed before the significant Figures of each of their Numerators.

Having found 27 several means between 10, and 1, placethem successively one under the other as in the Collum A, of the following Table; Then make another Collum (B) to contain the Respective logarithmes of those continual means.

And because biparting the logarithme of any number produceth the logarithme of the Square Root of that number, therefore take the logarithme of 10, which is 1.000000, and place it in the Collum B over against 10, then bipart it, (that is, divide it by 2) and you will have 0.500000 which is the logarithme of 3.16227766 0.16837 the Square Root of 10, then take half of that logarithme, viz. 0.250000 which is 0.500000, and place it for the logarithme of 1.778279410, &c. the second mean proportional, (or Square of Root of 3.162277660, &c.) and so by continual bipartition, you will at length find that 0.000000 0007450580, will be the logarithme of the last continual mean, viz. the logarithme of 1.00000000 171559, as in the following Table.

| A | B |
|---------------------|--------------------|
| Continual means. | their Logarithmes. |
| 10.0000000000000000 | 1.0000000000000000 |
| 3.16227766016837 | 0.5000000000000000 |
| 1.77827941003892 | 0.2500000000000000 |
| 1.33352143216332 | 0.1250000000000000 |
| &c. | &c. |
| 1.000000006862238 | 0.0000000029802322 |
| 1.000000003431119 | 0.0000000014901161 |
| 1.000000001715559 | 0.0000000007405805 |

III. Any

III. Any Number whatsoever being given, how to make the Logarithme thereof.

When it is required to make the Logarithme of any number, extract so many continual means between the given number and 1, until the mean which cometh nearest 1, may be a mixt number less than 2, and so near 1, that it may have 7 Cyphers placed before the significant figures of its Numerator, which being done, you may easily find out the Logarithme of that continual mean, by help of the foregoing Table; and then by doubling, and redoubling the Logarithme of the said continual mean, as many times as you found continual means by extraction, so shall you at last have the Logarithme of the given number.

You may make the Logarithme of any number whatsoever by this and the last Rule.

As for Example.

Let us pitch upon the number 2, and make its Logarithme.

To do which, annex to the number 2 a competent number of Cyphers, *viz.* 28, and extract the Square Root thereof, which you will find to be 1.41421356237309 for the first continual mean, to which said mean annex 14 Cyphers more, and extract the Square Root thereof, and so proceed, by annexing of Cyphers and extracting of Roots, till the nearest mean proportional number to 1, may have seven Cyphers placed before the significant Figures of its Numerator, which after 23 several Extractions you will

will find to be said to be 1.0000000862658.

Then to find out the logarithme of this continual mean, say by the Rule of 3 Direct.

As the significant Figures of the Twenty fifth mean proportional in the foregoing Table, viz. 6862238.

Is to its respective Logarithme, 29802322.

So are the significant Figures of the last continual mean found between 1 and 2, viz. 8262958.

To its respective Logarithme 35885571.

Now if you prefix before the Logarithme last found 8 Cyphers, it will be 0000000035885571, which being doubled and redoubled 23 times, (because there were 23 continual means found between 1 and 2) there will at last be produced 0.30102998797568, which is the logarithme of the number 2, which was Required, but because we intend the Table of Logarithmes to consist but of 7 places, and because 2 nines follow the sixth place therefore make the Figures 2 to be 3 and so shall the logarithme of 2 be 0.301030 cancelling the following Figures as superfluous.

The Logarithme of 2 being found, you may easily find the logarithmes of 4, 5, 8, 16, 20, 25, 32, 40, 50, 64, &c. by Artificial Multiplication and Division, which is by adding and subtracting of logarithmes; for if you take the logarithme of 2 out of the logarithme of 10, there will remain the logarithme of 5 and the logarithme of 2 Doubled gives you the logarithme of 4; then add the logarithme of 4, to the logarithme of 2, and you have the logarithme of 8, and to the logarithme of 8 add the logarithme of 2, and it gives you the logarithme of 16, and the logarithme of 5 added to the logarithme of 4, gives the logarithme of 20, and the logarithme of 6 doubled

doubled, gives the Logarithme of 25, &c.

In the next place you are to get the Logarithmes of 3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 89, 97, &c. by help of which all the rest may be Calculated.

IV. The first figure of every logarithme, which is separated from the rest by a point or prick is very properly called the Index, or Characteristick of the logarithme, which sheweth the Nature of the number by it signified, viz. whether it be positive, or negative, and if positive, of what number of places it doth consist, and if negative, what place of the Decimal Fraction the first figure of the number by it signified, shall possess, as in the following Table.

| | | | | | | |
|----------------------|---|-----------|---|---------|---|------------|
| So the logarithme of | { | 46768 | } | will be | { | 4.670134 |
| | | 4676.8 | | | | 3.670134 |
| | | 467.68 | | | | 2.670134 |
| | | 46.768 | | | | 1.670134 |
| | | 4.6768 | | | | 0.670134 |
| | | .46768 | | | | — 1.670134 |
| | | .046768 | | | | — 2.670134 |
| | | .0046768 | | | | — 3.670134 |
| | | .00046768 | | | | — 4.670134 |

Whereby you may perceive that the logarithmes of absolute and defective numbers are the same, only the Characteristick of a defective number is marked with the note of defection, for the logarithme of the absolute number 46768 is 4.670134, the Characteristick 4, shewing the number by it signified to consist of 5 places, as is already said in the fourth Rule of the se-

cond Chapter, and the Logarithme of the mixt number 46.768 is 1.670134 which is the same with the former, only the Characteristick is 1, which sheweth the Integral number by it signified, to consist of two places, the rest being a decimal Fraction. Likewise the Logarithme of the Decimal .46768 is — 1.670134, which is still the same with the former, only its Characteristick being marked with a note of defectiō sheweth it to be the Logarithme of a Decimal Fraction, and because the Characteristick is — 1, it sheweth that the first figure of the number by it signified doth possess the first place of the Decimal, or place of primes: Again the Logarithme — 46703 is still the same, and if you look for it in the Table of Logarithmes, not regarding the Index, you will find it to be the Logarithme of 46768, but because its Index is defective, I conclude it to be the Logarithme of a Decimal, and because the Index, or Characteristick is — 4, therefore I conclude that the first Figure of the number signified by it, must possess the fourth place of the Decimal, wherefore place 3 Cyphers before it, and you have .000 6768 for the Decimal signified by the Logarithme — 4.670134. This being well understood the rest will easily be attained by the following Directions.

CHAP.

CHAP. IV.

Of the use of the Table of Logarithmes.

THE use of the Table of Logarithmes is twofold, *viz.* First, To find therein the logarithme of any given number, or to find the number appropriated to any given logarithme.

Secondly, To resolve diverse necessary problems in Arithmetick, Geometry, Trigonometry, Astronomy, &c.

Concerning the first of these, I shall not meddle, because our Limits will not afford sufficient room to insert a Table of Logarithmes, and the Tables already published by others are sufficiently explained, in that point as Mr. *Briggs*, Mr. *Gunter*, Dr. *Newton*, Mr. *Wingate*, Mr. *Normood*, Mr. *Phillips*, &c. Every one shewing how by their own Tables to find the logarithme, of any number, or the number to any logarithme, therefore I shall proceed to shew their use in Arithmetick, *viz.* how to Multiply, Divide, and Extract Roots, &c. thereby, And First,

To Multiply by the Logarithmes.

In Multiplication by the logarithmes there are 3 Cases, *viz.* the Characteristicks of the logarithmes of the Factors are either both affirmative

mative, or both negative: or else they are the one affirmative, and the other negative?

I. When they are both Affirmative.

When the Characteristicks of the logarithmes of the Factors are both Affirmative, then the sum of those logarithmes is the logarithme of the fact or product.

Examples.

Multiply 34 ————— log. 1.531479

by 26 ————— log. 1.414973

Product 884 ————— log. 2.946452

Multiply 28.86 ————— log. 1.460296

by 8.9 ————— log. 0.949390

Product 2568.34 ————— log. 2.409686

Note that if you carry 10 to the Characteristick, it is affirmative, as in the last Example.

II. When they are both Negative.

When the logarithmes of the Factors have their Characteristicks both Negative, or defective, then the sum of their logarithmes is the logarithme of their product, the sum of their Characteristicks being also negative as in the following Examples.

Multiply .004 ————— log. ---3.602060

by .02. ————— log. ---2.301030

Product is .00008 ————— log. ---5.903090

Mul-

Multiply .025 ——— log. — 2.397940
by .42 ——— log. — 1.623249

Product is .01050 ——— log. — 2.021189

Multiply .093 ——— log. — 2.968483
by .058 ——— log. — 2.763428

Product .005394 ——— log. — 3.731911

And here note, that when you carry ten to the Characteristicks it is affirmative, and must be abated out of their sum as in the two last examples :

III. When they are Heterogeneous, *viz.* the one Affirmative, and the other Negative.

When the Characteristicks of the Factors are the one Negative, and the other Affirmative, then add the logarithmes together, and when you come to the Characteristicks, take their difference, and place it for the Characteristick of the Product, making it either Affirmative or Negative, according to the affection of that wherein lay the excess; and here note, that if you carry any thing to the Characteristicks, it is Affirmative, and must be added to the affirmative characterist. And in the following Examples.

Multiply 348 ——— log. 2.541579
by .64 ——— log. --- 1.806180

Product 222.72 ——— log. 2.347759

Multiply 348 ——— log. 2.541579
by .0064 ——— log. --- 3.806180

Product 2.2272 ——— log. 0.347759

Mul-

Multiply 3.48 ——— log. 0.541579
 by .0064 ——— log. 3.846180

Product 022273 ——— log. —2.347759

Multiply 3562 ——— log. 3.551693
 by .008 ——— log. 3.903089

Product 28.496 ——— log. 1.454782

CHAP. V.

Division by the Logarithmes.

I **T**O subtract the Logarithme of one number out of the logarithme of another is the same (and produceth the same effect) with Division in Natural Numbers, the Logarithme remaining being the Logarithme of the Quotient.

II. In Division by the Logarithmes there are three Cases, *viz.* First, when the Characteristicks of the Dividend, and of the Divisor are both Affirmative: Secondly, when they are both Negative. And Thirdly, when they are Heterogeneous, *viz.* the one Affirmative, and the other Negative. Of which in their order.

I. When

I. When they are both Affirmative.

III. When the Characteristicks of the Dividend, and of the Divisor, are both Affirmative, then if you subtract the Logarithme of the Divisor out of the logarithme of the Dividend, the remainder will be the logarithme of the Quotient. And if you borrow 10 from the Characteristicks, it is Affirmative.

Examples.

| | | |
|-------------|------|----------|
| Divide 468 | log. | 2.670246 |
| by 12 | log. | 1.079181 |
| <hr/> | | |
| Quotient 39 | log. | 1.591065 |
| Divide 144 | log. | 2.158362 |
| by 16 | log. | 1.204120 |
| <hr/> | | |
| Quotient 9 | log. | 0.954242 |

These Examples are so plain that they need no Explanation.

II. When they are both Negative,

IV. When the Characteristicks of the Dividend, and of the Divisor, are both Negative, subtract the Logarithme of the Divisor from the Logarithme of the Dividend, and the Remainder is the Logarithme of the Quotient, and if you borrow 10, it must be paid to the Index of the Divisor, affirmatively.

Examples.

Examples.

$$\begin{array}{rcl} \text{Divide} & .48 & \text{---} \text{---} \text{---} \log. \text{---} 1.681241 \\ (1) \text{ by} & .12 & \text{---} \text{---} \text{---} \log. \text{---} 1.079181 \end{array}$$

$$\text{Quotient } 4. \text{ ---} \text{---} \text{---} \log. \text{---} 0.602060$$

$$\begin{array}{rcl} \text{Divide} & .036 & \text{---} \text{---} \text{---} \log. \text{---} 2.556303 \\ (2) \text{ by} & .18 & \text{---} \text{---} \text{---} \log. \text{---} 1.255272 \end{array}$$

$$\text{Quotient } .2 \text{ ---} \text{---} \text{---} \log. \text{---} 1.301031$$

$$\begin{array}{rcl} \text{Divide} & .156 & \text{---} \text{---} \text{---} \log. \text{---} 1.193125 \\ (3) \text{ by} & .39 & \text{---} \text{---} \text{---} \log. \text{---} 1.591064 \end{array}$$

$$\text{Quotient } .4 \text{ ---} \text{---} \text{---} \log. \text{---} 1.602061$$

$$\begin{array}{rcl} \text{Divide} & .0171 & \text{---} \text{---} \text{---} \log. \text{---} 2.232996 \\ (4) \text{ by} & 9. & \text{---} \text{---} \text{---} \log. \text{---} 1.954242 \end{array}$$

$$\text{Quote } .019 \text{ ---} \text{---} \text{---} \log. \text{---} 2.278754$$

The first and second of the foregoing Examples are easily understood, and as for the third and fourth, all the difficulty therein is caused by borrowing 10 at the next figure to the Characteristicks, as in the third Example, in subtracting 5 out of 1. Now to make good the 10 borrowed, I pay 1 to the Characteristick of the Divisor, and because the said 1 is affirmative, and the said Characteristick negative, therefore subtract it from the Characteristick of the divisor, and there remains nothing; wherefore I take (0) out of the Characteristick of the Dividend, and there remains—1 for the Characteristick

stick of the Quotient. The same is to be understood in the fourth Example, and in all others of the same Nature.

A General Observation drawn from the third and fourth Rules foregoing.

V. If when the Characteristicks of the Dividend and Divisor be Homogeneous, (that is, both affirmative, or both Negative) the Characteristick of the Divisor is greater than the Characteristick of the dividend, then in this case subtract the Characteristick of the Dividend out of that of the Divisor, placing the remainder for the Characteristick of the Quotient, changing its sign, viz. if it be affirmative, make it negative, and if it be negative, make it affirmative. Remembring the Directions under the last Rule when you borrow from the Characteristicks.

Observe the following Examples.

$$\begin{array}{rcl} \text{Divide } 6.4 & \text{---} & \text{log } 0.806180 \\ (1) \text{ by } 80 & \text{---} & \text{log. } 1.903090 \end{array}$$

$$\text{Quotient } .08 \text{ ---} \text{---} \text{---} 2.903090$$

$$\begin{array}{rcl} \text{Divide } 6.4 & \text{---} & \text{log. } 0.806180 \\ (2) \text{ by } 800 & \text{---} & \text{log. } 2.903090 \end{array}$$

$$\text{Quotient. } .008 \text{ ---} \text{---} \text{---} \text{log. } 3.903090$$

$$\begin{array}{rcl} \text{Divide } 6.3 & \text{---} & \text{log. } 0.799340 \\ (3) \text{ by } 78.75 & \text{---} & \text{log. } 1.896251 \end{array}$$

$$\text{Quotient } .08 \text{ ---} \text{---} \text{---} \text{log. } 2.903089$$

Divide

$$(4) \quad \begin{array}{rcl} \text{Divide } 75 & \text{---} & \text{log. ---} 1.875061 \\ \text{by } .0015 & \text{---} & \text{log. ---} 3.176091 \end{array}$$

$$\text{Quotient } 500 \text{ --- log. --- } 2.698970$$

$$(5) \quad \begin{array}{rcl} \text{Divide } .64 & \text{---} & \text{log. ---} 1.806180 \\ \text{by } .008 & \text{---} & \text{log. ---} 3.903090 \end{array}$$

$$\text{Quote } 80 \text{ --- log. --- } 1.903090$$

$$(6) \quad \begin{array}{rcl} \text{Divide } 16.56 & \text{---} & \text{log. ---} 1.219060 \\ \text{by } 460 & \text{---} & \text{log. ---} 2.662758 \\ \text{Quote } .0036 & \text{---} & \text{log. ---} 2.556302 \end{array}$$

III. When they are Heterogeneous, *viz.* the one Negative, the other Affirmative.

VI. When the Characteristicks of the Dividend and the Divisor are Heterogeneous, proceed as in the two first Cases, till you come to the Characteristicks, and then instead of subtracting the one Characteristick from the other, add them together, so shall their sum be the Characteristick of the Quotient, and it is of the same kind with the Characteristick of the Dividend.

But here note that when you borrow 10 at the next figure to the Characteristick it must be paid to the Characteristick of the Divisor Affirmatively *viz.* If the Characteristick of the Divisor be affirmatively, then add 1 to it to that you borrowed, and if it be negative, subtract 1 from it. As in the following Examples.

$$(1) \quad \begin{array}{rcl} \text{Divide } 144 & \text{---} & \text{log. ---} 1.158362 \\ \text{by } 12 & \text{---} & \text{log. ---} 1.079181 \end{array}$$

$$\text{Quote } .012 \text{ --- log. --- } 2.069181$$

Divide

$$\begin{array}{rcl} \text{Divide} & 64 & \text{---} \text{---} \text{log. ---} 1.806180 \\ (2) \quad \text{by} & .08 & \text{---} \text{---} \text{log. ---} 2.903090 \end{array}$$

$$\text{Quote } 800 \text{ ---} \text{---} \text{log. } 2.903090$$

$$\begin{array}{rcl} \text{Divide} & .64 & \text{---} \text{---} \text{log. ---} 1.806180 \\ (3) \quad \text{by} & 800 & \text{---} \text{---} \text{log. ---} 2.903090 \end{array}$$

$$\text{Quote. } 0008 \text{ ---} \text{---} \text{log. ---} 4.903090$$

In the second of the foregoing Examples I borrow 1 (in the place next the Characteristick) by subtracting 9 out of 8, wherefore to make it good, I subtract 1 from —2 (the Characteristick of the Divisor,) because it is Negative, and the remainder which is 1 — I add to 1 (the Characteristick of the Dividend) and their sum is 2 for the Characteristick of the Quotient which is Affirmative, because the Characteristick of the Dividend is Affirmative.

And in the last Example, I likewise borrow 1 from the Characteristick, wherefore to make it good, I add 1 to the Characteristick of the Divisor, (because it is affirmative) and that makes it 3, which added to —1 (the Characteristick of the Dividend) makes —4 for the Characteristick of the Quotient, which here is negative, because the Characteristick of the Dividend is negative.

Other Examples for Exercise may be such as follow.

$$\begin{array}{rcl} \text{Divide} & 648 & \text{---} \text{---} \text{log. ---} 2.11575 \\ \text{by} & .36 & \text{---} \text{---} \text{log. ---} 1.556303 \end{array}$$

$$\text{Quote } 1800 \text{ ---} \text{---} \text{log. } 3.255272$$

Divide

Divide 6.4 ———— log. 0.806180
 by .08 ———— log. 2.903090

Quote 80 ———— log. 1.903090

Divide 68 ———— log. 1.832509
 by .08 ———— log. 2.903090

Quote 850 ———— log. 2.929419

CH A P. VI.

To raise the Powers of Numbers, *viz.* to find the Square, Cube, Biquadrate, or Squared Square, &c. of any number. Also to Extract the Square, Cube, Biquadrate, &c. Roots of any Number by the Logarithmes.

I. **B**Y the third Section of the Second Chapter of this Book it is evident, that if you add the Logarithmes of two numbers together, the Sum will be the Logarithme of their Product

duct ; And by the first and fourth Sections of the 9th Chapter of my Decimal Arithmetick, it appeareth that any number multiplied by it self, produceth its Square, wherefore if you double (or multiply by 2) the logarithme of any number, it will produce the logarithme of its Square, which if duly considered you will find that to Square, Cube, &c. any number, is nothing else but to multiply the Logarithme of the given number by the Index of the Power you would raise it to, viz. If you would find the Square of any number, multiply the logarithme thereof by 2, so shall the product thereof be the Logarithme of its Square ; and if you would find the Cube of any number, multiply its logarithme by 3, and the product thereof will be the Logarithme of its Cube ; and if you would find the Biquadrate of any number, multiply its Logarithme by 4, and it will produce the Logarithme of its Biquadrate, &c. As in the following Example.

Let it be required to find the Square of 12

The Logarithme of 12 is 1.079181

$$\begin{array}{r} 1.079181 \\ \times 2 \\ \hline 2.158362 \end{array}$$

Which being multiplied by 2, produceth 2.158362, which is the Logarithme of 144, viz. the Square of 12.

Again let it be required to find the Square of 94.

The Logarithme of 94 is ——— 1.9731278
2

3.9462556

Which being multiplyed by 2, produceth 3.9462556, which is the logarithm of 8836, which is the Square of 94.

II. But if the characteristick of the logarithme be negative, that is if the given number, whose Square, Cube, Biquadrate, &c. you would find to be a Decimal Fraction, observe, that in multiplying the next figure to the Characteristick the ten, or tens to be born in mind are affirmative, and are to be deducted out of the Product of the negative Characteristicks.

Observe the several Examples following

What is the Square of .7 log.—1.845098
.7

Facit .49 log.—1.690196

What is the Square of .09 log.—2.954242
.09

Facit. 0081 log.—3.908484

What

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What is the Cube of 12? ——— log. 1.079181

3

Facit 1728 for $12 \times 12 \times 12 = 1728$ log. 3.237543

What is the Cube of 05; ——— log. —2.698970

3

Facit.000125 for $.05 \times .05 \times .05 = .00125$ log. —4.096910

What is the Biquadrate of 9? — log. 0.954242

4

Facit 6561 for $9 \times 9 \times 9 \times 9 = 6561$ log. —3.816963

What is the Biquadrate of .08? log. —2.903090

4

Facit .00004096 for $.08 \times .08 \times .08 \times .08 = .00004096$ whose logarithme is } —5.612369

What is the fifth Power of 6? — log. 0.778151

5

Facit 7776 = $6 \times 6 \times 6 \times 6 \times 6$ ——— log. 3.890755

The like is to be observed of all others.

To Extract the Square, Cube, Biquadrate, &c. Roots of any given numbers by the Logarithmes.

III. From a due consideration of the first Section of this Chapter, it may easily be perceived, that

To Extract the Cube Root of any number is to bipart (or divide by 2) its logarithme, so shall

R 2

that

that biparted logarithme be the logarithme of the Square Root desired.

Example.

Let it be required to Extract the Square Root of 75832

$$q. = 75832 \text{ ————— } \log. 4. 879852$$

$$\sqrt{} = 275.37 \text{ ————— } \log. \text{Biparted } 2.439926$$

Let it be required to find the Square Root of 4489.

$$q. = 4489 \text{ ————— } \log. 3.652149$$

$$\sqrt{} = 67 \text{ ————— } \log. \text{Biparted } 1.826074$$

In the first of these Examples the logarithme of 75832 is 4.879852 which being biparted (or divided by 2 gives 1.826074 for the logarithme of (265.37) the Root required.

And in the second Example 3.652149 (the logarithme of 4489 being biparted gives 1.826074 for the logarithme of (67) its Square Root.

So will the Square Root of 36783 be found to be 191.789 *fere*, and the Square Root of 3866 will be 62.17717 *fere*. And the Square Root of 95 will be found to be 9.7468, &c.

To extract the Cube Root of any number is to tripart (or divide by 3) its logarithme, so shall this triparted logarithme be the logarithme of the Cube Root required.

Example

Examples.

Let it be required to Extract the Cube Root of
157464.

The Cube 157464 ———— log. 5.197181
 ³)—————
 $\sqrt[3]{c}$. .54 ———— log. triparted 1.732393

Let it be required to find the Cube Root of
187237601580329.

The Cube 187237601580329-log. 14.27239308
 $\sqrt[3]{c.}$ 47209 — its triparted — $\log. 4.75746436$

In the first of these Examples where it is required to extract the Cube Root of 157464, its logar. is 5.197181 which being divided by 3 hath for its third part 1.732393 which is the logarithme of (54) the Cube Root of 157464, which was required. And the same is to be observed in finding the Cube Root of 187237601580329 by the Logarithmes; or of any other positive number whatsoever as you may see by the following Examples.

The Cube = 8 ————— its log. 0.903090
 3) ————
 $\sqrt[3]{c} = 2$ ————— its log. 0.301030

The Cube 125 ————— its log. 2.096910
 ³⁾ —————
 $\sqrt[3]{c.} = 5$ ————— its log. 0.698970

R 3

The

The Cube 15.625 ————— its log. 1.193820

$\sqrt[3]{c.} = 2.5$ ————— its log. 0.397940

To Extract the Biquadrate Root of any given number, do thus, *viz.* Take
To extract the Bi- its logarithme, and divide
quadrate Root of it by 4, so shall the fourth part
any Number. thereof be the logarithme of
 the biquad. Root required, as
 in the following Example.

Let it be required to extract the Biquadrate Root of 256.

Biquadrate number given 256 — its log. 2.408239

4) —————
 Its $\sqrt[4]{\text{biquad.}}$ = 4 ————— its log. 0.602059

Here the log. of the given biquadrate number, *viz.* 256 is 2.408239 which being divided by 4, giveth 0.602059 for the *logarithme* of (4) the biquadrate Root required,

In like manner if you would extract the Root of the fifth Power of any number given, Divide its *logarithme* by 5, so shall the Quotient be the *logarithme* of its Root. And if you would find the Root from the sixth power of any Number, divide its *logarithme* by 6, and the Quote is the *logarithme* of the Root desired, &c.

IV. But here you are to observe in Extracting
 the Square, Cube, Biquadrate, or any other Root
To extract the Square, of a negative number; or
Cube, &c. Roots of ne- decimal by the *logarithmes*,
gative numbers by the that if you cannot evenly
Logarithmes. divide

divide the Index or Characteristick of the *logarithme* without the remainder, then add to the said Characteristick so many units till it may be divided without any remainder, and place the Quotient for a new *Characteristick*, (belonging to the Root;) Then look how many units you lent to the *Characteristick*, and esteem them so many tens to be prefixed to the *logarithmetical figure* immediately following the *Characteristick*, then proceed to finish the work, so shall this new *logarithme* be the *logarithme* of the Root required, which will also be negative.

Examples. follow.

What is the Square Root of .144?

Square given = .144 ———— its *log.* — 1.158362
2)

Its \sqrt{q} = 37947 ———— its *log.* biparted. -- 1.579181

What is the Square Root of .00324

Square given = .00324 ———— *log.* — 3.510545
2)

Its \sqrt{q} = 05621 ———— *log.* biparted. — 2.755272

What is the Cube Root of .000512

Cube given = 000512 ———— its *log.* — 4.709269
3)

Its $\sqrt[3]{c}$ = .05 ———— its *log.* triparted — 2.903089

In the first of these Examples, where it is required to Extract the Square Root of .144, its *logar.* is — 1.158362, which (according to the third Rule) I should bipart, (or divide by 2)

R 4

And

And because its negative *Characteristick* (-1) cannot be evenly divided by 2, I increase it by an unit, and it makes 1 , then will the quote be -1 for the Index of the *log.* of the *Root*, then do I proceed to the next figure, to the *Characteristick* which is 1 , and because it added 1 to the *Characteristick*, therefore I increase the next *Figure* by adding 10 to it. (or prefixing 1 before it) and then it is 11 , &c. So I find the *logarithme* of the *Root* to be -1.575572 , viz. $.37947$.

And in the third Example, where it is required to find the Cube Root of $.000512$, the Index of its *logarithme* is -4 , which cannot be evenly divided by 3, therefore I add 2 to it to make it 6, and the Quotient is -2 for the Index of the *logarithme* of the *Root*, then because I added to the Index -4 , therefore I increase the next *Figure* to it with 2 tens, making it 27 &c. So is the Cube Root required found to be 05.

Observe the like in extracting of any other Roots :
Otherwise, you may make use in the following
Table.

| | A | B |
|-------|-----------------------|----|
| 2 | { —1—2 | —1 |
| | { —3—4 | —2 |
| | { —5—6 | —3 |
| | { —7—8 | —4 |
| | { —9—10 | —5 |
| 3 | { —1—2—3 | —1 |
| | { —4—5—6 | —2 |
| | { —7—8—9 | —3 |
| 4 | { —1—2—3—4 | —1 |
| | { —5—6—7—8 | —2 |
| 5 | { —1—2—3—4—5 | —1 |
| | { —6—7—8—9—10 | —2 |
| 6 | { —1—2—3—4—5—6 | —1 |
| | { —7—8—9—10—11—12 | —2 |
| <hr/> | | |
| | 50. 40. 30. 20. 10. 0 | |
| <hr/> | | |

The use of the foregoing Table.

In the foregoing Table the *Figures* 2. 3. 4. 5. 6. placed on the left hand, are in the Indices of Powers, whose *Roots* are required to be extracted, or they are Divisors by which to divide the *logarithme* of any given power, in order to find out its Root : As the number 2 (which is the uppermost) is the Divisor for finding the *logarithme* of the Square Root of any number : And 3 the Divisor for the Cube ; and 4 for the Bi-quadrature, &c. The *Figures* placed between the perpendicular

pendicular line, and the several lines of connection, and under A are the *Characteristicks* of the *logarithmes* of Negative or Decimal Numbers, whose *Roots* are required to be extracted; And the Figures placed on the right hand of the perpendicular line under B, are the *Characteristicks* of the *Logarithme* of the several *Roots*: And the numbers at the bottom of the Table, viz. 50, 40, 30, &c are the numbers to be added, or rather prefixed to the first *Figure* of the *logarithme* next the *Characteristick* whose Negative Index is found in the same series or Collum even with the Divisor, &c.

Example.

Let it be required to extract the *Cube Root* of
405224.

The *Logarithme* of the given Number is
—1.6076951.

And the Divisor whereby to extract the *Cube Root* is 3, which I find in the foregoing Table on the left hand; then on the right hand of its line of connection, I find the *Characteristick* of the *Logarithme* —.6076951 which is —1, and just against it on the right hand under B I find —1 for the *Characteristick* of the *Logarithme* of the *Root*, and in the bottom line under the said —1, in the same series, I find 20 which is to be prefixed to the *Figure* in the *Logarithme* next the *Characteristick*, &c. and having finished Division, I find the *Logarithme* of its *Root* to be —1.8622317 which is .74.

So if the *Logarithme* —6.418426 were to be divided by 4, First the Divisor 4 on the left

left hand of the Table, and find the *Characteristick* —6, behind its line of connection just against which on the right hand of the perpendicular line you have —2 for the Index or *Characteristick* of the Quotient; and at the bottom, just underneath—6 you have 20, which being added to (4) the first figure of the Dividend next the *Characteristick* makes it 24, in which the Divisor 3 is contained 6 times, &c. See the work.

Biquad. proposed. 0000026207 —log. —6.418426

The $\sqrt{(4)} = 040235$ ————— —log. —2.604606

CH A P. VII.

Of the Use of Log. in Comparative Arithmetick.

PROP. I.

Having three Numbers given, to find a fourth proportional.

THIS is nothing else but the work of the Rule of 3, and it may be thus performed,
viz

Add

Add the *Logarithme* of the second and third Numbers together, and from their sum subtract the *Logarithme* of the first, so shall the remainder be the *Logarithme* of the fourth, as in the following Example.

The 3 given numbers are 3, 24, and 108 unto which it is required to find a fourth proportional.

$$3 : 24 :: 108 : 864$$

The Operation by the Logarithmes.

| | | | |
|--------------------------|-------|----------|-------|
| As 3—its <i>log.</i> | ————— | 0.477121 | |
| Is to 24 its <i>log.</i> | ————— | 1.380211 | } add |
| So 108 its <i>log.</i> | ————— | 2.033424 | |

The sum of the 2 last *log.* is 3.413635

From which if you subtract the *log.* Of the first, the remainder is 2.936514

Which is the *Logarithme* of 864. the fourth proportional required.

The former work may be somewhat shortned, if instead of the *Log.* of the first you take its Complement Arithmetical (which is nothing else but to set down what every *Figure* wants of 9, till you come to that next the right hand, and then set down what it wants of 10) and then add them all 3 together, and cancel the first *Figure* of the sum on the left hand, and then will the sum be the *Log.* of the Answer, as will appear by working the foregoing Examples.

As

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As 3 Comple. Arith. of its *log.* — 9.521879 }
 Is to 24 — its *log.* — 1.380211 } add
 So is 108 — its *log.* — 2.033424 }

 To 864 — its *log.* — 12.936514

P R O P. II.

Between two Numbers given to find a Mean proportional.

When the *Logarithmes* of the numbers propounded are homogeneal, viz. both Affirmative, or both Negative, add them together, then bipart that *Logarithmetical* sum, so have you the *Logarithme* of the mean proportional required, which *Logarithme* so found is of the same kind with the *Logarithmes* of the number given

Let it be required to find a mean proportional between 18 and 6.

18 — *log.* 1.25527
 6 — *log.* 0.77815

 2) 2.02342

 10.392 *log.* 1.01171

In this Example the *Logarithme* of 18, and 6 being added together make 2.02342 which is the *Logarithme* of 108, and that *Logarithme* being divided by 2 (which is the same with extracting the Square Root of its significant number;

ber, as in the third Rule of the sixth Chapter) the Quotient is 2.02342, the *Logarithme* of 10,392 which is the mean proportional required.

Example 2.

Let it be required to find a mean proportional between .018 and .006.

.018 its log. 2.25527
 .006 its log. 3.77815

.006 its log. ———— 3.77815

$$2) - 4.03342$$

.010392 log. ———— 2.01671

II. But when the *Characteristicks* of the *Logarithmes* of the given Numbers be Heterogeneous, viz. the one Affirmative, and the other Negative; add the *Logarithmes* together as before, till you come to the *Characteristick*, then subtract the lesser *Characteristick* out of the greater, (according to the third Rule of the fourth Chapter,) which being done, bipart the Logarithmetical production, so shall the Quote be the *Logarithme* of the mean proportional required, which will always be of the same kind with that *Logarithme* of the given Numbers, whose Index is greatest, as in the following Examples.

Example

Example 1.

What is the mean proportional between 36 and .5 ? Facit 4.2427.

Here the *Log.* of $\left\{ \begin{array}{l} 36 \\ .5 \end{array} \right\}$ is $\left\{ \begin{array}{l} 1.556302 \\ ---1.698970 \end{array} \right\}$

Their sum is $2) \underline{1.255272}$

which being divided by 2, gives
the *Log.* of the me. prop. which } 1.627636
is the *Log.* of 4.2427.

Example 2.

What is the mean proportional between 12 and .75 ? Facit 3.

The *Log.* of $\left\{ \begin{array}{l} 12 \\ .75 \end{array} \right\}$ is $\left\{ \begin{array}{l} 1.079181 \\ ---1.875061 \end{array} \right\}$

Their sum is $2) \underline{0.954242}$

which being divided by 2 }
gives the *Log.* of 3 the mean } 0.477121
proportional required.

for .75 : 3 :: 312 : 3

PROP.

P R O P. III.

Between 2 Numbers given, to find two mean Proportionals.

Whether the numbers given be Homogeneous, or Heterogeneous, subtract the *Logarithme* of the lesser extream from the *Logarithme* of the greater extream, then take $\frac{1}{3}$ of the difference of the said *Logarithmes*, and add it to the *Logarithme* of the lesser extream, so will the sum be the *Logarithme* of the lesser mean; then add the same Difference to the said *Logarithme* of the lesser mean, and the sum will be the *Logarithme* of the greater mean; still observing the Rules delivered in the fourth and fifth Chapters of this book, in adding and subtracting of *Logarithmes*.

Examples.

Ex. 1. Let it be required to find 2 mean proportionals between 144 and 12?

The Log. of $\left\{ \begin{array}{l} 144 \\ 12 \end{array} \right\}$ is $\left\{ \begin{array}{l} 2.158362 \\ 1.079181 \end{array} \right\}$

The Difference ——— 3) 1.079181

$\frac{1}{3}$ of the Differ. 0.359727

Lesser mean 12.473 log. 1.438908

Greater mean 62.899 log. 1.798635

Example

Example. 2. Let it be required to find 2 mean proportionals between .75 and .05?

The Log. of $\left\{ \begin{array}{l} .75 \\ .05 \end{array} \right\}$ is $\left\{ \begin{array}{l} -1.875061 \\ -2.698970 \end{array} \right.$

The Difference $\text{---} 3 \overline{) 1.176091}$
 $\frac{1}{3}$ part of the Diff. is 0.392030

Lesser mean is .12331 $\text{---} \log. 1.091900$

Greater mean is .30411 $\text{---} \log. 1.484030$

Example. 3. Let it be required to find 2 mean proportionals, between 125 and .05?

The Log. of $\left\{ \begin{array}{l} 125 \\ .05 \end{array} \right\}$ is $\left\{ \begin{array}{l} 2.096910 \\ -2.698970 \end{array} \right.$

Their Difference $\text{---} 3 \overline{) 3.397940}$

$\frac{1}{3}$ of their Differ. $\text{---} 1.132646$

The lesser mean is .67860 Log. $\text{---} 1.831616$

The greater mean is 9.2101 Log. 1.964262

P R O P. IV.

Three Numbers given to find a fourth in a Duplicate Proportion.

Take the Logarithmes of the two Numbers which have one and the same Denomination, and
 S sub-

subtract the lesser Logarithme from the greater, and double the remainder, (that is multiply it by 2.) Then if the first number be less than the second, add the said double difference to the Logarithme of the other Number, so will the sum be the Logarithme of the fourth number, or number required, as in *Example*.

The superficial content of a Circle whose Diameter is 14 Inches is 154 Square inches, I demand the Content of another Circle, whose Diameter is 25 Inches? Facit 491.07 square Inches. See the operation.

Diam. 14 Inches its *log.* ——— 1.146128

Diam. 25 Inches its *log.* ——— 1.397940

The Difference of the *log.* ——— -0.251812

2

Their Difference doubled ——— 0.503624

The given content its *log.* ——— 2.187521

The Cont. required 491.07. *log.* 2.691145

But if the first number be greater than the second, then instead of adding the doubled difference to the other number, subtract it therefrom, so shall the remainder be the Logarithme of the number required, as in the following *Example*.

There is a Circle whose Diameter is 8 Inches, and its superficial Content is 616 square Inches, I demand what is the Superficial Content of another Circle, whose Diameter is 25 Inches? Facit 491.07 Square Inches, as in the former *Example*

Diameter

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Diameter 28 Inches its *log.* ——— 1.447158

Diameter 25 Inches its *log.* ——— 1.397940

The Difference of the *log.* ——— 0.049218

2

The Difference doubled ——— 0.098436

The given Content 616, its *log.* 2.789581

Content required 491.07, its *log.* 2.691145

P R O P. V.

*Having 3 Numbers given to find a fourth
in a Triplicate Proportion.*

Triple the Difference of the Logarithmes of the two given Terms, which have the same Denomination. Then if the first Term be less than the second, add the said Triple Difference to the Logarithme of the other Term, so shall the sum be the Logarithme of the fourth Term required, as in the following Example.

There is a Bullet whose Diameter is 4 Inches, and its weight is 9 l. I demand what weight a Bullet of the same metal will be, whose Diameter is 8 Inches ? Facit 72 l. view the following work.

Diameter 4 Inches ——— *log.* — 0.602060

Diameter 8 Inches ——— *log.* — 0.903090

The Difference of the *logar.* ——— 0.301030

3

The Difference Tripled is ——— 0.903090

The given weight 9 *l.* — *log.* — 0.954243

The weight required is 72 *l.* — *log.* 1.857333

But if the first term be greater than the second then subtract the said Tripled Difference from the logarithme of the other term, so shall the remainder be the logarithme of the fourth number required. As in Example.

There is a Bullet whose Diameter is 8 Inches, and its weight is 72 pounds, I demand the weight of another Bullet of the same Mettal, whose Diameter is 4 Inches? Facit 9 *l.* See the operation, it being the converse of the former.

Diameter 8 Inches, its *log.* ——— 0.903090

Diameter 4 Inches, its *log.* ——— 0.602060

The Difference of the *log.* is ——— 0.301030

3

The Difference Tripled is ——— 0.903090

The given weight 72, its *log.* ——— 1.857333

The weight required 9 *l.* its *log.* 0.954243

C H A P.

C H A P. VIII.

Of Anatocisme, or Compound Interest, wherein is shewed how by the Logarithmes to answer all Questions concerning the Increase, or present worth of any Sum of Money or Annuity, for any Term of Years, or at any Rate of Interest. According to the six Fundamental Theorems invented and laid down by Mr. Oughtred in his Treatise *De Anatocismoſive Uſura Compoſita*, annexed to his *Clavis Mathematicæ*.

I. **W**hen any Question in Compound Interest is proposed, it will fall under one of the six Cases following, viz.

S 3

I. To

1. To find the Increase or amount of any sum of money put out at Compound Interest for any number of years, and at any Rate of Interest propounded.

2. To find the present worth of any sum of money due at the end of any number of years to come, Rebate being allowed at any Rate of Compound Interest.

3. To find the increase, or amount of an Annuity being forborn for any number of years at any Rate of Compound Interest.

4. To find what Annuity any sum of Money due at the end of any Term of years to come will purchase at any Rate *per Cent*.

5 To find the present worth of any Annuity to continue any number of years, allowing Rebate at any Rate *per Cent*.

6. To find what Annuity any Sum of Money will purchase for any number of years, and at any Rate of Interest proposed.

II. When any Question in Compound Interest is propounded, find out the Interest of 1 *l*. and let 1 *l*. with its interest be the Rate of Interest implied in the Question, as if any Question were proposed at 8 *per Cent*. the Int. of 1 *l*. for a year is .08 and the rate of Interest is 1.08, if at 6 *per Cent*. the Rate is 1.06, &c. of which find out the Logarithme.

III. When any Annuity or Debt the payments be half yearly, Quarterly, or Monthly, &c. you are to divide the Logarithme of the said Rate by 2, 4, or 12, &c. so shall that Quotient be the Logarithme of the Rate, as suppose any Question were propounded at 8 *per Cent*, the
Rate

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Rate of Interest here implied is 1.08, for

$$100 : 108 :: 1 : 1.08$$

Which said Rate is for yearly payments, the Logarithme whereof is 0.0374204, but if the payments are to be half yearly, then if you divide the said Logarithme by 2, it will give you 0.0187102 for the Logarithme of the Rate, and if the payments be Quarterly, divide the said Logarithme of 1.08 by 4, and it will give you 0.0093551 for the Log. of the Rate, and if the payments be monthly, then if you divide the said Log. of 1.08 by 12, it will give you 0.0031183 for the Log. of the Rate, &c. and this is generally the first thing to be observed in every Question, as you will find by the following Examples.

CASE I.

To find the Increase or Amount of any Sum of Money put out at Compound Interest for any Term of years, and at any Rate of Interest propounded.

Quest. 1. if 50 l.—16 s. be put out at 8 per Cent. Compound Interest. for 7 years, I demand how much will then be due to the Creditor?
Facit 87 l.—01 s.—02 $\frac{3}{4}$ d.

IV. Multiply the Logarithme of the Rate by the number of years, and the product will give you the Logarithme of the Amount of 1 l. for the proposed time, to which if you add the Log. of the Sum propounded, the sum will be the Log. of the Answer.

S 4

The

The Operation by the Logarithmes.

The log. of 1.08 the Rate prop. 0.0334237

The numb. of years propounded. 7

| | | |
|---|-------------|-------|
| The log. of the Increase of 1 l for 7 years. _____ | } 0.2339659 | } add |
| The log. of (50.8 l.) the sum proposed. _____ | | |

The log. of (87.061) the answer. } 1.9398296

which is 87 l. — 01 s. — 2 $\frac{3}{4}$ d. fere

Quest. 2. What is the amount of 76 l. — 04 s.
for 3 $\frac{1}{2}$ years at 8 per Cent? Facit 97 l. — 17 s. — 01 d.

The Operation by the Logarithmes.

The log. of (1.08) the given
Rate per Annum ? } 0.0334237

| | |
|---|-------------|
| 4) _____ | } 0.0083559 |
| which divided by 4, gives the log. of the Quarterly Rate | |

13

0.0250677

083559

| | |
|--|-------------|
| which multiplied by 13 the Quarters in 3 years give the | } 0.1086267 |
| log. of the increase of 1 l. | |

the log. of (76.2) the given sum 1.8819547

the log. of (97.854) the Answ. 1.9905814

Quest.

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Quest. 3. If 50 *l.* be put forth at Interest for 20 years at $6\frac{1}{4}$ *per Cent.* I demand how much it will be increased to at the end of the said time. Facit 168 *l.*—01 *s.*—10 *d.*

The rate of Interest here proposed is $6\frac{1}{4}=6.25$ *per Cent.* therefore to find out the Rate of 1 *l.* for a year, say by the Rule of proportion.

$$\begin{array}{ccccccc} l. & & l. & & l. & & l. \\ 100. & : & 106.25 & : : & 1 & : & 1.0625 \end{array}$$

So that the Rate of Interest implied in the Question fit for Calculation by the Log. is 1.0625 according to the second Rule of this Chapter, behold.

The Operation.

$$\begin{array}{lcl} \text{The log. of (1.0625) the given} & \} & \\ \text{Rate} \text{-----} & \} & 0.0263289 \\ \text{The number of years propound.} & & 20 \\ \text{The log. of the amount 1 } l. \text{ in} & \} & \\ \text{20 years} \text{-----} & \} & 0.5265780 \\ \text{The log. of (50) the sum pro-} & \} & \\ \text{posed,} \text{-----} & \} & 1.6989700 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{add}$$

$$\text{The log. of 168.090} \text{-----} 2.2255480$$

Which is 168 *l.*—01 *s.*—10 *d.* very near, and so much will 50 *l.* be increased to in 20 years at 6 *l.*—5 *s.* *per Cent.*

C A S E 2.

To find the present worth of any sum of money due at the end of any number of years to come, Rebate being allowed at any Rate of Compound Interest.

V. When it is required to find the present worth of any sum of Money, first find the amount of 1 *l.* for the proposed time, and at the Rate of Interest propounded, then find the Logarithme of the sum proposed to be Rebated, and from it subtract the Logarithme of the amount of 1 *l.* (found as before) and the remainder will be the Logarithme of the present worth of the sum proposed. As in the following Example.

Quest. 4. What is 30 *l.* that is due 7 years hence worth to be paid presently, allowing Rebate at 8 per Cent? Facit 17 *l.* — 10 *s.* — 01½ *d.* as you may perceive by

The Operation by the Logarithmes.

The log. of (1.08) the proposed Rate — 0.033424
The time proposed ————— 7

The log. of the amount of 1 *l.* for 7 years 0.233968
The log. of 30 ————— 1.477121

The log. of (17.506) the Answer ——— 1.243153

which is 17 *l.* — 10 *s.* — 01½ *d.* and so much is the present worth of 30 *l.* due 7 years hence.

Quest. 5.

Quest. 5. What is the present worth of 120 l. due 2 years hence, allowing Rebate at 6 per Cent?
Facit 106 l.—15 s.—11 d.

The Operation of the Logarithmes.

The log. of (1.06) the proposed Rate—0.025306
 The time proposed ————— 2

The log. of the amount of 1 l. for 2 years, 0.050612
 The log. of 120 ————— 2.079181

The log. of (106.79) the Answer. — 2.028569

which is 106 l.—15 s.—11 d. and so much is the present worth of 120 l. due 2 years hence.

C A S E. 3.

To find the Increase, or Amount of an Annuity, being forborn any number of years, at any Rate of Compound Interest.

VI. For Resolving Questions concerning the forbearance of Annuities, you are (by the fourth Rule) first to find out the Amount of 1 l. for the Time, and at the Rate of Interest propounded.

Secondly, Find out the Logarithme of the said amount made less by 1, and also the log. of the Rate made less by 1, and subtract the latter from the former, so shall the remainder be the log. of the amount of 1 l. Annuity for the term of years propounded, to which if you add the Logarithme of the proposed Annuity, the sum will be the Logarithme of the Amount, or Increase of the said Annuity. As in the following Example.

Quest.

Quest. 6. What will be the Amount, or Increase of 48 *l.*—16 *s.* *per an.* for 7 years, Compound Interest being Computed at 8 *per Cent.* Facit 435 *l.*—08 *s.*—05 $\frac{1}{2}$ *d.* *fere.* See

The Operation by the Logarithmes.

The log. of 1.08 the given rate is 0.033424 } mult.
The time propounded ————— 7 }

The log. of (1.7138) the amount } 0.233968
of 1 *l.* for 7 years. ————— }

1.7138—1=7138 its log. — 1.853577 } subtr.
1.08—1=08 its log. — 2.903090 }

The difference of the log. which }
is the increase of 1 *l.* annuity } 0.950487 } add
is ————— }

The log. of (48.8) the Annuity } 1.688419 }
proposed ————— }

The log. of (435.422) the amount } 2.638906 }
of the proposed Annuity ————— }

which is 435 *l.*—8 *s.*—5 $\frac{1}{2}$ *d.* very near, and so much will be the Increase of an Annuity of 48 *l.* 16 *s.* in 7 years, at 8 *per Cent.* Compound Interest.

Quest. 7. There is an Annuity of 50 *l.* forborn to the end of 10 years, I demand how much is then due, Compound Interest being computed at 6 $\frac{1}{4}$ *per Cent.*? Facit 666 *l.*—16 *s.* as you will find by

The

The Operation by the Logarithmes.

The log. of (1.0625) the Rate 0.0263289 }
The term of years ————— 10 } mult.

The log. of (1.8335) the amount of 1 l. for 10 years. } 0.2632890

1.8335 — 1 = .8335 its log. — 1.9209056 }
1.0625 — 1 = .0625 its log. — 2.7958800 } subtr.

The log. of the amount of 1 l. annuity for 10 years ————— } 1.1250256 }
The log. of (50) the Annuity proposed. ————— } 1.6989700 } add

The log. of (666.80) the amount of the annuity proposed ————— } 2.8239956 }
which is 666 l. — 16 s. and so much will be due at the end of the said time.

C A S E.

To find what Annuity any sum due at any time to come will purchase to continue for any time, and at any Rate of Interest proposed.

VII. The Operation in this Case is the same in every respect with that in the former Case, only whereas in the last case you subtracted the log. of the rate less 1, from the log. of the increase of 1 l. less 1, so in this you must subtract the log. of the increase of 1 l. less 1, from the log. of the rate less 1, as in the following Example.

Quest.

Quest. 8. There is 705 *l.* due at the end of 7 years to come, I demand what Annuity to continue 7 years, the same will purchase, Compound Interest being allowed at 8 *per Cent*? Facit 79.015 *l.* = 79 *l.* — 00 *s.* as you may find if you observe

The Operation by the Logarithmes.

The log. of (1.08) the proposed
Rate ————— } 0.033424 } mult.
The proposed Time ————— 7

The log. of the increase of 1 *l.* }
for 7 years 1.738 ————— } 0.233968
1.08 — 1 = .08 its log. — 2.903090 } subtr.
1.7138 — 1 = .7138 its log. — 1.853577 }

The log. of the value of 1 *l.* — 1.050513
The log. of (705) the purchase }
money. ————— } 2.848189

The log. of the purchase (79.015) 1.898702
which is 79 *l.* — 00 *s.* — 4 *d.* *fere.*

Questions of this Nature may be solved at two Operations by the second and sixth Cases; First by the Rule in the second Case find the present worth of the sum propounded, then by the sixth, find what Annuity such a sum will purchase.

C A S E. 5.

To find the present worth of an Annuity to continue any Term of years, howsoever payable, viz. either yearly, half yearly or Quarterly, Rebate being allowed at any rate per Cent.

VIII. Find out the Logarithme of the Rate, and multiply it by the number of Years or Quarters,

ters, according as the Annuity is payable, and that will produce the Logarithme of the increase of 1 *l.* for the proposed time, to which add the Log. of the Rate made less by 1, and subtract that sum from the Log. of the increase of 1 *l.* made less by 1, so shall the remainder be the Log. of the present worth of 1 *l.* annuity for the time proposed to which add the logarithme of the proposed Annuity, and the sum will be the *Logarithme* of the present worth of the given Annuity. As in Example.

Quest. 9. What is the present worth of an Annuity of 30 *l.* payable by yearly payments, and to continue 30 years, allowing Rebate after the Rate of 8. *per Cent. per Annum*?

Facit 337 *l.*—14 *s.*—09 $\frac{1}{2}$ *d.* as appears in

The Operation by the Logarithmes.

The log. of (1.08) the propos. rate 0.033424 } mult.
The term of years ————— 30 }

The log. of (10.063 *l.*) the in- } 1.002720 }
crease of 1 *l.* for 30 years. } add
The log. of (80) the rate less 1 — 2.903090 }

—————
— 1.905810 subtr.

The log of 10.063 — 1 = 9.063 0.957272

The log. of the present worth }
of 1 *l.* Annuity ————— } 1.051462 }
The log. of (30) the proposed }
Annuity ————— } 1.477121 } add

The log. of (337.74) the present }
worth of the proposed An- } 2.528583 }
nuity. ————— }

which is 337 *l.*—14 *s.*—09 $\frac{1}{2}$ *d.*

CASE

CASE 6.

To find out what Annuity to continue any term of years any given sum of Money will purchase at any Rate of Compound Interest.

IX. When you would know what Annuity any given sum will purchase, first (as in the foregoing Rules) find out the Logarithme of the Rate, which multiply by the proposed time, so will that product be the Logarithme of the encrease of 1 *l.* to which add the Log. of the rate made less by 1, and from that sum subtract the Log. of the said increase of 1 *l.* made less by 1, so will the remainder be the Log. of what 1 *l.* will purchase for the proposed Time, to which if you add the Log. of the given purchase money, the sum will be the Log. of the Annuity that the given sum will purchase. As in Example,

Quest. 10. What Annuity to continue 7 years, and payable by Quarterly payments will 246 *l.* purchase. Allowing Rebate at 8. *per Cent*? Facit 12. 297 *l.*

The Operation, by the Logarithmes.

The log. of 1.08 the given Rate *per An.* — 0.033424

which divided by 4 gives the log. of } 0.008356 } mult.
(1.0194) the Rate *per Quarter.* — }
The Quarters in 7 years — 28

66848

16712

The Log. of the encrease of 1 *l.* for 28 } 0.233968 } add
Quarters, *viz.* 1.7138 — }
1.0194 = .0194 its log. — 2.287801

Sum — 2.521769

1.7138 — 1 = .7138 its log.

— 1853577 } subtr.

The log. of the purchase of 1 *l.* — 2.668192 } add
The log. of the proposed Sum 264 — 2.421604

The log. of (12.297) the Annuity } 1.089796 }
which the said sum will purchase }

which is 12 *l.* — 05 *s.* — 11 $\frac{1}{2}$ *d.*

More variety of Questions might be stated, but these to the Ingenious are sufficient.

FINIS.

ALGEBRAICAL DEFINITIONS.

CHAP. I.

Concerning the construction of Collick Powers, and the way of expressing them by Letters, together with the signification of all such Characters or Marks as are used in the ensuing Treatise.

1. **T**HE Analytical Art generally called Algebra, is that by which, when a Problem, or hard Question is propounded, we assume the Quantity
- T 2
- or

or Number sought, as if it were really known; and, with this assumed Quantity, and the Quantity and Quantities given, we proceed by undeniable Consequences, until the Quantity first assumed is found to be equal to some quantity or, quantities really known, and is therefore it self also known

II. Algebra is either Numeral, or Literal.

III. Numeral Algebra is so called, because all the given Quantities in any Question are expressed by Natural Numbers, and the number or quantity sought is solely represented by some Letter or Character taken at the pleasure of the Artist.

IV. Literal Algebra is so called, because when a question is resolved after this method, the known or given quantities as well as the unknown, are all expressed by Letters of the Alphabet, or some other Convenient marks or Characters, and this is also called, Specious Algebra; and when a question is resolved after this manner, at the end of the operation, there is discovered, a Canon, directing how the question proposed, or any other of the like nature may be solved, and therefore is Literal Algebra, accounted more excellent than Numeral Algebra, for that produceth not a Canon without extraordinary difficulty; because the numbers first given are by Arithmetical operations so interwoven and confounded, that it may seem a task too tedious for the most ingenious Artist to trace out their footsteps.

V. The

V. The Doctrine of Algebra consists in the knowledge of certain quantities called *Cosick Powers*, which we shall immediately explain.

VI. In a series, or rank of Geometrical proportionals continued, proceeding from Unity or one, whether they be ascending, or descending, all the numbers or Terms except the first (which is supposed to be unity) are called *Cosick Numbers*, or Powers, as for Example, in this rank of continual proportionals, *viz.* 1, 2, 4, 8, 16, 32, 64, 128, 156, &c. the second Term (2) is called the root or first Power, the third term (4) is called the Square or second power, the fourth Term (8) is called the Cube, or third power; the fifth (16) is called the Biquadrate, or fourth power; (32) is the fifth power, (64) is the sixth power, (128) is the seventh power, &c.

In like manner if you take a rank of Geometrical proportionals continued, and descending from unity *viz.* 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, &c. or 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c. or 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c. The second Term is called the root or first Power, the third term is called the second Power, the fourth term is called the third Power, &c.

VII. Whence it is evident that the Square or Second Power is generated by the Multiplication of the root or first Power into it self, and the Cube or third power is generated by multiplying the second Power by the root, or by multiplying the root 3 times into it self, and the Biquadrate or fourth power is produced by multiplying of the third power by the root, or by multiplying the root 4 times into it self, and the fifth power is produced by multiplying the fourth power by the root, &c. As for Example.

T 3

If

If you take 2 for a Root, multiply it by its self, it produceth 4 for the Square or second power of the Root 2: Again, multiply 4 by the Root 2, and it produceth 8 for the third power, or Cube of the Root 2: Again, if you multiply the Cube 8 by the Root 2, it produceth the fourth Power, or Biquadrate of the Root 2, &c.

In like manner if 3 were proposed for a Root, it being multiplied by it self, produceth 9 for the Square, or Second Power of 3, and 9 being multiplied by the Root 3, produceth 27 for the Cube, or third Power of the Root 3, &c.

And also if $\frac{1}{2}$ be proposed for a Root, and it be multiplied by it self it produceth $\frac{1}{4}$ for the Square or Second power of (the Root) $\frac{1}{2}$, and $\frac{1}{4}$ (the Square) being multiplied by (the Root $\frac{1}{2}$), it produceth $\frac{1}{8}$ for the Cube, or third Power of (the Root) $\frac{1}{2}$ &c.

Whence it is evident that the 4, 6 or 7 powers of any Roots may be found out. without any respect at all had to the intermediate Powers between the Root and the power required; as suppose there were given the Root 3, and it were required to find the fifth power of it. I take 3, and set it down 5 times in order thus, 3, 3, 3, 3, 3, and multiply them all into each other, according to the rule of continual multiplication, and the last product (which is 243) is the fifth power of the Root 3, which was required.

Again let it be required to find the fourth power of 5, I take 5, and set it down 4 times thus, 5, 5, 5, 5, then do I multiply them continually, and find the last product to be 625, which

which is the fourth power (of the given Root) as was required. The like may be observed in the finding of any other power of any other given Root.

VIII. If there be a series of Geometrical proportionals continued, and against each power there be placed numbers orderly representing the number or degree of distance of each power from the Root, such numbers are called the Indices or exponents of the powers, because they shew how often the Roots is involved into it self for the production of such a power, as in the Rank, or Scale of Algebraical powers placed in the margent, proceeding from the root 2, to the tenth power thereof, which is 1024, under which is written the word Powers, and then against each particular power, on the left hand thereof, is expressed Index, or Exponent of that Power, shewing how often the Root is involved or multiplied into it self to produce that Power: As for Example, against the number 64, is placed the number 6, which sheweth that 64 is the sixth power of its Root, or that its Root is multiplied 6 times into it self to produce the number 64. The like is to be understood of any other,

| 1 | 2 |
|----------|---------|
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |
| Indices. | Powers. |

Likewise if any two or more Indices, or Exponents

ponents be added together, their sum will be an exponent shewing what power will be produced by the multiplication of those Powers belonging to those Exponents or Indices which you add together; as in the foregoing Table let it be required to find out what power of the Root 2 will be produced by multiplying 128 (its seventh power) by 8 (its third power,) in order to which I take 3 and 7, the respective Indices of the given powers, and add them together, and their sum is 10, which sheweth that the third power, and the seventh power of any Number, or Root, being multiplyed together, will produce the tenth power of that Root; so in our example 128 being multiplyed by 8, produceth, 1024, which is the tenth power of the Root 2.

In like manner, the Indices 3 and 5 being added together, make 8 for a new Exponent, which sheweth that 32 and 8 (the powers belonging to those Exponents) being multiplyed together, will produce the eighth power, *viz.* 256, as appears by the said Table, the like of any other

So that you see that the addition of Indices answers to the multiplication of their Correspondent powers.

And in like manner will the subtraction of Indices, or Exponents, answer to the Division of their correspondent powers, observing always to make the power corresponding the subtrahend (or Index to be subtracted) to be the Divisor.

IX. When

IX. When a Question is propounded, and its solution is to be searched out by the Algebraick Art, the number or magnitude sought is generally called a Root, and it must be represented or signified by some Character or Symbol, as must be also all the powers proceeding from the said Root according to the tenure of the Question, in order to which there may be taken some letter of the Alphabet at the pleasure and discretion of the Artift, as *a, b, c, or d, &c.* to express the said Root, but to avoid confusion in operation, by the commixture of known with unknown Quantities, our Modern Analysts have been accustomed to assume vowels to represent unknown Quantities, and to put Consonants to signify known or given quantities.

X. If for the number or quantity sought there be put or assumed the Vowel *a*, then its Square will be *aa*, that is, *a* being multiplied by it self, produceth *aa*, that is *a* squared, or the square of *a*, for *a* time *a* is *aa*, and the Cube or third power raised from the Root *a*, is *aaa*, that is, *a* times *aa*, is *aaa*, and the fourth power accordingly is *aaaa*, and after the same manner may any higher power of *a* be signified :

In like manner if for the quantity or number sought there be assumed, the letter *e*, then shall the Square raised therefrom be *ee*, and the third power *eee*, and the fourth power *eeee*, and the fifth power *eeeee*, &c.

Also if *b*, or any other Consonant, be put for a given or known Quantity, then its Square will be, *bb*, its Cube *bbb*, and its biquadrate *bbbb*, &c.
but

But by some Analysts the powers of *a*, or any other letter, or Vowel, or Consonant, are expressed by placing the Index or Exponent of the power in a small Character, just after the Symbol, even with the head thereof, viz. $a, a^2, a^3, a, \&c$ signify the Root *a*, its Square, its Cube, and its Biquadrate, &c. which may be further exemplified by the following Table.

A Table

A Table shewing the Powers of Numbers, and how to express the Simple Powers by Alphabetical Characters.

| | | | | | | |
|--------------------------------|---|------|-------|---------|------------|-----------------|
| The Root, or first Power | 1 | 2 | 3 | 4 | a | a |
| The Square, or second Power | 1 | 4 | 9 | 16 | aa | a ² |
| The Cube, or third Power. | 1 | 8 | 27 | 64 | aaa | a ³ |
| The Biquadrate or fourth power | 1 | 16 | 81 | 256 | aaaa | a ⁴ |
| The fifth Power. | 1 | 32 | 243 | 1024 | aaaaa | a ⁵ |
| The sixth Power. | 1 | 64 | 729 | 4096 | aaaaaa | a ⁶ |
| The seventh Power. | 1 | 128 | 2187 | 16384 | aaaaaaa | a ⁷ |
| The eighth Power. | 1 | 256 | 6561 | 65536 | aaaaaaaa | a ⁸ |
| The ninth Power. | 1 | 512 | 19683 | 262144 | aaaaaaaaa | a ⁹ |
| The tenth Power. | 1 | 1024 | 59049 | 1048576 | aaaaaaaaaa | a ¹⁰ |

XI. The numbers made use of in solving of Algebraical Questions, are either absolute Numbers, or Numbers prefixt.

Absolute numbers are those which are disjunct from any kind of Magnitude or Quantity, either known or (unknown) required, but stand simply of themselves, without having Relation to any thing else, as 5, 10, 20, 100, $\frac{1}{4}$, and $\frac{1}{2}$ are called absolute numbers.

Numbers prefixt are such as are immediately prefixt to some letter or letters, signifying an Algebraical quantity, either known, or required such as are $2a$, $4a$, $10a$, $100a$, $\frac{1}{2}a$, $\frac{3}{4}a$, $3aa$, $5bbb$, $3a$, $5b^3$; which numbers so prefixed, shew how often the quantity to which they are prefixed, is to be taken, as $4a$, signifieth that a is to be taken 4 times, and $5bbb$, or $5b^3$, signifieth that the Cube or third power of b is to be taken 5 Times, $\frac{1}{2}a$ is half of a , and $\frac{2}{3}b$ is two thirds of b ; The like is to be understood of any other. And,

Note, that when you have any Algebraical Quantity or Letter, or Character, not having any number prefixed to it, then 1, or unity must be imagined to be prefixed, as a , or $1a$, b , or $1b$, &c.

XII. As in Vulgar and Decimal Arithmetick, in Algebraical Arithmetick, the operations are performed either by Absolute Numbers, or by Alphabetical characters, in all the fundamental rules, viz. Addition, Subtraction, Multiplication, Division, and the Extraction of Roots: and note that

Where it is required to perform the work by absolute numbers, that the operation is in every respect the same as in Common Arithmetick.

tick. But where it is performed by Alphabetical Letters, there is an absolute necessity of using some Characters, to signifie the Operation, an explanation of which Characters take as followeth.

XIII. The Character (+) is a sign of affirmation or Addition, which it is placed between two quantities, signifying that the 2 numbers or quantities between which it is placed, are to be added together, and is as much as to say *plus*, as $3 + 6$ signifieth the sum of 3 and 6, is as much as to say 3 *plus* 6, or 3 more 6, which is 9 and $4 + 7 + 9$ signifieth the sum of 4, 7, and 9, which is 20; so $a + b + c$ signifieth the sum of a , b , and c .

And here note, that when there is no Mark, or Character before any Letter or quantity, then is it Affirmative, and the Mark (+) is supposed to stand before it; as a , is $+a$ or $+1a$, and b is $+b$, or $+1b$, and bcd is $+bcd$ the like of others.

XIV. This Character (−) is a negative sign, and always belongeth to the quantity or Number which followeth it denying it to be, and signifieth a fictitious Number, or quantity less than nothing.

So -7 is a feigned number, less than nothing by 7, *viz*, as the height of the Sun above the Horizon may be affirmed to be 7 *deg.* or $+7 \text{ deg.}$ so when it is depressed 7 degrees below the Horizon, its height may be said to be -7 deg. that is, 7 *deg.* less than nothing.

But when the said sign or Character is placed between two numbers or Quantities, it signifies that the number or quantity which followeth

eth it, is to be subtracted out of some Number or Quantity going before it, as $12-8$ signifieth that 8 is to be subtracted out of 12, or it signifieth the excess of 12 above 8, or the Difference between 12 and 8, which is 4, so $a-b$ signifieth the excess of a above b , and it is as much as to say (a less b ,) so $a+b-c$ signifieth that c is to be from the sum of a and b .

XV. This Chapter (\times) is the sign of Multiplication, and signifieth that the Numbers or Quantities between which it is placed, are to be multiplyed together, as 4×5 signifieth the product of 4 and 5, which is 20; so $3\times 5\times 8$ signifieth the product of the continual multiplication of 3, 5, and 8, viz. 120.

Likewise $b\times c$ signifieth the product of the multiplication of b by c , and $b\times c\times d$ signifieth the product made by the continual multiplication of b , c , and d , into each other.

But for the most part Analysts signifie the multiplication of literal Quantities by setting the letters together like letters in a word, as ab is the same with $a\times b$ and abc is the same with $a\times b\times c$ and this indeed is to be preferred before the other as most convenient and fittest for operation.

XVI. This Character (\simeq) signifieth the Difference between the two quantities between which it is placed, when it is not known in which of them the excess lyeth. So $b\simeq c$ signifieth the Difference between b and c , which it is not known whether b be greater or lesser than c .

XVII. The

XVII. The said 4 Characters defined in the 13, 14, 15, and 16 Sections foregoing, *viz.* $+$, $-$, \times and \cup , may oftentimes have Relation to such a Compound Quantity following the Character, as hath a line drawn over each part of it, as for example, $c + \overline{b \cup d}$, by which you are to understand that the Quantity (c) is to be added to the difference between the Quantities (b and d) in which of them soever, the excess lyeth.

Likewise $a - \overline{b \cup c}$ which signifieth that the difference between b and c is to be subtracted from the Quantity expressed by a .

Also $a \times \overline{b + c}$ signifieth that the sum of b and c is to be multiplied by the quantity a , where take notice that in regard there is a line drawn over the two quantities b and c the sign \times hath reference to the multiplication of a into the quantity c as well as the quantity b , which immediately followeth it, but if the said line were omitted, and the quantities were thus expressed, $a \times b + c$, it would signifie the quantity c to be added to the product of the multiplication of a and b .

Furthermore $b - \overline{c + d}$ signifieth that the quantities c and d are or must be subtracted from the Quantity b , whereas if there were not a line over the quantities c and d , it would signifie that the quantity d is to be added to $b - c$.

And $c \cup \overline{a + e}$ signifieth the difference between the quantity c , and the sum of d and e , whereas if the line were not over d and e , it would signifie the quantity e to be added to the difference between c and d .

XVIII. This Character ($\sqrt{}$) is a radical sign, and signifieth that the Square Root of the quantity or quantities following it, is to be extracted as $\sqrt{36}$, signifieth the Square Root of 36, viz. 6.

So \sqrt{ab} signifieth the Square Root of the product of the quantities a and b , and \sqrt{abc} is the Square Root of the product of the continual multiplication of the quantities of, a , b , and c .

But when you would represent the Root of a Power that is higher than a Square; then immediately after the said Radical sign, expresses the index, or exponent of its power in a parenthesis, as followeth, viz. $\sqrt{(3)} 64$, signifieth the Cube Root of 64, which is 4, $\sqrt{(4)} 81$ signifieth the Biquadrate Root of 81, viz. 3.

Also $\sqrt{(3)} ab$, signifieth the Cube Root of the product of the multiplication of the quantities, a , and b , and $\sqrt{(4)} cd$ signifieth the Biquadrate Root of the product of the multiplication of the quantities, c and d .

And the said Radical sign doth oftentimes belong to such a Compound Quantity following it, as hath a line over every part of it. As for Example, $\sqrt{b+c}$ signifieth the Square Root of the sum of the Quantities b and c . So $\sqrt{(3)} \overline{a+b-c}$ signifieth the Cube Root of the remainder, when the quantity c is subtracted from the sum of the quantities, a and b , and $(\sqrt{(4)} \overline{aa+b-c})$ signifieth the Biquadrate Root of the remainder, when the quantity c is subtracted from the sum of the Square of a added to b .

Likewise $a - \sqrt{bb+c-d}$ signifieth that to the quantity a is to be added the Square Root of the remainder, when the quantity d is subtracted from the sum of the Square of the quantity b ,
and

and the quantity c : And these and such like are by Analysts generally called universal Roots.

After the same manner may be expressed the universal Square Root of $b + \sqrt{aa + c}$ thus, viz. $\sqrt{(2)b + \sqrt{aa + c}}$ which signifieth the Square Root of the sum when b is added to the Square Root of $aa + c$.

XIX. This Character $(=)$ signifieth an Equation, or equality of the magnitudes, or quantities between which it is placed, and imports as much as these words, viz. (is equal to) as in the following Example, viz. $3 + 4 = 7$, which is as much as to say, the sum of 3 and 4, or 3 plus 4 is equal to 7; so $7 + 9 = 12 + 4 = 16$ imports that the sum of 7 and 9 is equal to the sum of 12, and 4 which is equal to 16; and $9 = 12 - 3$, signifieth that 9 is equal to the excess of 12 above 3.

Also $4 \times 5 = 2 \times 10 = 16 + 4 = 20$ signifieth that the Rectangle or Product of 4 by 5 is equal to the Rectangle or Product of 10 by 2, which is equal to the sum of 16 and 4, equal to 20.

Likewise $\frac{24}{6} = \frac{8}{2}$ signifieth that the Quotient of 24 divided by 6, is equal to the Quotient of 8 divided by 2.

Again $a + b = c - d$ signifieth that the sum of a and b is equal to the excess c above d and $c + d = \frac{f}{g}$ signifieth that the sum c and d is equal to the Quotient of f divided by g ; and $b \times c = r - s$ signifieth that the Rectangle of b and c is

c is equal to the excess or r above s , and $a = \sqrt{cc - \frac{1}{4}cc - \frac{1}{2}c}$ signifieth that a is equal to the remainder, when $\frac{1}{2}c$ or $\frac{c}{2}$, is subtracted from the universal square Root $cc - \frac{1}{4}cc$ this will be made plain and easie to the ingenious practitioner by the ensuing Example of this Treatise.

XXI. This Character (\sqsupset) stands for the word (greater) signifying the number, or quantity standing on the left hand of the said Character to be greater than that on the right hand thereof; as $8 \sqsupset 3$ signifieth that 8 is greater than 3; also $a + b \sqsupset c$ signifieth that the sum of a and b is greater than c , &c.

XXII. This Character (\sqsubset) stands for the word less) and it signifieth that the number or quantity standing on the left hand thereof is lesser than that on the right hand. As $4 + 3 \sqsubset 20 - 8$ signifieth that the sum of 4 and 3 is less than the excess of 20 above 8. Likewise $c - d \sqsubset b + e$ is thus read, viz. the Remainder of d being subtracted from c is lesser than the sum of b and e .

XXIII. This Character, ($::$) is always placed in the middle between 4 Geometrical proportionals, as in the following Examples, viz. $2 : 4 :: 9 : 18$ is thus to be read, viz. as 2 is to 4, so 9 is to 18; or after the manner of the Rule of 3, if 2 require 4, 9 will require 18. Also $b : c :: d : e$ is thus read, as b is to c ,

so is d to e . And $a + e : b :: c + b$
 is as much as to say as the Compound Quantity $a + e$ is to the quantity b , so is the Compound Quantity $c + b$
 Quan

Quantity $c+b$ to the Quotient of the Compound Quantity $bc+bb$ being divided by $a+e$.

XXIV. This Character (\div) placed after any number of quantities exceeding two, declareth the said numbers or quantities after which it is placed to be continual Geometrical proportionals, so 2, 4, 8, 16, 32, 64 \div signifieth the said numbers to be continual proportionals Geometrical, for, as 2 is to 4, so is 4 to 8, and so is 8 to 16, and so is 16 to 32, and so is 32 to 64, &c. also these quantities, viz. a, b, c, d, e , \div are continual proportionals Geometrical, for, as a is to b , so is c to d , and so is d to e .

CHAP. II.

Addition of Algebraical Integers.

I **A**S in Common Arithmetick, so in Algebraical, Addition finds out the aggregate, or sum of two or more given quantities however expressed numerally or literally.

II. When the quantities given to be added are alike, and have like signs, collect the numbers prefixed to each quantity into one sum, and

U 2

there

thereto annex the letter, or letters of any one of the given quantities, and then prefix the sign of Affirmation or Negation, *viz.* $+$ or $-$ so shall the quantity thus found be the sum desired.

And here note that every quantity which hath no number prefixed to it, is supposed to have the number 1 prefixed, so is $a=1a$, and $b=1b$.

Example.

What is the sum $3b-b+2b$? Facit $6b$, for the sum of the numbers prefixed to each quantity, *viz.* 3, 1, and 2 is 6, to which if I annex the Character b , it will be $6b$, which must have the sign $+$ prefixed to it, or else it must be imagined so to be, then will $+6b$ be the sum of the given quantities. So if $5ab$ the sum of $3ab+2ab$. And $-4cd$ the sum of $-3cd$.

More Examples of this Rule.

| | | | | | | |
|----------------------------|---|-------|--|--------|--|----------|
| Quantities to be added. | { | $3aa$ | | $-4aa$ | | $+15abc$ |
| | | $2aa$ | | $-aa$ | | $+11abc$ |
| | | aa | | $-2aa$ | | $+25abc$ |
| <hr/> | | | | | | |
| Sum | | $6aa$ | | $-7aa$ | | $+51abc$ |
| <hr/> | | | | | | |

III. When the quantities given to be added together are like, but have unlike signs, then subtract the lesser number prefixed from the greater, and to the remainder annex the letter or letters by which any one of the given quantities is expressed; and thereto prefix the sign of $+$ or $-$ according to the sign of that prefixed num-

number wherein lay the excess, so shall this new quantity be the sum of the quantities propounded.

Example.

Let it be required to add $+5cd$ to $-2cd$, the sum will be found to be $+3cd$; for, first, I subtract -2 from -5 and there remains 3 , to which I annex cd , so will there be $3cd$, to which I prefix the sign $+$ because it belongs to the number 5 , wherein lay the excess, so have I $+3cd$ for the sum required. See the work.

$$\text{add } \begin{cases} +5cd \\ -2cd \end{cases}$$

$$\text{Sum } +3cd$$

Again, if it were required to add $-4aa$ to $-7aa$ the sum would be found to be $-3aa$, because the sign $-$ belongs to the number 7 wherein lay the excess, see the work in the Margent.

More Examples of the last Rule.

| | | | | | |
|--------------|---|---------|---------|----------|----------|
| To be added. | { | $5abcd$ | $5aae$ | $-15ggg$ | $-30bcd$ |
| | | $-abcd$ | $-9aae$ | $11ggg$ | $14bcd$ |
| | | <hr/> | | | |
| sum | | $4abcd$ | $-3aae$ | $-4ggg$ | $-16bcd$ |
| | | <hr/> | | | |

And here note that if the numbers prefixed to the given quantity be equal, and they have different signs, their sum will be 0, so if it were required to add $+8bcd$ to $-8bcd$ their sum will be 0, the negative sign destroying the affirmative.

IV. When the quantities given to be added are more than 2, and have different signs, then according to the second Rule of this Chapter, bring the quantities having like signs into one sum, that is; the affirmative quantities into one sum, and the negative into another, then by the foregoing third Rule add those two quantities together, so shall their sum be the number sought.

Example.

Let it be required to add the sum of $3aa + 7aa - 2aa - 5aa$. First by the said second Rule I find the sum of $3aa + 7aa$ to be $10aa$; and the sum of $-2aa - 5aa$ to be $-7aa$, then by the said third Rule I find the sum of $10aa - 7aa$ to be $3aa$, or $+3aa$, so that I conclude the sum of $3aa + 7aa - 2aa - 5aa$ to be $+3aa$.

More Examples of this Rule follow.

| | | | | |
|--------------|---|--------|--------|-----------|
| To be added. | { | $+3cc$ | $-2cd$ | $-13bcd$ |
| | | $-7cc$ | $-7cd$ | $+17bcd$ |
| | | $+8cc$ | $+5cd$ | $+7bcd$ |
| | | $-4cc$ | $+9cd$ | $-8bcd$ |
| Sum | | 0 | $+5cd$ | $3 + bcd$ |

V. When

V. When the simple quantities given to be added together, be unlike, then (how many soever there be) set them one after another in the same line without altering their signs.

Example.

What is the sum of $4b$ added to $3cd$? Facit $4b+3cd$ for the sum.

More Examples of this Rule follow.

$$\begin{array}{r|l|l|l}
 \text{To be ad-} & \left\{ \begin{array}{l} 2b \\ aa \end{array} \right. & \left\{ \begin{array}{l} 3bc \\ 2cd \end{array} \right. & \left\{ \begin{array}{l} 8ab \\ -3be \end{array} \right. \\
 \text{ded.} & & & \\
 \hline
 \text{Sum} & 2b+aa & +3bc+2cd & +8ab-3be
 \end{array}$$

$$\begin{array}{r|l|l|l}
 \text{To be ad-} & \left\{ \begin{array}{l} 3e \\ 2b \\ -ae \end{array} \right. & & \left\{ \begin{array}{l} -2fg \\ -3gh \\ -4rs \end{array} \right. \\
 \text{ded.} & & & \\
 \hline
 \text{Sum} & 3e+2b-ae & +2fg-3gh-4rs
 \end{array}$$

Algebraical Addition of Compound Integers.

VI. The Addition of Compound Algebraical Integers is easily performed by the help of the foregoing Rules of this Chapter, whether the Compound quantities to be added are alike, or unlike; as you may easily perceive by the work of the following Examples.

Let it be required to find the sum of $3a+b$ and $5a+3b$. their sum will be $8a+4b$ for

$3a + 5a = 8a$, and $3b - b = 4b$, whose sum is $8a + 4b$ by the second Rule of this Chapter.

Also the sum of $6cd + 3bb$ and $2cd - 5bb$ will be found to be $8cd - 2bb$ for (by the second Rule of this Chapt.) $6cd + 2cd = 8cd$, and by the third Rule the sum of $3bb - 5bb = -2bb$ which two sums added together by the fifth Rule of this Chapter will be $8cd - 2bb$.

Moreover if it were required to find the sum of these Compound Quantities, viz. $15gg + 8a - 20$ and $2gg - 3a + 12$ it will be $18gg + 5a - 8$ for $15gg + 3gg = 18gg$ by the second Rule, and the sum of $8a - 3a = 5a$ by the third Rule, and by the same $12 - 20 = -8$, the sum of which 3 sums is $18gg + 5a - 8$ by the fifth Rule of this Chapter.

And the sum of $8b - 16 + 2cd$ and $24 - 5b + 3cd$ is $3b + 8 + 5cd$. And here note that in setting down Compound quantities to be added together, it matters not which of them you set first, so that to every quantity there is prefixed its proper sign; as $3a + b = cc$ is the same with $b + 3a - cc$ and with $-cc + b + 3a$, &c.

More Examples of the Addition of Compound Algebraical Integers.

| | | | |
|-----------|------|---|---|
| To be ad- | ded. | $\begin{array}{r} 4ccc - 18 + 3ab \\ 8ccc + 16 - 8ab \\ 3ccc - 12 + 6ab \\ \hline 9ccc + 10 + ab \end{array}$ | $\begin{array}{r} 10ab + 12aa - 6d \\ - 8ab = 8aa + 9d \\ - 2ab - 2aa - 2d \\ \hline + 2aa + d \end{array}$ |
| | | <hr style="width: 100%;"/> | |

To

$$\begin{array}{r|l}
 \text{To be ad-} & \begin{array}{l} 16cde + 4db + 5p \\ -8cde - 2db \\ -3cde - db \end{array} \\
 \text{ded.} & \\
 \hline
 \text{Sum} & \begin{array}{l} 5cde + db + 5p \\ c^4 + 5ef - ab \end{array}
 \end{array}$$

CHAP. III.

Subtraction of Algebraick Integers.

I shall not here need to give you a definition of the nature of subtraction, but shall only give you a general Rule for finding out of the remainder, excess, or difference of any two quantities, and that in all cases whatsoever.

I. When a Quantity Single or Compound, is given to be subtracted from another, then change the sign, or signs of the quantity to be subtracted, into the contrary signs, that is $+$ into $-$, and $-$ into $+$; which being done, add the two given quantities together by the Rules of the foregoing second Chapter, so shall their sum be the difference, or remainder sought.

Example.

Example 1.

Let it be required to subtract $3a$ from $8a$.

The quantity here given to be subtracted is $3a$, which according to the second rule of the second chap. is $+3a$, therefore must its sign $+$ be changed into $-$, so will it be $-3a$, which being added to $8a$ (by the third Rule of the second Chap.) their sum will be $5a$, for, $8a - 3a = 5a$, and $8a$ and is the difference between the quantities so much $3a$.

Example 2.

Let it be required to subtract $-3bc$ from $4bc$.

Here because $-3bc$ is the quantity to be subtracted, therefore must its sign $-$ be changed into $+$, so will it be $+3bc$, which being added to $4bc$, by the second Rule of the second Chapter, their sum is $7bc$, for, $3bc + 4bc = 7bc$, and so much is the remainder when $-3bc$ is subtracted from $+4bc$,

Example 3.

Let it be required to subtract $-3bde$ from $-9bde$.

Here because $-3bde$ is the quantity to be subtracted, therefore must its sign $-$ be changed into $+$, so will it be $+3bde$ which being added to $-9bde$ according to the third Rule of the second Chapter their sum will be $-6bde$ for $+3bde - 9bde = -6bde$, and so much is the remainder when $-3bde$ is subtracted from $-9bde$.

Example

Example 4.

Subtract $3cd$ from $8de$. The sign of $3cd$ being changed, it will be $-3cd$, which being added to $8de$ by the fifth Rule of the second Chapter, their sum will be $8de - 3cd$ which is the remainder when $3cd$ is subtracted from $8de$.

Example 5.

What is the remainder when $-3bc$ is subtracted from $2cd$? Facit $3bc + 2cd$, or $2cd + 3bc$.

In all which Examples you see that the sign of the quantity given to be subtracted is changed into the contrary sign.

More Examples of Subtraction of Simple Algebraical Integers.

Example 6.

Example 7.

$$\begin{array}{r|l} \text{From} & 3cd \\ \text{Subtract} & cd \end{array} \quad \begin{array}{l} -5bc \\ -8bc \end{array}$$

$$\text{Remainder } 3cd - cd \mid 5bc - 8bc$$

$$\text{Remainder } \left. \begin{array}{l} \\ \text{contracted} \end{array} \right\} 3cd \mid 3bc$$

Example

Example 8.

Example 9.

$$\begin{array}{r}
 \text{From} \quad +3da \mid -cde \\
 \text{Subtract} \quad -da \mid +2cde \\
 \hline
 \text{Remainder} \quad 3da+da \mid cde-2cde \\
 \hline
 \text{Remainder} \quad \left. \begin{array}{l} \\ \text{contracted} \end{array} \right\} -4da \mid -3cde \\
 \hline
 \end{array}$$

And when it is required to subtract a Compound Integer from a Compound Integer, the operation will not in any wise differ from the former, observing always to change $+$ into $-$, and $-$ into $+$, as will appear by the following Examples.

Examples.

From $3a \ 4b$ let it be required to subtract $2a-b$. Here $2a-b$ being the quantity to be subtracted from the other, its signs must be changed into the contrary signs. And then instead of $2a-b$ you will have $-2a+b$, which being added to $3a \ 4b$ the sum will be $a+5b=3a \ 4b-2a \ b$, and so much is the remainder, when subtraction is performed according to the tenure of the Question. See the work laid down as followeth.

Example 3

From

$$\begin{array}{r} \text{From } 3a+4b \\ \text{Subtract } 2a-b \end{array}$$

$$\text{Remainder } 3a+4b-2a+b$$

$$\text{Remainder } \left. \begin{array}{l} \text{contracted} \end{array} \right\} a+5b$$

Example II.

From $3aa-2dc+ab$ let it be required to subtract $aa+3ab-3dc$. The quantity here given to be subtracted is $aa+3ab-3dc$, whose signs being changed, it will then be $-aa-3ab+3dc$, which being added to $3aa-2dc+ab$, the sum will be $3aa-2dc+ab-aa-3ab+3dc$, which according to the sixth Rule of the second Chapter is equal to $2aa+dc-2ab$. See the following operation.

$$\begin{array}{r} \text{From } 3aa-2dc+ab \\ \text{Subtract } aa+3ab-3dc \end{array}$$

$$\text{Remain. } 3aa-2dc+ab-aa-3ab+3dc$$

$$\text{Remainder } \left. \begin{array}{l} \text{contracted} \end{array} \right\} 2aa+dc-2ab$$

But when the given quantities are unlike, then place the quantity to be subtracted immediately after the quantity out of which it is to be subtracted in the same line, changing its signs, which new quantity when the said quantity is so annexed, is the remainder required, which will admit of no Contraction, because the quantities are unlike.

Example.

Example 12.

Let it be required to subtract $3ab + 2aa$ from $7bc + 6cd$, the quantity given to be subtracted is $3ab + 2aa$ which annexed to the other given quantity, changing its signs, will give $7bc + 6cd - 3ab - 2aa$, which is the remainder required. See the following work.

$$\begin{array}{r} \text{From } 7bc + 6cd \\ \text{Subtract } 3ab + 2aa \end{array}$$

$$\text{Remainder } 7bc + 6cd - 3ab - 2aa$$

More Examples of subtraction in Compound Algebraick Integers.

$$\begin{array}{r|l} \text{From } 3aa + 2bc & 8rd - 3dc \\ \text{Subtract } 2aa + 4bc & -2rd - 9cd \end{array}$$

$$\text{Remain. } 3aa + 2bc - 2aa - 4bc \quad | \quad 8rd - 3dc - 2rd + 9cd$$

$$\begin{array}{r|l} \text{Rem. } \left. \begin{array}{l} 3aa - 2bc \\ \text{cont.} \end{array} \right\} & 10rd + 6cd \end{array}$$

$$\begin{array}{r} \text{From } 6ace + 3cd - bc \\ \text{Subtract } 4ace - cd - 5bc \end{array}$$

$$\text{Remain. } 6ace + 3cd - bc - 4ace + cd + 5bc$$

$$\begin{array}{r} \text{Rem. } \left. \begin{array}{l} 2ace + 4cd + 5bc \\ \text{cont.} \end{array} \right\} \end{array}$$

From

From $3aa - 2bc + 5ab$

Subtract $2aa - 2bc - ab$

Remain. $3aa - 2bc + 5ab - 2aa + 2bc + ab$

Rem. } $aa + 6ab$
cont. }

From $8a^4 + 3bc - 3d$

Subtract $5a^4$

$| 3cd + 5a$

$| 2ab - 3c$

Remain. $8a^4 + 3bc - 3d - 5a^4 | 3cd + 5a - 2ab + 3c$

Rem. } $3a^4 + 3bc - 3d$
cont. }

As in Natural Arithmetick, the remainder and the sum subtracted being added together will be equal to the number from which the subtraction is made; so is it likewise in Algebraical Arithmetick, for if you add the remaining Quantity to the quantity subtracted, the sum will be equal to the quantity out of which the subtraction is made. As in the first Example of this Chapter, where it is required to subtract $3a$ from $8a$, and the remainder is $5a$, now if to $5a$ you add $3a$, the sum will be $5a + 3a = 8a$; And in the twelfth Example, where it is required to subtract $3ab + 2aa$ from $7bc + 6cd$; the remainder is found to be $7bc + 6cd - 3ab - 2aa$, to which if you add the number subtracted, viz. $3ab + 2aa$, the sum will be $7bc + 6cd$ equal to the given quantity out of which subtraction is made, for $2ab + 1aa$ being added to $-3ab - 2aa$ they destroy

stroy each other, because their signs are unlike, and this may serve for a sufficient (and indeed the only) proof of the work.

CHAP. IV.

Multiplication in Algebraick Integers.

I. **I**N Multiplication of Algebraical Quantities, there are always two quantities given, to find out a third.

II. The two quantities given are called the Factors, and the third quantity invented, or found by the said Factors, is called the Product, Fact, or Rectangle.

III. When the given Factors are single quantities alike, or unlike, if they have not natural numbers prefixed to them, the fact is discovered at first sight, and is performed by joyning both the quantities together in one, without any Character between them, like letters in a word.

But special regard must be had to the signs of the given quantities, in all kinds of Multiplication,

cation, whether by simple or Compound Quantities, and whether with, or without numbers prefixed to them: the nature of the product wholly depending thereupon, *viz.* If the signs of the quantities to be multiplied together be alike, that is both $+$ or both $-$, then the sign of the product or fact will be $+$, but if they be of different kinds, *viz.* the one $+$, and the other $-$, then the sign of the product will be $-$, as you will find by the several examples following.

Example 1. What is the Product of a multiplied by b ? Facit ab .

Here because both the Factors are signed with $+$, therefore the sign of the product is $+$.

In like manner, if the given Factors had been $-a$ and $-b$ the product would have been (ab or ba) the same as before, because the signs of the Factors are both alike, *viz.* both $-$.

But if the given Factors had been $+$ a and $-b$, or $-a$ and $+$ b then the product or fact would have been $-ba$ or $-ab$, because the signs of the Factors are unlike, *viz.* the one $+$ and the other $-$, Observe the like in all cases whatsoever.

Example 2. What is the product of abc multiplied by cd ? Facit $abccd$ or $+$ $abccd$.

And if you had been to multiply $-abc$ by $-cd$, the product would have been the same, *viz.* $abccd$, or $+$ $abccd$.

But if the Factors had been $-abc$ by $+$ cd , or $+$ abc by $-cd$, then the Fact would have been $-abccd$, because the signs of the Factors are unlike.

More Examples of the like Nature.

| | | | |
|--------------|-----------|------------|-------------|
| Multiplicand | $+ab$ | $ -ac$ | $ -aef$ |
| Multiplyar | $+ade$ | $ -ac$ | $ +aaf$ |
| <hr/> | | | |
| The Product | $- aabde$ | $ - aacc$ | $ - aaeff$ |
| <hr/> | | | |

IV. When the Quantities given to be subtracted are Single, or Simple Quantities, (whether alike, or unlike) having natural numbers prefixed to them, then in such cases let the natural numbers be multiplied together, and to their product annex the product of the given Algebraical Quantities, they being multiplied together as in the last Rule, so shall this new quantity found be the product required. As in the following Examples.

Example 1. Let it be required to multiply $3a$ by $9a$.

First, I multiply the numbers prefixed to both quantities, the one by the other, viz. 9 by 3, and their product is 27, to which I annex the Letters contained in both quantities, viz. aa , and they make $27aa$, which is the product, or Fact required.

Example 2. Let it be required to multiply $3aa$ by $4b$. Here first I multiply the numbers prefixed together, viz. 3 and 4, and they make 12, to which product I annex the Letters of both quantities given, viz. aa and b , and they make $12aab$ for the product required.

Example

Example 3. What is the product of $-3abc$ by $-5cd$?

Here first I multiply the given numbers prefixed, viz. 3 and 5, and they produce 15, to which I annex the Letter in both the quantities, viz. abc and cd , and they make $15abcc$, to which I prefix the sign $+$, because the signs of the given quantities were both alike, viz. $-$ and then will the product or fact be $+15abcc$.

Example 4. What is the product of $+6ab$ multiply by $-3cd$?

First, multiply the numbers prefixed, viz. 6 and 3, and the product is 18, to which I annex the Letters in both the given quantities, ab and cd , and it makes $18abcc$, to which I prefix the sign $-$ (because the signs of the Factors were unlike, viz. the one $+$, and the other $-$) and then the product will be $-18abcc$.

More Examples of the like Nature.

| | | | | |
|--------------|--------------------|--------------------|-------------------|-------|
| Multiplicand | 8fg | 48bc | 20dff | 14ghk |
| Multiplyar | 6rs | 6f | 3ff | 6 |
| Product | 48fg _{rs} | 288bc _f | 60df _f | 84ghk |

V. When Compound quantities are to be multiplied, the operation (in effect) is the same with multiplication of Simple Quantities delivered in the foregoing Rules, for you are to multiply every particular quantity in the multiplicand by each particular quantity in the multiplyar, (not regarding whether you begin the work at the right hand or the left) and then let the several products be joyned together according

according to the Rules of the Algebraical Addition, and that sum will be the product required. The following Examples will make the Rule plain.

Example 1.

In the first place let it be required to multiply a Compound quantity by a simple, (*viz.*) $ab + d$ by a . And in order thereto, First, I multiply a into ab , and the product is aab , and then into d , and it produceth ad , so is $aab + ad$ the product required, each member of the product being affirmative, because all parts of the Factors were Affirmative.

Example 2.

Let it be required to multiply $aa + ab + c$ by b . The Product of aa by b is aab , and the product of ab by b is abb , and the product of c by b is cb , all which particular products being joyned, and one Compound quantity composed thereof, it will give $aab + abb + bc$ for the product required. See the work in the margin.

Example

Example 3. Let it be required to multiply the Compound quantity $ac+dg$ by the Compound quantity $c+d$.

Multiply $ac+dg$
by $c+d$

$$\begin{array}{r} acc+cdg \\ acd+ddg \\ \hline \end{array}$$

Product $acc+cdg+acd+ddg$

First, Multiply each member of the multiplicand by c , and the product is $acc+cdg$, then multiply each member of the multiplicand by d , and the product is $acd+ddg$, which two products being joyned together by the Rule of Algebraical Addition, the sum is $acc+cdg+acd+ddg$, which is the product required, as appears by the operation.

Example 4. What is the product of $da+bc$ multiplied by $da-ab$?

Multiply $da+bc$

by $da-ab$

$$\begin{array}{r} ddaa+dabc \\ -daab-abbc \\ \hline \end{array}$$

Product $ddaa+dabc-daab-abbc$

First, Multiply the Multiplicand $da+bc$ by da (the first member of the multiplier) and it produceth $ddaa+dabc$, then multiply the said Multi-

plicand by $+ab$ (the second member of the multiplier) and it produceth $-aadb - abbc$, which two quantities being joyned together give $daaa + dabc - aadb - abbc$ for the product required. As you may see in the Operation.

Example 5. What is the product of $a + b - c$ multiplied by $a + b - c$?

$$\begin{array}{r}
 \text{Multiply} \quad a + b - c \\
 \text{by} \quad a + b - c \\
 \hline
 aa + ab - ac \\
 + ab + bb - bc \\
 + ac + bc - cc \\
 \hline
 \text{Product} \quad aa + 2ab - 2ac + bb - 2bc + cc
 \end{array}$$

First, Multiply each member of the Multiplicand by a (the first member of the multiplier) and it produceth $aa + ab - ac$, then multiply each said member in the multiplicand by b , (the second member of the multiplier) and it produceth $ab + bb - bc$, then multiply each member of the said multiplicand by $-c$ (the third and last member of the multiplier) and the product is $-ac - bc + cc$; which said three products being joyned together according to the Rules of Algebraical Addition, will give $aa + 2ab - 2ac + bb - 2bc + cc$ which is the Square of $a + b - c$ or product required, as appears by the whole operation.

And if there are natural numbers prefixed to any of the Compound Quantities, the operation will not be different from the foregoing Examples of this Rule, regard being had to the fourth

Rule

Rule of this Chapter, as will appear by the following Example.

Example 6. What is the product of $3b+2c$ multiplied by $4b-3c$?

Multiplycand $3b+2c$
 Multipliar $4b-3c$

$$\begin{array}{r} \hline -12bb+8bc \\ 9bc-6cc \\ \hline \end{array}$$

Product $12bb-bc-6cc$

First, By the fourth rule multiply each member of $(3b+2c)$ the multiplicand by $4b$, and the product is $12bb+8bc$; then multiply the said multiplicand by $-3c$, and the product is $-9bc-6cc$, which being added to $12bb+8bc$, the sum will be $12bb-bc-6cc$ which is the product required.

Example 7. What is the product of $2a+2c-8$ multiplied by $2a-5$?

Multiply $2a+2c-8$
 by $2a-5$

$$\begin{array}{r} \hline 4aa-4ac-16a \\ -10a-10c+40 \\ \hline \end{array}$$

Product $4aa-4ac-26a-10c+40$

First, $2a+2c-8$ being multiplied by $2a$, produceth $4aa-4ac-16a$, for $2a \times 2a = 4aa$ and
 $\times 4$ $2a \times 2c = 4ac$

$2a \times 2c = 4ac$ and $2ax - 8 = -16a$ which is marked with $-$, because the signs of the Factors are unlike, viz. -8 and $+2a$. Secondly $2a + 2c = 8$ being multiplied by $-5x$ produceth $-10a - 10c - 40$; for $-5x + 2a = -10a$ and $-5x + 2c = -10c$, and $-5x - 8 = -40$, all which quantities being joyned together by the Rules of Algebraical Addition will give $-4aa - 4ac - 26a - 10c - 40$ which is the product required. See the work.

More Examples in Multiplication of Compound Algebraical Integers.

$$\begin{array}{r} \text{Multiplicand} \quad 3b + 2c \quad | \quad 3cd + 3b - 6 \\ \text{Multipliar} \quad 3b - 2c \quad | \quad 8 \end{array}$$

$$\text{Product} \quad 9bb + 6bc \quad | \quad 24cd + 24b - 48$$

$$\text{Multiplicand} \quad 2ab + bcd$$

$$\text{Multipliar} \quad 3ab - bcd$$

$$6aabb - 3bbacd$$

$$-2bbacd - bbbccdd$$

$$\text{Product} \quad 6aabb - bbacd - bbbccdd$$

$$\text{Mult.} \quad 3a + bb - 16$$

$$\text{Mult.} \quad 4a - 6c - 4$$

$$12aa + 4abb - 64a$$

$$18ac - 6bbc + 96c$$

$$-12a - 4bd + 64$$

$$\text{Product} \quad 12aa + 4abb - 16aa - 18ac - 6bbc + 96c - 4bb - 64$$

Multiplicand

Multiplicand $3ab + 4cc - 2a$

Multiplyar $2ab - 3cc - a$

$$6aabb + 8ccab - 4aab$$

$$- 9abcc - 12c^4 + 6acc$$

$$- 3aab - 4acc - 2aa$$

Product $6aabb + 7aab - abcc - 12c^4 + 2acc + 2aa$

CHAP. V.

Division in Algebraick Integers.

I. **I**N Division Algebraical (as in Division in Common Arithmetick) there are two quantities given to find out a third; which quantity sought is called the Quote, or Quotient; and of the quantities given that which is to be divided, is called the dividend, and the quantity by which it is to be divided, is called the Divisor.

II. when it is required to divide one number or quantity by another, if you place the Dividend for the Numerator, and the Divisor for the Denominator of a Fraction, that Fraction so composed is equal to the Quotient that would arise

arise by the real Division of the one by the other.

For if it were required to divide 4 by 5, the quotient would be $\frac{4}{5}$, or if it were required to divide 12 by 7, the *Quotient* would be $1\frac{5}{7} = 1\frac{5}{7}$. The reason of which is the ground of the general part of Division in Algebra; for when one quantity is to be divided by another, set the quantity that is Dividend for the Numerator of a Fraction, and the quantity that is the Divisor set for the Denominator.

So if it were required to divide the quantity b by the quantity a , I would place them thus, *viz.* $\frac{b}{a}$ which signifieth the Quotient of b divided by a .

In like manner if it were required to Divide abc by cd , the Quotient would be $\frac{abc}{cd}$. And if $5ac$ were to be divided by $3cd$, the Quotient would be $\frac{5ac}{3cd}$. And if bcd were to be divided by 7, the Quotient would be $\frac{bcd}{7}$.

The same is to be observed in Division of Compound Algebraick Integers, for if it were required to divide $a+b$ by c , the Quotient would be $\frac{a+b}{c}$, and if $5b$ were to be divided by

$3a+bcd$, the Quotient would be $\frac{5b}{3a+bcd}$.

is to be divided, is called the dividend, and the quantity by which it is to be divided, is called the Divisor.

11. When it is required to divide one number or quantity by another, if you place the Divisor for the Numerator, and the Dividend for the Denominator of a Fraction, that Fraction is equal to the Quotient that would arise

More

More Examples of Division according to the foregoing Rule.

$$\begin{array}{r|l} \text{Dividend } cd & 3dce \\ \text{Divisor } fg & 15b \end{array} \quad \begin{array}{r|l} 3aa + e - f \\ a + b + c \end{array}$$

$$\begin{array}{r|l} cd & 3dce \\ \hline \text{Quotient} & \end{array} \quad \begin{array}{r|l} 3aa + e - f \\ \hline \end{array}$$

$$\begin{array}{r|l} fg & 15b \\ \hline \end{array} \quad \begin{array}{r|l} a + b + c \\ \hline \end{array}$$

$$\begin{array}{r|l} \text{Dividend } a^4b & 15bb \\ \text{Divisor } 3b^3 & 7cc \end{array} \quad \begin{array}{r|l} 6aa \\ 3 \end{array}$$

$$\begin{array}{r|l} a^4 & 15bb \\ 3b^3 & 7cc \end{array} \quad \begin{array}{r|l} 6aa \\ 3 \end{array} \quad \begin{array}{r|l} 2aa \\ \hline \end{array}$$

II. When in any quotient that is expressed according to the foregoing rule, there are the same Letter or Letters repeated in every part or member of the Numerator and Denominator, you may cancel such Letter or Letters, but be sure that what you cancel in one part, to cancel the very same in all the rest. So shall this new Quantity be a true quotient, equivalent to what it was before the said Letters were cancelled.

Example. What is the Quotient of bd divided by b ? According to the foregoing Rule the Quotient is $\frac{bd}{b}$ but because the Letter b is found both in the Numerator and in the Denominator, therefore cancel b in both of them

them, and then you will find the Quotient to be d , for $\frac{bd}{b} = d$.

Again let it be required to divide $ab + ad$ by acd , the Quotient is $\frac{ab+ad}{acd}$, and because the letter a is found in every member of the Numerator and Denominator, cast it out of every one, and then you will have $\frac{b+d}{cd}$ for the Quotient.

Likewise if you were to divide $ab + abc + abe$ by $abd + abf$ the Quotient you would find to be $\frac{ab+ab+abe}{abd+abf}$ which being contracted by cancelling ab in each member of the Numerator and Denominator, there will be found $\frac{1+c+e}{a+f}$ for the Quotient.

And $bb + b$ being to be divided by b , the Quotient will be $\frac{bb+b}{b}$ and by cancelling b in every part, there will be $b+1$ for the Quotient abbreviated, for $\frac{bb+b}{b} = \frac{1bb+1b}{1b}$ and by cancelling b in every part, there will be $\frac{1b+1}{1}$ and $\frac{1b+1}{1} = b+1$.

The same is to be observed whether the signs be $+$ or $-$; so if it be required to divide $abc + che$, by $bde - bce$ the Quotient will be $\frac{abe+be}{bde-bce}$. And because be is found in each quantity, I cancel it, and the Quotient contracted, or abbreviated will be found to be $\frac{a+c}{a+c}$.

Example. What is the Quotient of ab divided by b ? According to the foregoing Rule the Quotient is $\frac{bd}{b}$ but because the letter b is found both in the Numerator and in the Denominator, therefore cancel b in both of them.

More

More Examples of Contractions or Abbreviations in Division of Algebraick Integers according to the foregoing Rule.

$$\begin{array}{r|l|l}
 \text{Dividend} & aaa & ad - ak + a \\
 \text{Divisor} & aa & a \\
 \hline
 \text{Quotient} & aaa & ad - ak + a \\
 \hline
 \text{Quotient} & aa & a \\
 \text{contracted} & \left. \begin{array}{l} \text{Quotient} \\ \text{contracted} \end{array} \right\} a & \left. \begin{array}{l} ad - ak + a \\ d - k + 1 \end{array} \right\}
 \end{array}$$

IV. If when it is required to divide a Simple or Compound Algebraick Quantity by a Simple Quantity, there be prefixed to every member a number, or numbers, that may be divided by any other number without any remainder, then instead of the given prefixed numbers prefix the Quotient of each of the said numbers divided by the said Common measurer, not neglecting to cancel any letter that may be found in each part of the Numerator and Denominator, according to the foregoing third Rule. As for Example.

Divide $16bc$ by $4bd$. Here according to the foregoing Rule, the Quotient is $\frac{16bc}{4bd}$, but because the prefixed numbers 16 and 4 will admit of 4 for a common measure, therefore I divide them both by 4, and the Quotients are 4 and 1, which I prefix to the given Quantities instead of 16 and 4, and then the Quo-

Quotient will be $\frac{4bc}{1b}$, or $\frac{4bc}{b}$, and because bc is contained both in the Numerator, and the Denominator, cancel it, so have you $\frac{4c}{1} = 4c$ for the Quotient required.

Moreover $15abc \div 12bd$ being given to be divided by $3bg$, the Quotient will be found to be $\frac{5ac+4d}{g}$.

For, I first discover that the prefixed numbers 15, 12 and 3, have 3 for their common measure, by which they being severally divided, give 5, 4, and 1, which being prefixed to the said Quantities after b , (which is found in every quantity is cancelled) there will be $\frac{5ac+4d}{g}$ for the true Quotient required.

More Examples of Contraction in Division, according to the two last Rules.

| | | | | | |
|---------------------|-------|--|----------------|--|-----------------|
| Divide | $8bc$ | | $28cd$ | | $32abc$ |
| by | $4b$ | | $16d$ | | $12acd$ |
| | | | | | |
| Quotient | $2bc$ | | $7cd$ | | $8abc$ |
| | $4b$ | | $16d$ | | $12acd$ |
| | | | | | |
| Quotient contracted | $2c$ | | $\frac{7c}{4}$ | | $\frac{8b}{3d}$ |
| | | | | | |

V. When in Compound quantities one or more letter or letters is repeated in every member, then will the remaining letters in each quantity evenly

evenly divide the said Compound quantity with out any remainder, and the quotient will be the Letter or letters repeated in each number as aforesaid. As for Example.

1. What is the Quotient of $ba+ca$ divided by $b+c$? Here it is evident that the quotient will be a ; for proof whereof take the divisor $b+c$, and multiply by the quotient a , according to the fifth rule of the fourth Chap. and the product will be $ba+ca$ equal to the dividend.

2. Likewise if it were required to divide $bad+cad$ by $b+c$, the quotient will be found to be ad .

3. Also by the same reason if you divide $2ab-2acd-2adb$ by $b-cd-ab$, the quotient will be $2a$, and if you divide the same dividend by $2b-2cd-2ab$ the quotient will be a .

More Examples of the like nature.

| | |
|--------------------------------|--------------------------------|
| Dividend $6ab+2adc$ | $4bc+3cd-c$ $4b+3d-1$ |
| Dividend $6b+2dc$ | $4b+3d-1$ |
| <hr style="width: 100%;"/> a | <hr style="width: 100%;"/> c |

The reason why -1 is the last number of the last example is, because $-c$, or $-1c$ is the last number of the dividend, for according to the thirteenth Rule of the first Chapter, when a quantity hath no number prefixed to it; it is supposed to have the number 1 before it.

And here note, that as in Multiplication of Algebraick Integers $+$ by $+$, and $-$ by $-$ produceth $+$, and $+$ by $-$ produceth $-$, so in Division, if you divide $+$ by $+$, or $-$ by $-$, the sign

sign of the Quote will be $+$, but if you divide $-$ by $+$, or $-$ by $-$, the sign of the Quote will be $-$; so if $3ab$ be divided by $3a$, the Quote will be b , or $+b$, and if $-3ab$ be divided by $-3a$, the Quote will be $+b$, for if you multiply $-3a$ by $+b$, the product will be $-3ab$ by the third Rule of the fourth Chapter foregoing. Also if you divide $+3ab$ by $3a$, or $-3ab$ by $-3a$, the Quotient will be $+b$, for $+3a$ being multiplied by $+b$, produceth $+3ab$, and $-3a$ being multiplied by $-b$, produceth $+3ab$.

VI. From a due consideration of the manner of operating the Examples of the last Rule, a way may be discovered to divide a compound Quantity by a Simple, or Compound Quantity, and to find out the true Quotient when it likewise will be a Compound Quantity, the practice of which will be made plain by the following Examples.

Example 1. Let it be required to divide $ba+ca$, by a . Having placed the Dividend and Divisor as is usual in vulgar Arithmetick, and as you see in the following operation.

$$a) \overline{ba+ca} \quad (b+c$$

$$\underline{-ba}$$

$$+ca$$

$$\underline{-ca}$$

$$(0)$$

Then do I seek how often a is contained in ba (the first member of the Dividend) and the answer

swer is b times, therefore I put b in the Quo-
 tient, and thereby I multiply (a) the Divisor
 and the product is $+ba$, which must be subtracted
 from ba in the Dividend, and therefore I change
 its sign into $-ba$ by the first Rule of the third
 Chapter, and there remaineth 0 , then do I
 bring down $+ca$ the next member of the Di-
 vidend, and divide it by a . and the Quotient is
 $+c$, by which I again multiply (a) the Divisor,
 and the product is ca , which subtracted from
 ca there remaineth 0 , and so the work of Di-
 vision is ended, and I find the Quotient of
 $ba+ca$ divided by a to be $b+c$, for proof where-
 of if you multiply $b+c$ by a (the Divisor) the
 product will be $ba+ca$ equal to the given Divi-
 dend.

Example 2. Let it be required to divide
 $ba+ce+be+ce$ by $b+c$

Having disposed of the Dividend and Divisor
 in order to the work with a crooked line behind
 which to place the Quotient, as in Common
 Arithmetick; then first I seek how often b (the
 first member of the Divisor) is contained in ba ,
 (the first member of the Dividend) and there aris-
 eth a , which I put in the quotient, and there-
 by multiply each member of the Divisor, viz.
 $b+c$, and the product is $ba+ca$, which place un-
 der the two first quantities of the dividend to-
 wards the left hand, viz. under $ba+be$, and by
 the first Rule of the second Chapter subtract
 it therefore, so will the remainder be 0 ; to
 which I bring down the remaining part of the
 dividend, viz. $be+ce$ and divide be by e , and
 there ariseth in the quotient e , by which I multi-
 ply the whole Divisor $b+c$, and the Product is
 $be+ce$, which subtracted from the Divi-
 dend

dend $be+ce$, the Remainder is 0. See the whole work as followeth.

$$\begin{array}{r}
 \text{Quotient} \\
 b+c) \quad ba+ca+be+ce \quad (a+e \\
 \underline{ba+ca} \\
 0 \quad 0+be+ce \\
 \underline{be+ce} \\
 (0 \quad 0)
 \end{array}$$

So that the Quotient is $a+e$, now to prove the work, multiply the *Divisor* $b+c$ by the *Quotient* $(a+e)$ according to the fifth Rule of the fourth Chapter, and the *Product* you will find to be $ba+ca+be+ce$ which is equal to the given *Dividend*; and therefore I conclude the operation to be truly performed.

Example 3. In like manner, if you divide $ba+bd+ca+cd-ac-de$ by $a+d$, the *Quotient* will be found to be $b+c-e$ according to the following work.

$$\begin{array}{r}
 \text{Quotient} \\
 a+d) \quad ba+bd+ca+cd-ac-de \quad (b+c-e \\
 \underline{ba+bd} \\
 0 \quad 0+ca+cd \\
 \underline{ca+cd} \\
 0 \quad 0-ac-de \\
 \underline{-ac-de} \\
 (0 \quad 0)
 \end{array}$$

The work of the last Example explained.

In the foregoing Example, first I divide ba (the first member of the Dividend) by a the first quantity or member of the Divisor, and there ariseth a in the *Quotient*, which is $+a$, (because the signs of the *Dividend* and *Divisor* are $+$) and thereby I multiply the *Divisor* $a+d$. and the *Product* is $ba+bd$, which I place under the two first members of the *Dividend* as you see in the work, and subtract it therefrom, and the remainder is 0 , to which I bring down the two next quantities, viz. $+ca+cd$.

Then do I divide $+ca$ by a , and there ariseth in the *Quotient* $+c$, because the *Dividend* and *Divisor* are both signed $+$,) by which I multiply the said *Divisor*, and the product is $+ca+cd$ which I place under the *Dividend*, and subtract it therefrom, and there remaineth 0 , to which I annex the two next and last members of the *Dividend*. viz. $-ae-de$; and divide $-ae$ by $+a$, and the *Quotient* is $-e$, (because the signs of the *Dividend* and *Divisor* are different, viz. the one $+$, and the other $-$) and thereby I multiply the whole *Divisor*, and the *Product* is $-ae-de$, which subtracted from $(-ae-de)$ the *Dividend*, the remainder is 0 , and so the work is finished, and the *Quotient* arising by this Division is $b+c-e$, as you may prove at your leisure.

If the quantities or members of the *Dividend* of the foregoing Example are not placed in the same order that is there expressed, the effect of the operation will be the same, as you may see by the following work.

| <i>Divisor</i> | <i>Dividend</i> | <i>Quotient</i> |
|----------------|-------------------------------|-----------------|
| $a + d$ | $ba + ca - ae + bd + cd - de$ | $(b + c - e$ |
| | $ba + bd$ | |
| | $ca - ac$ | |
| | $ca + cd$ | |
| | $-ae - de$ | |
| | $-ac - de$ | |
| | (00) | |

First, I divide ba by a , and the Quote is b , by which I multiply the *Divisor* $(a + d)$ and the product is $ba + bd$ which I subtract from $ba + ca$ and the remainder is (by the Rule of the third Chapter) $ba + ca - ba - bd$ which being contracted + by the Rules of Addition is $ca - bd$, (for $+ba$ and $-ba$ expunge each other.) then to this remainder do I bring down the two next quantities of the *Dividend*, viz. $-ae + bd$ which being annexed to the said remainder $ca - bd$ it then (makes for a new dividual) $ca - bd - ce + bd$, but $-bd$ and $+bd$ destroy each other, and therefore the dividual contracted is $ca - ce$, which I divide by $a + d$ as before, and the Quotient is $+c$, by which I multiply the *Divisor*, and the Product is $ca + cd$, which being subtracted from the said dividual $ca - ae$, the remainder is $ca - ae - ca - cd$ which being contracted, is $ae - cd$, to which I joyn the two next quantities in the *Dividend*, viz. $-cd - de$, and it makes $(ae - cd + cd - de) = -ae - de$ (for $-cd$ and $+cd$ destroy each other for a new dividual, which I divide by the said *Divisor* $a + d$, and the Quote is $-e$, by which I multiply the *Divisor*

visor, and the *Product* is $-ae-de$, which subtracted from $-ae-de$ (the *dividual*) the remainder is $-ae-de-ae-de=0$, and so the work is finished, and I find the *Quotient* to be $b+c-e$ as before.

But here note by the way that it doth not always fall out that you are to divide the first member of the *dividual* by *Note.* the first of the *Divisor*, but by some other member which you can discover will do the work without making a *Fraction*. As in the following Example.

Example 4. Let it be required to divide $aa-ce$ by $a+e$?

First, I divide aa by a , and there ariseth in the *Quotient* a , by which I multiply the *Divisor* $a+e$, and the *Product* is $aa+ae$, which subtracted from the *Dividend* ($aa-ce$) the remainder is $-ce-ae$ for a *dividual*, and then do I not seek how often a is contained in ce , for then the answer would be a *Fraction*, but I divide ce by its correspondend *Divisor* $+e$, and there ariseth $-e$, to be written in the *Quotient* next after a , but not $+e$, because $-$ divided by $+$ quotes $-$. Then I multiply the whole *Divisor* $a+e$ by $-e$. and the *product* is $-ae-ee$, which subtracted from the said *dividual* $-ce-ae$, the remainder is 0, so is the work ended, and I find the *Quotient* to be $a-e$. See operation.

$$\begin{array}{r}
 a + e \quad aa - ee \quad (a - e \\
 aa + ae \\
 \hline
 -ee - ae \\
 -ae - ee \\
 \hline
 (0 \quad 0)
 \end{array}$$

Example 5. If it were required to divide $aaa + abd + baa + bbd$ by $aa + bd$ by the Quotient would be found to be $a + b$, as appears by the work.

$$\begin{array}{r}
 aa + bd \quad aaa + abd + baa + bbd \quad (a + b \\
 \underline{aaa + abd}
 \end{array}$$

$$\begin{array}{r}
 0 \quad 0 - baa + bbd \\
 \underline{baa + bbd} \\
 0 \quad 0
 \end{array}$$

Example 6. If $aaa - abb + abd + baa - bbb + bbd$ be divided by $a + b$ the Quote will be $aa - bb + bd$, as by the operation.

$$\begin{array}{r}
 a + b \quad aaa - abb + abd + baa - bbb + bbd \quad (a - b + bd \\
 \underline{aaa + baa} \\
 -abb + abd \\
 -abb - bbb \\
 \hline
 +abd + bbd \\
 +abd + bbd \\
 \hline
 0 \quad 0
 \end{array}$$

Example

Example 7. Let it be required to divide $18abc + 9bcc + 24bcc + 24abc + 16cc$ by $3bc + 4c$, the quotient will be found to be $6ab + 3bc + 4c$. See the following operation.

| Divisor | Dividend | Quotient |
|------------|---------------------------------------|-------------------|
| $3bc + 4c$ | $18abc + 9bcc + 24bcc + 24abc + 16cc$ | $(6ab + 3bc + 4c$ |
| | $18abc + 24bcc$ | |
| | <hr/> | |
| | $+ 9bcc + 24bcc$ | |
| | $+ 9bcc + 12bcc$ | |
| | <hr/> | |
| | $+ 12bcc + 16cc$ | |
| | $+ 12bcc + 16cc$ | |
| | <hr/> | |
| | 0 0 | |

When Algebraical Division according to the Rules before delivered, will not exactly perform the work without any remainder, then you may place the Dividend and Divisor Fraction-wise, which is indeed the most general practice amongst Algebraists; or else proceed in Division as far as you can by the preceding method; and then place the remainder for a Numerator over the Divisor, as in the following Example, where $aa - bb + ac$ is divided by $a + b$, and the quotient is $a - b + \frac{ac}{a+b}$, the remainder being $+ac$.

$$\begin{array}{r}
 a + b \overline{) aa - bb + ac} \quad (a - b + \frac{ac}{a+b} \\
 \underline{aa + ab} \\
 -bb - ab + ac \\
 \underline{-ba - bb} \\
 \hline
 0 \quad 0 + ac
 \end{array}$$

C H A P. VI.

The Doctrine of Algebraical
Fractions. And First,*Of Reduction.*

I. **H**E that intends a considerable proficiency in this mysterious Art, must be very well acquainted with the Doctrine of Vulgar Fractions, a mean knowledge therein not being sufficient for all operations whatsoever in Algebraick Fractions have their dependence thereupon, being wrought in every respect as vulgar Fractions, they are by the help of the Rules contained in the several Chapters foregoing, and there are very few questions solved Algebraically, but what have one or more Fractions concerned in its operation.

To reduce Fractions, having unequal Denominators to Fractions of the same value having a common Denominator.

II. When you would reduce algebraical Fractions to a common Denominator, multiply the Numerator of the first Fraction into the Denominator or denominators of the rest, so shall the Product be a Numerator equal to the Numerator of

of the first Fraction, likewise multiply the Numerators of the second, third, &c. Fractions, into all the Denominators except its own, and the several products shall be so many new Numerators; then multiply all the Denominators continually, so shall the product be a common Denominator to all the Numerators found out as before.

Example 1. Reduce $\frac{a}{b}$ and $\frac{d}{c}$ to a common Denominator. Multiply the Numerator a (of the first Fraction) into the Denominator (c) of the second Fraction, and the product is ac , for a Numerator $=a$, then multiply (d) the Numerator of the second Fraction into (b) the Denominator of the first, and the product is db , for a Numerator $=d$, then multiply the Denominators together, viz. b into c , and the product bc is the Denominator common to both the Numerators, so will the 2 new Fractions be $\frac{ac}{bc}$ and $\frac{db}{bc}$ for $\frac{a}{b} = \frac{ac}{bc}$ and $\frac{d}{c} = \frac{db}{bc}$.

Example 2. What Fractions are $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ having an equal or common Denominator?

Facit $\frac{aad, bcd}{cad cad}$ and $\frac{cca}{cad}$, for $a \times a \times d = aad$, = the Numerator a , and $b \times c \times d = bcd$ = the Numerator b , and $c \times c \times a = cca$ = the Numerator, and $c \times a \times d = cad$ which is the common Denominator.

To reduce an Algebraical Fraction to its lowest Terms equivalent.

III. When in the Numerator and Denominator of an Algebraical Fraction, the same letter or letters is contained, then cancel the same in both and if there be any numbers prefixt, if you can discover any number that will divide them both without any remainder, then prefix those quotients instead of the numbers prefixed before ; so shall this new Fraction be of the same value with the Fraction proposed.

So will $\frac{cbd}{cbb}$ be reduced to $\frac{d}{b}$, by cancelling cb in the Numerator and Denominator.

Also $\frac{24bde+32bda}{8bdd}$ by being reduced to its lowest terms will be $\frac{3e+4a}{d}$ by cancelling bd in every part, and dividing the prefixed numbers by 8. More Examples follow.

| | |
|-------------------------------|--|
| $\frac{cbs}{cde} \frac{b}{e}$ | $\frac{16rbsd}{2orsgq} \frac{4bd}{sgq}$ |
| $e + \frac{ghm}{gh} = e + m$ | $\frac{24adc}{12ade+16adb} - 8c = 3e+4b$ |

Or if you can (in a Compound Algebraical Fraction) discover a quantity that will divide the Numerator and Denominator without any remainder, (according to the sixth Rule of the fifth Chapter) then shall the Quotients be a new

new Numerator and a new Denominator equal to the Fraction in its given Terms. As in the following Examples.

$$\begin{array}{c} \frac{ba+da}{b+d} = a \quad | \quad \frac{aa+2ae+ee}{a+e} = a+e = e \\ \hline \frac{aa-2ae+ee}{a-e} = ae \quad | \quad \frac{aaaa-bbbb}{aa-bb} = aa-bb \end{array}$$

To reduce an Integral quantity to an Algebraical Fraction.

IV. Multiply the given quantity by the intended Denominator, so shall the Product be the Numerator required. As in the following Examples.

Let it be required to reduce the quantity b to a Fraction, having ad for its Denominator. To do which I multiply the given quantity b by ad , and the product is the Numerator, viz. bad , so shall $\frac{bad}{ad}$ be the Fraction required, for $\frac{bab}{ad} = b$

Also if it were required to reduce the quantity e to a Fraction, whose Denominator should be $b+c$, it would be $\frac{be+ce}{b+c}$

V. If it be required to reduce a mixt quantity to a Fraction, multiply the Integral quantity by the Denominator of the Fractional part, and joyn the product to the Numerator of the Fractional part, so shall the sum be the Numerator. As in Example.

Reduce

Reduce $a + b + \frac{c}{d}$ to an improper Fraction Mult.
 the Integral part $a + b$ by the Denominator d ,
 and the Product $da + db$ which being added to the
 numerator c , makes $da + db + c$ for the numerator,
 to which placing d for a Denominator, it gives
 $\frac{da + db + c}{d} = a + b + \frac{c}{d}$ for the answer.

VI. When you are to express an Algebraick In-
 teger Fraction-wise, without an Assigned Deno-
 minator, then make the given quantity the Nu-
 merator, and 1 the Denominator.

So will ab be $\frac{ab}{1}$ and cd will be $\frac{cd}{1}$ and $a + b$
 will be $\frac{a+b}{1}$ &c.

These things are so plain that they need no
 farther explanation by examples.

C H A P. VII.

Of Addition and Subtraction of Algebraical Fractions.

I. **W**hen the Fractions given to be added
 together have an equal or common
 Denominator, add the Numerators together,
 and place their sum for a Numerator over the com-

common Denominator, which new Fraction shall be the sum of the given Fractions, but if they have not a common Denominator, reduce them by the second Rule of the sixth Chapter, and then proceed as before.

Example 1. What is the sum of $\frac{a}{b}$ and $\frac{ac}{b}$? Facit $\frac{a+ac}{b}$ the sum of the Numerators, viz. a and ac is $a+ac$ which placed over the Denominator b , gives $\frac{a+ac}{b}$ for the sum required.

Example. 2. So also the sum of $\frac{abc}{f}$ and $\frac{dg}{f}$ will be $\frac{abc+dg}{f}$

Example 3. And the sum of $\frac{a+b-c}{a}$ $\frac{a+b+c}{a}$ and $\frac{2a-2b}{d}$ will be found to be $\frac{4a}{d}$ for the sum of the Numerators is $a+b-c+a+b+c+2a-2b=4a$.

Example 4. What is the sum of $\frac{a}{b}$ and $\frac{c}{d}$? Facit $\frac{ad+cb}{bd}$ the given Fraction being reduced to a common Denominator by the second Rule of the sixth Chapter, are $\frac{ad}{bd}$ and $\frac{cb}{bd}$ whose sum is $\frac{ad+cb}{bd}$

Example 5. What is the sum of $\frac{a}{b}$ and $\frac{3f}{d}$? Facit $\frac{acd+bbd+3fghc}{bcd}$

II. When it is required to gather mixed quantities into one sum, then add the fractional parts together by the foregoing Rule, and likewise bring the Integers into one sum, and the sum of these two sums will be the sum required.

Example.

What is the sum of $bb + \frac{ab}{c}$ and $cd + \frac{a+b}{d}$?

The Sum of the Fractions being added by the foregoing Rule is

$$\frac{abd+ca+cb}{cd}$$

To which sum if you add the Integral parts of the propounded mixed Quantities, the sum required will be

$$bb + cd + \frac{abd+ca+cb}{cd}$$

Subtraction of Algebraical Fractions.

III. If the 2 given Fractions have not a common Denominator, then reduce them to such by the second Rule of the sixth Chapter, then (by the Rule of the third Chapter) subtract the Numerator of the Fraction to be subtracted from the Numerator of the other Fraction, and place the remainder for a Numerator over the common Denominator, which new Fraction shall be the remainder sought. As in the following Examples.

If you would subtract $\frac{ab}{e}$ from $\frac{bc}{e}$ Take the Numerator ab from the Numerator bc , and the remainder is $bc-ab$ which being placed over the Denominator

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Denominator e , it will give $\frac{bc-ab}{e}$ for the remainder, or difference sought.

Also let it be required to subtract $\frac{ab+c-18}{b+c}$ from $\frac{bc+5c-24}{b+c}$.

The Difference of the Numerators of the given Fractions is.
$$\left. \begin{array}{l} bc+5c-24-ab-c+18 \\ =bc+4c-6-ab \end{array} \right\}$$

Which remainder or difference being made a Numerator to the common Denominator will give the difference sought, which is
$$\left. \begin{array}{l} bc+4c-6-ab \\ b+c \end{array} \right\}$$

And if it were required to subtract $a+\frac{cd-bc}{c}$ from $aa+\frac{ab}{c}$ The given mixt quantities will (by the fifth Rule of the sixth Chapter) be reduced to $\frac{caa+ab}{c}$ and $\frac{ae+cd-be}{e}$.

Which will be reduced to these Fractions of the same value, having a common Denominator, viz.
$$\left\{ \begin{array}{l} \frac{caae+abe}{ce} \\ \text{and} \\ \frac{cae+ccd-ccb}{ce} \end{array} \right.$$

And if from the Numerator $caae+abe$ you subtract the Numerator $cae+ccd-ccb$ there will be given for the remainder sought, viz.
$$\frac{caae+abe-cae-ccd+ccb}{ce}$$

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In like manner if from a it be required to subtract $\frac{a+c}{b-c}$ First by the fourth Rule of the sixth Chapter, reduce the quantity a to the improper Fractional Quantity $\frac{ab-ac}{b-c}$ and therefrom subtract the given Fraction $\frac{a+c}{b-c}$ so will you have the remainder sought which is $\frac{ab-ac-a-c}{b-c}$

CH A P. VIII.

Multiplication and Division of Algebraical Fractions.

I. **W**hen it is required to multiply two Algebraical Fractions the one by the other, the work is the same as in Vulgar Fractions, for if you multiply the Numerators of the given Fractions together, and likewise their Denominators together, and place their Respective Products for a new Numerator, and a new Denominator, that new Fraction shall be the Product required.

Example.

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Example 1. What is the Product of $\frac{9a}{b}$ multiplied by $\frac{ab}{c}$ Facit $\frac{9aab}{cb}$ for $9a \times ab = 9aab$, which is the Numerator, and $c \times b = cb$ the Denominator.

Example 2. What is the Product of $\frac{3a+2c}{3cb}$ multiplied by $\frac{4cd}{b+c}$ Facit $\frac{12cda+8ccd}{2cbb+2ccb}$; for $4cd \times 3a+2c = 12cda+8ccd$, which is the Numerator.

Example 3. What is the Product of $a+\frac{bc}{d}$ multiplied by $8c+\frac{ad}{b}$ Facit $\frac{8dacb+8ccbb+aadd+bdc}{db}$ for $a+\frac{bc}{d} = \frac{da+bc}{d}$ and $8c+\frac{ad}{b} = \frac{8cb+ad}{b}$ by the Rule of the sixth Chapter, and $\frac{da+bc}{d} \times \frac{8cb+ad}{b} = \frac{8dacb+8ccbb+aadd+bdc}{db}$ which is the Product required.

Example 4. What is the product of ab multiplied by $\frac{a+b}{c}$ Facit $\frac{aab+abb}{c}$; for $ab = \frac{ab}{1}$ and $\frac{ab}{1} \times \frac{a+b}{c} = \frac{aab+abb}{c}$ which is the Product required.

II. If it so chance that you have a Fraction to be multiplied by an Integer that is equal to the Denominator of the Fraction, then take the Numerator for the Product.

Example. What is the Product of $\frac{aa-2ae+ee}{a-c}$ being multiplied by $a-c$? Facit $aa-2ae+ee$.
Z The

The reason of which is plain, for the Numerator being multiplied by the Integer, and the same Integer being put as a Denominator to the Product, the Quotient arising by the Division of the Numerator by the said Denominator, will be equal to the Numerator of the given Fraction;

$$\text{so } \frac{a}{b} \times b = \frac{ab}{b} = a.$$

III. When it is required to divide one Algebraical Fraction by another, if they have a common Denominator, cancel the Denominator, and divide the Numerator of the Dividend by that of the Divisor, so shall that Quote be the Quotient sought.

So if it were required to divide $\frac{abc}{d}$ by $\frac{bcc}{d}$ the Quotient will be found to be $\frac{abc}{bcc} = \frac{a}{c}$ for having cast away the common Denominator d , and divided (abc) the Numerator of the Dividend by (bcc) the numerator of the Divisor, the Quotient will be $\frac{abc}{bcc}$ which is $\frac{a}{c}$ by cancelling bc in the Numerator and Denominator.

IV. When the given Algebraical Fractions have not a common denominator, then multiply the denominator of the divisor into the Numerator of the dividend, and the product is a new numerator; also multiply the numerator of the divisor into the denominator of the dividend, and the product is a new denominator; which new Fraction is the quotient sought, and this is a general Rule in all cases whatsoever, and is the same with division in Vulgar Fractions, only keeping to the Algebraick Rules.

Example

Example. What is the quotient of $\frac{a}{b}$ being divided by $\frac{a}{b}$; Facit $\frac{ca}{ba} = \frac{c}{b}$, for $c \times a =$, (the new Numerator,) and $a \times b = ba$ (the new Denominator).

Likewise, if it were required to divide $\frac{2a+b}{a+b}$ by $\frac{2bb}{a+b}$ the Quotient would be $\frac{3aa+4ab+bb}{2bbc}$ for the numerator of the dividend is $3a+b$, and the denominator of the divisor is $a+b$, and $\frac{a+b}{a+b} \times \frac{3a+b}{a+b} = 3aa+4ab+bb$ which is the Numerator; and the numerator of the divisor is $2bb$, and the denominator of the dividend is c , and $2bb \times c = 2bbc$, which is the Denominator. The like is to be observed in all cases both in Multiplication, Division of Algebraical Fractions.

CHAP. IX.

The Rule of Three in Algebraick Quantities.

I. **T**HE Rule of Three in Algebraical quantities represented by Letters, (whether it be Direct or Inverse) differs not from the Rule

Z. 2

of

of Three in Vulgar Arithmetick, Respect being had to the Rules of Algebraical Multiplication and Division, before delivered in this Book, for (in a direct proportion) if you multiply the second term by the third, and divide the Product thereof by the first the quotient will be the fourth quantity sought in proportion.

Example 1. If b gives c , what will d give?

Facit $\frac{cd}{b}$.

In this Example the second and third quantities, are c and d , which being multiplyed together, produce cd by the Third Rule of the fourth Chapter, which being divided by (b) the first quantity, the quotient is $\frac{cd}{b}$ which is the fourth proportion sought for.

$$b : c :: d \frac{cd}{b}$$

Which may be proved according to the proof of the Rule of Three Direct laid down in the Tenth Chapter of my Vulgar Arithmetick: For,

The *Product* of the second and third Terms is cd . And

The *Product* of the first and fourth Terms is $\frac{bcd}{b}$

$$bcd = cd$$

And by the 3d Rule of the 6th Chap. $\frac{bcd}{b}$

(the first Term) which was to be proved.

Example

Example 2. If $b+c$ require d , what will $d+e$ require? Facit $\frac{dd+de}{b+c}$ For,

$$b+c : d :: d+e : \frac{dd+de}{b+c}$$

Example 3. If 12 require 36, what will $4ab$ require? Facit $\frac{144ab}{12} = 12ab$. For,

$$12 : 36 :: 4ab : \frac{144ab}{12} = 12ab$$

II. Nor will the operation be different from the former, if any of the 3 given quantities be a *Fraction*, or if they be all *Fractions*, observing the Rules of multiplication and division in Algebraick quantities; or when any of the given Terms is a mixt quantity, let it be reduced to the form of a *Fraction*, by multiplying the Integral part by the denominator, and joyning the *Product* to the *Numerator* of the *Fractional* part, and then multiply and divide as before.

Example 4. If $b+\frac{c}{d}$ require d , what will $\frac{f}{g}$ require? Facit $\frac{ddf}{dbg+gc}$? For if you first reduce

$b+\frac{c}{d}$ to the form of a *Fraction*, it will be $\frac{db+c}{d}$ and the second Term d being set *Fraction-wise*, will be $\frac{d}{1}$ then if you multiply ($\frac{f}{g}$) the third

Term by $(\frac{d}{1})$ the second Term, the *Product* will be $\frac{df}{g}$, which being divided by $(\frac{db+c}{d})$ the first Term, the Quotient will be $\frac{ddf}{dbg+gc}$, which is the fourth proportional sought. For,

$$\frac{db+c}{d} : \frac{d}{1} :: \frac{f}{g} : \frac{ddf}{dbg+gc}.$$

I shall not need here to give any Examples in the Inverse Rule of proportion in the Algebraick quantities, the manner of the operation being the same with the former, only the proportion flows backward, as in the Rule of Three Inverse in Vulgar Arithmetick.

CHAP. X.

A Collection of some easy Questions wherein the Rules hitherto delivered are Exercised, taken out of Mr. Oughtred's *Clavis Mathematica*, Chap. 11. Sir Jonas More's *Arithmetick in Spices*, Chap. 10, and Mr. Kersey's *Elements of Algebra*, Chap. 10. of the First Book.

I. **T**Here are two Quantities or numbers, whereof the greater is a ($=4$) and the lesser is e ($=2$). What is their sum? What their difference? What the product of their multiplication? What the Quotient of the greater divided by the lesser? What the Quotient of the lesser divided by the greater? What the sum of their Squares? What the difference of their Squares? What is the sum of their sum, and difference? What is the difference of their sum and differences? What is the Product of their sum and difference? What the Square of their sum?

Z 4

What

What the Square of their difference? What the Square of their Product?

| | |
|---|-----------------|
| 1. The sum of the quantities proposed is | $a + c$ |
| 2. Their difference is | $a - c$ |
| 3. Their Product by multi- plication | ac |
| 4. The quote of the greater divided by the lesser, | $\frac{a}{c}$ |
| 5. The quote of the lesser by the greater | $\frac{c}{a}$ |
| 6. The Sum of their Squares | $aa + cc$ |
| 7. The difference of their Squares | $aa - cc$ |
| 8. The sum of their sum and difference | $2a$ |
| 9. The difference of their sum and difference | $2c$ |
| 10. The Product of their sum and difference | $aa - cc$ |
| 11. The Square of their sum. | $aa + 2ac + cc$ |
| 12. The Square of the difference | $aa - 2ac + cc$ |
| 13. The Square of their product | $aacc$ |

II. There are two quantities whose sum is b ($=12$) and the greater of them is a ($=8$) I demand what is the lesser? What their difference? What is the product of their multiplication? What is the sum of their Squares? What the difference of their Squares?

| | |
|------------------------|-----------|
| 1. The lesser is | $b - a$ |
| 2. Their difference is | $2a - b$ |
| 3. The Product is | $ab - aa$ |
| 4. The | |

- | | |
|---------------------------------------|---------------|
| 4. The sum of their Squares is | $2aa--2ba+bb$ |
| 5. The difference of their Squares is | $2ba--bb$ |

III. There are two Quantities or Numbers whose difference is d , ($=4$) and the greater of them is a ($=8$) I demand what is the lesser? What is their sum? What their Rectangle or Product? What the sum of their Squares? What the difference of their Squares?

- | | |
|---|---------------|
| 1. The difference, or excess } being subtracted from the greater, gives the lesser. | $a-d$ |
| 2. Their sum is | $2a-d$ |
| 3. Their Product or Rectan- } gle is | $aa-da$ |
| 4. The sum of their Squares is | $2aa--2ad+dd$ |
| 5. The difference of their Squares is | $2ad-ad$ |

IV. There are two Numbers, Magnitudes, or Quantities, whereof the Ratio of the greater to the lesser is as r to s , (or as 3 to 2) and the greater of them is a ($=12$.) I demand what is the lesser? What is their Sum? What their difference? What their Rectangle, or Product? What the sum of their Squares? And what the difference of their Squares?

- | | |
|------------------------------------|------------------|
| 1. The lesser is by the Rule of 3. | $\frac{sa}{r}$ |
| 2. Their sum is | $a+\frac{sa}{r}$ |
| 3. Their difference is | $a-\frac{sa}{r}$ |
| 4. Their | |

4 Their Rectangle or Product is }

$$\frac{saa}{r}$$

5 The sum of their Squares is

$$aa + \frac{ssaa}{rr}$$

6 The difference of their Squares is }

$$aa - \frac{ssaa}{rr}$$

But if the Ratio between the lesser and the greater had been given as s to r , (or as 2 to 3) and the lesser had been given e ($=8$) then,

1 The greater by the Rule of 3 would be }

$$\frac{re}{s}$$

2 Their sum

$$\frac{re}{s} + e$$

3 Their difference

$$\frac{re}{s} - e$$

4 Their Rectangle, or Product

$$\frac{rce}{s}$$

5 The sum of their Squares

$$\frac{rree}{ss} + e^2$$

6 The difference of their Squares, }

$$\frac{rree}{ss} - e^2$$

V. There are two numbers or Quantities whereof the Rectangle or Product is b ($=96$) and the greater quantity is a ($=12$) What is the lesser? What their sum? What their difference? What the sum of their Squares? And what the difference of their Squares?

1 The Product given being dividend by (a) the lesser quantity is }

$$\frac{b}{a}$$

2 Their

2 Their sum

$$a + \frac{b}{a}$$

3 Their difference

$$a - b$$

4 The sum of their Squares

$$aa + \frac{bb}{aa}$$

5 The difference of their Squares.

$$aa - \frac{bb}{aa}$$

But if the Rectangle had been given b , as before, and the lesser quantity had been given e ($=8$) Then

1 The greater would have been found by Division to be

$$\frac{b}{e}$$

2 Their Sum

$$\frac{b}{e} + e$$

3 Their difference

$$\frac{b}{e} - e$$

4 The Sum of their Squares.

$$\frac{bb}{ee} + ee$$

5 The difference of their Squares.

$$\frac{bb}{ee} - ee$$

CHAP. XI.

Reduction of Equations.

I. **A**N Equation is an equality between two quantities of different names, whether the

the comparison of Equality be between Simple, or Compound Quantities, or both; between which two Quantities there is always this Character, *viz.* =.

So in this following Equation, *viz.* $a = 3c$, a is said to be the first part, and $3c$ the second part of the Equation, and signifieth that some Number or Quantity represented by a is equal, is three times another Number or Quantity represented by c .

So $a = b + c$ signifieth that some quantity represented by a is equal to the sum of two other Numbers or quantities represented by b and c .

The manner of composing an Equation will be understood by solving of the several questions contained in this and other following Chap. But when known, are mingled with unknown quantities, in an equation they must be so separated or reduced that the unknown quantity or quantities may remain intire on the one side, or part, and the known or given quantities on the other side or part of the Equation, which to perform is the work of Reduction, and which is contained in the several following Rules of this Chapter.

Here note, that the Quantity unknown or sought in every Equation is represented by the Letter a , or some other Vowel, and the quantity or quantities known or given are represented by Consonants, as b, c, d, f , &c.

Reduction of Addition.

II. If equal numbers or quantities be added to equal numbers or quantities, the sums or totals will be equal, and therefore.

If

If it be granted that $a - 8 = 20$

Then by adding $+8$ to each part of the Equation there ariseth $a - 8 + 8 = 20 + 8$

Then because in the first part of the Equation there is -8 and -8 , they destroy each other by the Third Rule of the Second Chap. and it followeth that $a = 28$

Again let this Equation be proposed to be reduced, viz. $a - b = d + b$

Then by adding b to each part of the Equation, there ariseth $a - b + b = d + b + b$

And because $-b$ and $+b$ are in the first part of the equation, they destroy each other, and the Equation is Likewise if $a a - b - c = ff$

Then by adding $b + c$ to each part of the equation there ariseth $aa = ff + b + c$

Now from a due consideration of the premises it followeth that if in an Equation there be any Number or Quantity proposed with the sign -- before it, then if it be transferred to the other side of the equation, and cancelled on the side or part, where it now standeth, the effect will be the same as the adding of that Quantity to each part of the Equation, and this

this by Artifts is called Transposition. As in the first of the foregoing Examples, where it is granted

That $a - 8 = 20$
 And by transposing -8 on the
 other side of the Equati-
 on, making it there $+8$ it
 giveth $a = 28$

And in the second Example where $a - b = d - b$
 By transposing $-b$, cancelling
 it on the first side of the
 equation, and making it
 $+b$ on the other, it is $a = d + 2b$

And let it be granted that $a - bb - d = cc$
 Then by transposing of $-bb$
 and $-d$ there ariseth $a = cc + bb + d$

Reduction by Subtraction.

III. If in any Equation there be any number or quantity signed with $+$ (on which side of the equation soever) if it be cancelled on that side, and placed on the other side with the sign prefixed to it, the work of Reduction is truly performed, and this is also called Transposition, and is only the converse of the foregoing Rule. Examples.

Let it be granted that $a + 8 = 36$
 Then if $+8$ be cancelled, and
 placed on the other part of
 the equation with the sign
 $-$ it will give $a = 36 - 8$
 Which equation being contracted is $a = 28$

Again,

Again let be given

By the Transposition of $+b$ }
on the first side the Equa- }
tion it is }

$$2aa + b = aa + cc$$

$$2aa = aa + cc - b$$

And by Transposition of aa }
on the second side of the }
equation it is }

$$aa = cc - b$$

Also if

By Transposition of $b - c$ }
to the second side of the }
equation it is }

$$aa + b + c = ba + dd$$

$$aa = ba + dd - b - c$$

And by the Transposition }
of ba to the first side of }
the equation it is }

$$aa - ba = dd - b - c$$

Which method (in reducing of the premised Equation) is deduced from this general Axiom, viz.

If from equal Numbers or Quantities, equal Numbers or Quantities are subtracted, the remainder shall be equal.

So in the second Example }
there is given this equation, }
viz. }

$$2aa + b = aa + cc$$

First by subtracting b from }
each part of the equation, }
there is }

$$2aa = aa + cc - b$$

Then I subtract aa from each }
part, and there remaineth }

$$aa = cc - b$$

Reduction by Multiplication.

IV. When in an Equation one or both parts are Fractions, then let them be reduced to a common denominator by the, 2d, 4th, and 5th Rules of

of the sixth Chapter, and then casting away the Denominator, use only the Numerators, so shall Equations exprest by Algebraical Fractions be reduced to other Equations, consisting altogether of Integers. As in the following Examples.

If

$$\frac{a}{8} = 9$$

Then by reducing 9 in the second part of the Equation to a Fraction, having 8 for its Denominator, it is

$$\frac{a}{8} = \frac{72}{8}$$

And by casting away the Denominator which is common to both, it is

$$a = 72$$

Again, if

$$a = \frac{bcd}{a+b}$$

Then by reducing a ; on the first side of the Equation to a Fraction, having $a+b$ for its Denominator, it is

$$\frac{aa+ba}{a+b} = \frac{bcd}{a+b}$$

And by casting away the common Denominator $a+b$ the Equation is

$$aa+ba = bcd$$

Likewise, if

$$\frac{ab}{c} = \frac{dc}{b}$$

The quantities being reduced to a common Denominator, are

$$\frac{abb}{cb} = \frac{dcc}{cb}$$

And the common Denominator cb being cast away, the Equation is

$$abb = dcc$$

V. When

V. When either part of an Equation is Composed of a mixed Quantity or Quantities, let the Integral part or parts be reduced to a Fraction or Fractions, and then proceed as in the last Example.

It is granted that

$$b + c + \frac{a}{a} = cd + \frac{bc}{a}$$

First, it is reduced to $\frac{bb + bc + a}{b} = \frac{cda + bc}{a}$

Which Fractional Equation

being reduced according
to the foregoing Rule, is

$$bba + bca + aa = bcad + bbc$$

VI. When some power or degree of the number or quantity sought is multiplyed into each part, and each member of an Equation, then let that degree or power be cancelled in each part and member, so will it quite vanish, and the Equation will be reduced to more Simple Terms. As for Example.

Let it be granted that

$$aa - ba = ca$$

Forasmuch as a is a Factor in each part and member of the equation, therefore it being expunged in each, there ariseth this equation

$$a - b = c$$

VII. When (according to the second, third, forth, and fifth Rules) an Equation is reduced, and that some known Number or Quantity is multiplyed into the quantity sought, then divide each part of the Equation by that known Quantity, to the end that the quantity sought may

A a

have

have no quantity multiplied into it but 1 (or unity.) As in Example,

If it be granted that
Then because the Quantity
sought is (a) multiplied
by b , divide each part of
the equation by b , and
there ariseth

$$ba = cd$$

$$a = \frac{cd}{b}$$

VIII. When any one part of an Equation is composed of a furd quantity, (*viz.* such as hath the radical sign, prefixed to it) and the other part is a rational quantity: then let that rational quantity be raised to the power signified by the Radical sign, and then cast away the said radical sign, so shall both parts of the Equation be a rational quantity. As,

If it be proposed that
Square 8 and place its Square
in the room of it self, cast-
ing away the radical sign
from the first part of the
Equation, and then it will be

$$\sqrt{a} = 8$$

$$a = 64$$

Likewise if
Then by raising the second
part of the Equation to its
Square, and casting away the
radical sign from the first
part, there ariseth this Equa-
tion, *viz.*

$$\sqrt{a} = cd$$

$$a = ccdd$$

Again,

Again, if

$$\sqrt{a} = b + c$$

The second part of the Equation being squared, and the radical sign cancelled in the first, there ariseth

$$a = bb + 2bc + cc$$

Reduction by Division.

IX. If equal Quantities be divided by equal Quantities, the Quotients thence arising will be equal. For,

If

$$aa = 10a$$

Then by dividing each part of the Equation by a , there ariseth this Equation.

$$a = 10$$

And if

$$aaa = bba + daa$$

Then by dividing each part of the Equation by a , there ariseth

$$aa = bb + da$$

And da in the second part of the Equation being transposed by the third Rule of this Chapter, there ariseth this Equation, viz.

$$aa - da = bb$$

And if

$$ba - ca = dd$$

Then by dividing each part of the Equation by $b - c$, it is

$$a = \frac{dd}{b-c}$$

C H A P. XII.

To Convert Analogies into Equations, and Equations into Analogies.

I. **T**HIS is deduced from this universal Theorem, *viz.* That if four quantities are Proportionals, the product of the two Means is equal to the product of the two Extrems; and if three numbers are Proportionals, the product of the two Extrems is equal to the Square of the Means.

1. Let there be proposed } $a : b :: c : d$
 these four Porportionals. }
 Then by the said Theorem }
 this Equation will follow, } $ad = bc$
viz. }

2. Let there be proposed } $a, b, c ::$
 these three continual Pro- }
 portionals, *viz.* }
 That is to say } $a : b :: b : c$
 Whence there followeth this } $ac = bb$
 Equation, *viz.* }

II. From a due consideration of the Premises it is evident that Equations may oftentimes be resolved into Proportionals, *viz.* when the Product of two quantities is found equal to the product

duct of two other quantities : Then as any one of the Factors in the first side of the Equation is to any one of the Factors in the second part of the Equation, so is the remaining Factor of the second part, to the remaining Factor in the first part : And the Converse,

• Suppose that $bc=ad$
 From thence may be drawn } $b : a :: d : c$
 this Analogy,

The truth of which may be proved by the first Rule of this Chapter, for thereby the said Analogy may be reduced to the given Equation, viz.
 $bc=ad$.

Again if $3ba=3dc$
 Then from thence may be deduced this Analogy, viz. } $3b : c :: 3d : a$
 or } $3b : 3c :: d : a$
 or } $b : 3c :: d : 3a$

Likewise if $bc=ca-d$
 Then may that Equation be } $c+d : b :: c : a$
 resolved into these Proportional,

And if $da=6ba$
 Then it will be found that $6b : d :: d : a$

III. When it happens that there is an Equation between an Algebraical Fraction, and an Integer, if the Numerator of the said Fraction can be resolved into two such quantities, as being multiplyed the one by the other, will produce the said Numerator, then will the said Equation

tion produce this proportion, *viz.*

As the Denominator of the Fraction is to one of the Factors of which the Numerator is produced, so is the other Factor to the Integer, unto which the said Fraction it equal. Examples.

If it be granted, that

$$\frac{bc}{a} = a$$

Then may that Equation be }
resolved into this Analogy.

$$d : c :: b : a$$

For,

$$d : c :: b : \frac{bc}{a} (a)$$

Again, if

$$\frac{cd}{b+a} = a$$

Then may that Equation be }
resolved into this Analogy,

$$b+d : c :: d : a$$

And also if

$$dd = \frac{ab + ad}{cc}$$

Then may the said Equation }
be resolved into this Analogy, *viz.*

$$cc : b+d :: a : dd$$

The Practice of the two last Rules will be plainly discovered in the next Chapter (in the resolution of Questions producing simple Equations) to be of most excellent use in discovering or laying down of Theorems for the ready solution of the Question proposed, or any other of the same nature, which Theorems are to be kept reserved in store for the finding out of new, and the confirmation of old Truths.

C H A P. XIII.

The Resolution of Arithmetical Questions (Algebraically) which produce Simple Equations.

I. **A**N Equation is two-fold, *viz.* First, Simple, and secondly, Adfectèd or Compounded.

II. A Simple Equation is when the Quantity sought (solely possessing one part of the Equation) is either expressed by a Single or Simple Root, as a , or by a Single or Simple Power as aa , or aaa , &c. as in these Equations, *viz.* $a=32$, and $ac=64$, or $4aaa=256$, and such like.

III. When a Question is propounded, and to be resolved Algebraically, then for the Answer put a , and for each of the given Numbers put Consonants, then proceed according to the Tenure of the Question, by Addition, Subtraction, Multiplication, or Division, until an Equation is Composèd; and when the Equation is composèd, then proceed to reduce it (according to the Rules contained in the Eleventh Chapter) until the Quantity unknown (being a or some power of a) do solely possess one part of

A a 4

the

the Equation, and the known or given quantities the other part, and then will the quantity sought be also known.

IV. I shall in the Resolution of every *Question* proceed (*gradatim*) step by step, according to the method used by Mr. *Kersey*, each step being numbred orderly in the margin, from the beginning to the end, by 1, 2, 3, 4, &c. And I shall only proceed in the operation literally, because otherwise this Treatise would swell to a bigger Volume than is at present intended; but I shall give the Learner a taste of Numeral Algebra, in the solution of two or three of the first *Questions* thereby.

Quest. 1. There are two Numbers whose sum is 48 (or b) and the excess of the greater above the lesser is 14 (or c) I demand what are the Numbers.

The Solution literally.

- | | |
|---|-------------------|
| 1. For the greater number put | a |
| 2. From which if you subtract the difference c you will have the lesser, which is | $a - c$ |
| 3. The greater and lesser being added together, will be equal to (b) the sum whence this Equation | $2a - c = b$ |
| 5. And by the Transposition of $-c$ the Equation is | $2a = b + c$ |
| 5. Then dividing each part of the Equation by 2, it is | $\frac{b + c}{2}$ |

6. And

6. And if from $\frac{b+c}{2}$ you subtract (c) the excess of the greater above the lesser, the lesser will be $\frac{b+c}{2} - c$

So that the Numbers sought are 31 and 17, for by the fifth step (a) the greater is found to be $=$ to $\frac{b+c}{2}$ and b is given 48, and c is given 14, the sum of which is 62, which divided by 2, gives 31, for the greater, and by the sixth step, if from the greater you subtract the difference (c) the remainder will give the lesser, which is 17, for $\frac{b+c}{2} - c = 17$.

Now if the fifth and sixth steps are duly considered, they will present you with this

Theorem,

The sum of the sum and difference of any two Numbers being divided by 2, will give the greater Number; and the difference of any two Numbers being subtracted from half the sum of the sum and difference, the remainder will give the lesser number.

The Solution Numerally.

1. For the greater number put a
2. From which if you subtract the difference (14) the lesser is $a - 14$

3. Which

3. Which added together, will be the sum, whence this Equation. $\left. \begin{array}{l} 2a - 14 = 48 \end{array} \right\}$
4. And by transposition of -14 it will be $\left. \begin{array}{l} 2a = 62 \end{array} \right\}$
5. And both parts of the Equation being divided by 2, will give the value of (a) the greater. $\left. \begin{array}{l} a = 31 \end{array} \right\}$
6. From which if you subtract (14) the Difference, the remainder will give the lesser by the second step. $\left. \begin{array}{l} 17 \end{array} \right\}$

So that the Numbers sought are 31 and 17, which will satisfie the conditions of the Question.

Question 2.

There are two Numbers whose Sum is 56 (or b) and the lesser hath such proportion to the greater, as 2 to 5, (or c to d) I demand what are the Numbers?

1. For the lesser number put a
2. Then by the Rule of Three find the greater, viz. $\left. \begin{array}{l} c : d :: a : \frac{da}{c} \end{array} \right\} \frac{da}{c}$
3. Wherefore the sum of the two numbers sought is $\left. \begin{array}{l} a + \frac{da}{c} \end{array} \right\}$
4. Which sum must be equal to the given sum, whence this Equation. $\left. \begin{array}{l} a + \frac{da}{c} = b \end{array} \right\}$

5. Which

5. Which Equation being reduced by the fourth and fifth Rules of the eleventh Chap. the value of a will be found to be

$$\frac{cb}{c+d}$$

6. And by the first, second, and fifth steps the greater number will be discovered to be

$$\frac{db}{c+d}$$

So that the numbers sought are 40 and 16, for (a) the lesser is found to be by the fifth step $\frac{cb}{c+d}$ viz. the Product of (cb) 56 by 2 divided by ($c+d$) the sum of 2 and 5, viz. 7, which is 16, &c.

And if (according to the third Rule of the twelfth Chap.) the two last steps be turned into proportionals, it will give this

Theorem.

As the sum of the Terms which represent the Ratio of two Numbers, is to the sum of the numbers themselves, so is the lesser term to the lesser number; and so is the greater Term to the greater Number.

Therefore if the sum of two Numbers is given, and also their Ratio, the Numbers themselves are also given by this *Theorem*.

The

The same Question solved Numerically.

1. For the lesser number put a
2. Then by the Rule of Three
the greater number is found } $\frac{5a}{2}$
viz. $2 : 5 :: a : \frac{5a}{2}$
3. Then will their sum be $a + \frac{5a}{2}$
4. And according to the te-
nure of the Question, their
sum must be equal to the gi-
ven sum, whence this equa-
tion } $a + \frac{5a}{2} = 56$
5. And that Equation being
reduced by the fifth and
sixth Rules of the eleventh
Chap. the value of a will be
found to be } $\frac{112}{7} = 16$
6. Which being subtracted }
from the given sum, the
greater number is } 40

Quest. 3.

A Gentleman asked his Friend (that had four Purfes in his hand) what Money he had in each Purfe? To whom he answered, that he knew not, but (quoth he) this I know, that in the second Purfe there are 8 or (b) Crowns more than in the first or least Purfe, and in the third 8 Crowns more than the second, and in the fourth or biggest Purfe there are 8 Crowns more than in the third, and twice as many as in the first or least, I demand what number of Crowns he had in each Purfe?

1. For

1. For the number of Crowns }
in the first Purse put a
2. Then in the second there is $a + b$
3. And in the third there is $a + 2b$
4. And in the fourth $a + 3b$
5. Which according to the te- }
nure of the Question is dou- }
ble to that in the first, }
whence this Equation $a + 3b = 2a$
6. Then by the transposition }
of a from the first side of the }
Equation, it is $3b = a$

which discovereth the value of a to be $3b$, or 3 times 8, which is 24, &c. which is the number of Crowns in the first Purse, and consequently the number of crowns in each Purse, is 24, 32, 40, and 48, which will satisfy the conditions of the Question.

The same Question solved Numerically.

1. For the Crowns in the first }
purse put a
2. Then in the second there is $a + 8$
3. And in the third $a + 16$
4. And in the fourth $a + 24$
5. Which is double to the }
number of Crowns in the }
first, whence this equation, }
 $a + 24 = 2a$
6. Which Equation being re- }
duced by the transposition }
of a , discovers the value of }
 a , viz. $a = 24$

Quest.

Quest. 4.

Three men build a Ship which cost them 2700 *l.* (or *b*) Pounds, of which *B* must pay double to what *A* must pay, and *C* must pay three times as much as *B*, I demand the share that each must pay.

1. For the sum to be paid by $\left. \begin{array}{l} A, \text{ put} \end{array} \right\} a$
2. Then *B* must pay $2a$
3. And *C* must pay $6a$
4. The sum of these three quantities are equal to the total charge, whence this Equation $9a = b$
5. Which being reduced, divides $= \frac{b}{9}$
 covers the value of *a*, viz. $\frac{b}{9}$
 which is the sum that *A* must pay, viz. 300 *l.*
 Therefore *B* must pay $\frac{2b}{9} = 600$ *l.* which is twice as much as *A*, and *C* must pay $\frac{6b}{9} = 1800$ *l.* which is three times as much as *B*.

Quest. 5.

There is a Fish whose head is supposed to be 9 (or *b*) inches, and his Tail is as long as his Head and half his Body, and his Body is as long as his Head and his Tail; I demand the length of such a Fish?

1. For the length of the Body put a
2. Then will the Tail be $\frac{a}{2} + b$
3. Then

3. Then if to the Tail you
add the length of the Head, } $\frac{a}{2} + 2b$
viz. b , the sum is
4. Which according to the te-
nure of the Question is e- } $a = \frac{a}{2} + 2b$
qual to the length of the
Body, whence this Equation
5. And the second part of the
Equation being clear'd of
the unknown quantity a by
Reduction, gives the value
of a the length of the Body, } $a = 4b$
viz.
6. Then according to the Te-
nure of the Question, if there-
from you subtract (b) the
length of the head, the re- } $3b$
mainder will be the length
of the Tail, which is

By the fifth Step the length of the Body is found to be $4b = 36$, and by the sixth step the length of the Tail is discovered to be $3b = 3 \times 9 = 27$. So that the length of the head is (given) 9 inches, the length of the Tail 27 inches, and the length of the Body 36 inches, which numbers will satisfy the conditions of the Question.

For, $36 = 27 + 9$ the Body,
And $-\frac{3}{2} + 9 = 27$ the Tail.

So that the whole length of the Fish is
 $9 + 27 + 36 = 72$ Inches.

QUEST.

QUEST. 6.

A Father lying at the point of death, left to his three Sons *A*, *B*, and *C* all his Estate in Money, and divided it thus, viz. to *A* he gave $\frac{1}{2}$, wanting 44 (or *b*) pounds, and to *B* he gave $\frac{1}{3}$ and 14, (or *c*) pounds over, and to *C* he gave the rest, which was 82 or *d*) pounds less than the share of *B*. Now I demand what was the Father's Estate?

1. For the Father's Estate put a
 2. Then will the share left to $\left. \begin{array}{l} A \\ B \end{array} \right\}$ $\frac{a}{2} - b$
 3. And the share of *B* $\frac{a}{3} + c$
 4. And by the third step the $\left. \begin{array}{l} \text{share of } C \text{ is} \\ \text{The Quantities in the three} \end{array} \right\}$ $\frac{a}{3} + c - d$
 5. last steps being added together, give $\frac{7a}{6} + 2c - b - d$
 6. Which must be equal to the $\left. \begin{array}{l} \text{Father's Estate, whence this} \\ \text{Equation.} \end{array} \right\}$ $\frac{7a}{6} + 2c - b - d = a$
- Which Equation after due reduction and transposition of Quantities, the value of *a* is $\left. \begin{array}{l} \\ \end{array} \right\}$ $a = 6b + 6d - 12c$ discovered to be

And $6b = 6 \times 44 = 264$, and $6d = 6 \times 82 = 492$, and $12c = 12 \times 14 = 168$, now $264 + 492 - 168 = 588$, so that the Father's Estate was 588 pounds, of which *A* had 250*l.* *B* 210*l.* and *C* 128, which Numbers do answer the conditions of the Question.

Quest.

Quest. 7.

Two persons thus discoursed together concerning their Money, quoth *A* to *B* give me 3 (or *b*) of your Crowns, and I shall have as many as you; nay quoth *B* to *A*, but if you will give me 3 of your Crowns, I shall have 5 times as many as you. Now I demand how many Crowns had each person?

1. For the number of Crowns which *A* had put a
2. Then forasmuch as adding 3 (or *b*) Crowns to *A* will be equal to the Crowns remaining to *B* after he had given 3 Crowns to *A* therefore *B* will then have left. $a + b$
3. And consequently if you add thereto the 3 (or *b*) Crowns which he gave to *A* the sum will be the number of Crowns which *B* had at first, which is $a + 2b$
4. Then if from the number of Crowns *A* had at first (*a*) you subtract 3 (or *b*) crowns, there will remain to *A* $a - b$ crowns, and giving the same to *B* he will then have $a + 3b$
5. Which according to the tenure of the Question is five times as much as what *A* had left, whence there ariseth this Equation. $5a - 5b = a + 3b$

B b

6. Which

6. Which equation being reduced by the second and seventh Rules of the eleventh Chapter, the value of a is discovered to be

$$a = 2b$$

7. And by the sixth and third steps the number of Crowns which B had at first are found to be

$$a + 2b = 4b$$

So that it is found that A had 6 Crowns, and B had 12 Crowns, which numbers will satisfy the conditions of the Question. For,

$$9 + 3 = 12 - 3 = 9$$

And,

$$12 + 3 = 5 \times 6 - 3 = 15$$

Quest. 8.

A Labourer had 576 (or b) pence for threshing 60 (or c) Quarters of Corn, viz. Wheat and Barly; for the Wheat he had 12 (or d) pence per Quarter, and for the Barly he had 6 (or f) pence per Quarter, I demand how many Quarters of each he threshed?

1. For the quarters of Wheat } which he threshed put a

2. Then the quarters of Barley will be $c - a$

3. The quantity of Wheat in the first step being multiplied by its price produceth da

4. The

4. The quantity of Barly in the second step being multiplied by its price, produceth

$$fc - fa$$

5. The sum of the quantities in the two last steps must be equal to the given price of the 60 quarters, whence this equation

$$da - fa + fc = b$$

6. Which being reduced by the second, third, and fifth Rules of the eleventh Cha. the quantity of Wheat will be discovered to be

$$a = \frac{b - fc}{d - f}$$

7. And by the second and fifth steps the quantity of Barly is discovered to be

$$c = \frac{b - fc}{d - f}$$

So that the quarters of Wheat which he threshed were 36, and the quarters of Barly 24.

The Proof.

$$12 \times 36 = 432$$

And

$$6 \times 24 = 144$$

And

$$432 + 144 = 576, \text{ which was to be proved.}$$

Quest. 9.

A Gentleman bought a Cloak of a Sales-man, which cost him 3*l.*—10*s.* or 70 (or *b*) shillings, and desiring the Salesman to tell him what he

B b 2

gained

gained thereby, he said he gained $\frac{1}{4}$ (or c) of what it cost him, the question is what the Cloak cost the first penny?

1. Suppose the Cloak cost
2. Then he gained
3. The first and second steps being added together, their sum will be equal to the sum which the Gentleman gave for it, whence this equation
4. Which Equation being reduced by the ninth Rule of the eleventh Chap. the value of a will be discovered to be

 a ca

$$ca + a = b$$

$$a = \frac{b}{b+1}$$

So that it cost 56 shillings, $\frac{1}{4}$ of which is 14 shillings, and $56 - 14 = 70$.

And if the quantity in the fourth step be duly considered, you will find that if the gain had been any other part or parts of the first cost, if the price it was sold for had been divided by the Fraction representing part of the gain, increased by 1, the quote would have been the answer.

Question 10.

A Gentleman hired a Labourer to work for him for 40 (or b) days, and made this agreement with him that for every day he wrought he should have 20 (or c) pence, and for every day that he played he should forfeit 8 (or d) pence, and at the end of the said 40 days he received

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184 (or f) pence, which was his full due. Now I demand how many days he wrought, and how many days he played?

1. For the number of days he wrought, put a
2. Then the number of days he played will be $b - a$
3. And if the time he wrought (in the first step) be multiplied by 20 (c) it will produce the total he gained by work, viz. ca
4. And if the time he played (in the 2d step) be drawn into 8 (d) the product will be what he lost by play $db - da$
5. And if the total loss (in the fourth step) be subtracted from the gain (in the third step) the remainder will be what he received, whence this Equation $ca - db + da = f$
6. Which being reduced by the second and ninth Rules of the eleventh Chapter, it will discover the value of a to be eighteen which is the days that he wrought. $a = \frac{f + db}{c + d}$
7. And from the sixth and second steps the number of days he played are discovered to be 22 days, viz. $b - \frac{c + d}{f + d}$

So that by the sixth step it appears he wrought 18 days, and by the seventh step it appears that he played 22 days.

The proof.

$$18 \times 20 = 360 \text{ and}$$

$$22 \times 8 = 176 \text{ and}$$

$$360 - 176 = 184$$

Quest. 11.

A person (in the Afternoon) being asked what a Cloak it was, answered that $\frac{3}{4}$ (or b) parts of the time from Noon was equal to $\frac{5}{8}$ (or c) parts of the time remaining to midnight, now, (supposing the time from Noon to Midnight to be divided in 12 (or d) equal parts or hours) I demand what was the present hour of the day?

1. For the hour sought put a
2. Then the time to midnight } $d - a$
will be
3. Then will $\frac{3}{4}$ (or b) parts of } ba
the Hour from Noon be
4. And $\frac{5}{8}$ (or c) parts of the } $cd - ca$
time remaining till midnight
will be
5. Therefore from the third } *viz.* $ba = cd - ca$
and fourth steps there ariseth this equation.
6. Which equation being reduced according to the second and ninth Rule of the eleventh Chapter gives the value of a (to be $6 \frac{48}{55}$ the hour sought) *viz.* $a = \frac{cd}{b - c}$

- So

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So that the hour sought was $6\frac{48}{32}$, and consequently the time remaining till midnight was $5\frac{281}{32}$ hours, which two numbers will answer the conditions of the question, for, $\frac{3}{4}$ parts of $6\frac{48}{32}$, which is $3\frac{3}{4}$ is equal to $\frac{3}{8}$ parts of $5\frac{281}{32}$, as you may prove at your leisure.

Moreover, If the last step be converted into proportionals by the third Rule of the twelfth Chap. it will give this

Theorem.

As the sum of the parts of any two Numbers (wherein there is an equality) is to the sum of those Numbers, so is the given parts of any one of those Numbers, to the other Number.

As suppose it were required to find out two Numbers, whose sum is 27, and such, that $\frac{3}{4}$ of the one may be equal to $\frac{3}{8}$ of the other, the same may be found out by the said Theorem. For,

$$\frac{3}{4} + \frac{3}{8} : 27 :: \frac{3}{4} : 15.$$

which number so found is the number sought, whereof $\frac{3}{4}$ is to be taken; and the other is $27 - 15 = 12$, or it may be found by the following proportion, *viz.*

$$\frac{3}{4} + \frac{3}{8} : 27 :: \frac{3}{8} : 12$$

Quest. 12.

One asked a Shepherd what was the price of his hundred Sheep, quoth he, I have not an hundred, but if I had as many more, and half as many

ny more, and $7\frac{1}{2}$ (or b) sheep, then I should have just 100 (or c) I demand how many sheep he had?

1. For the number of sheep he had, put

2. Which being doubled is

3. And if to the second step you add half the first, it is

4. And if to the third step there be added $7\frac{1}{2}$ (or b) the sum is

5. Which quantity in the fourth step is equal to 100 (or c) whence this equation

6. Which equation being reduced by the 5th and 7th Rules of the 11th Chap. the value of a will be discovered to be 37, viz.

So that the number of sheep he had were 37 for $37 + 37 + 7\frac{1}{2} + 7\frac{1}{2} = 100$

C H A P. XIV.

How to Extract the Root of a Square formed from a Binomial, and how by having any two of the Members of such a Square given to find out the third.

I. **A** Binomial is a quantity consisting of two names or parts, as $a+b$, or $a-b$, $aa+cc$, $b+d$, &c. And when a Square is formed from such a Root, it will consist of three members or parts, viz. two Affirmative Squares of the parts of which the Binomial is composed, and the double Rectangle of those parts, which double Rectangle is sometimes affirmative, and sometimes negative, viz. Affirmative, when the parts of the Binomial are both affirmative, or both negative, that is, when they are both signed with $+$, or both with $-$; and negative, when one of the parts of the Binomial Root is signed with $+$, and the other with $-$.

So if $a+b$ were given for a Root, its Square would be $aa+2ab+bb$ which is composed of (aa and bb) the Squares of the parts of which the Root is composed, and of ($2ab$) the double Pro-

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Product, or Rectangle made by the multiplication of the said parts (a and b) one by the other. See the work.

$$\begin{array}{r}
 a+b \text{ the Root} \\
 a+b \\
 \hline
 aa+ab \\
 ab+bb \\
 \hline
 aa+2ab+bb \text{ the Square.}
 \end{array}$$

So if it were required to find the Square of the Binomial $a-b$, or $b-a$ it (being multiplied by it self would be $aa-2ab+bb$, which is composed of (aa and bb) the sum of the Squares of the parts, and their double Rectangle, as before, but ($2ab$) the Double Rectangle of the parts is signed with—, so that the Squares of the difference of any two numbers or quantities is equal to the sum of the Squares of the said quantities or numbers made less by their double Rectangle. As by the work,

$$\begin{array}{r}
 a-b \text{ Root} \qquad \qquad b-a \text{ the Root} \\
 a-b \qquad \qquad \qquad b-a \\
 \hline
 aa-ba \qquad \qquad \qquad bb-ba \\
 -ba+bb \qquad \qquad -ba+aa \\
 \hline
 aa-2b+bb \text{ Square } bb-2ba+aa \text{ the Square}
 \end{array}$$

So if the Number 10 were divided into 8 and 2, viz. $8+2$, its Square would be $64+32+4=10 \times 10=100$. And the Square of $8-2$ is $64-32+4=6 \times 6=36$ for $8-2=6$ and $6 \times 6=36$.

Note,

Note, That a Binomial Root having one of its parts signed with—, is by some Authors called a Residual Root, as $a-b$, and $c-d$, &c. are Residuals.

II. From what hath been said concerning the Square of a Binomial, may be inferred this

Theorem.

If a Compound quantity consisting of 3 members, whereof two are Squares of different names, with the sign+ prefixed to them, and the third is the double Rectangle of the Roots of those Squares, having also the sign+ prefixed to it, then shall the Square Root of such a compound quantity be the sum of the Square Roots of the said two simple Squares; but if the said double Rectangle hath the sign— prefixed to it, the Square Root of the said Compound quantity, shall be the difference of the said Roots.

So the Square Root of $aa+2ab+bb$ will be found to be $a+b$, for the Square Root of aa is a , and the Square Root of bb is b , which two Roots added together, give $a+b$.

Also the Square Root of $aa+8a+16$ will be found to be $a+4$, the 2 Squares in the given quantity are aa and 16 , and $8a$ is the double product of (a and 4) the said Roots being multiplied the one by the other.

Likewise the Square Root of $aa-2ab+bb$ is $a-b$, or $b-a$, not $a-b$, because the double Rectangle ($2ab$) is signed with—.

Furthermore the Square Root of $9aa+12ba+4bb$ is $3a+2b$: The two Square quantities in the said Compound Square are $9aa$, and $4bb$, whose

whose Roots are $3a$ and $2b$, and $12bb$ is the double Product of $3a$ and $2b$ being multiplied together.

And the Square Root of $aa - 20a - 100$ is $a - 10$, for the two Squares in this Compound Square Quantity are aa and 100 , whose Square Roots are a and 10 , and $20a$ is the double Rectangle of 10 and a , they being multiplied together.

The foregoing Theorem being well understood will be of excellent use in the Resolution of Questions, producing Quadratick Equations, as you will find by the Questions contained in the next Chapter.

III. When it is required to extract the square Root of quantity whose Root cannot be exactly extracted, then prefix the radical sign to it, which shall represent its Square Root. So the Square Root of bc is \sqrt{bc} , or $\sqrt{2) bc}$, and the Square Root of $\frac{aa+cc}{aa+bb}$ is thus represented, viz. $\sqrt{\frac{aa+cc}{aa+bb}}$, or $\sqrt{(2) \frac{aa+cc}{aa+bb}}$, &c.

IV. From a due consideration of the foregoing Theorem, a way is discovered how by having any two of the members of a Square formed from a binomial Root, the third member may be found out. For,

When two Affirmative Square Quantities are given for two of the members of a Square formed from a binomial Root, then take the Roots of those two Squares and multiply them the one by the other, and double the Product, so shall that Product being doubled be the third member, which being annexed to the two given Squares, either by $+$, or $-$, it will make an exact Compound Square, whose Root shall be a Binomial.

So

So if $aa - bb$ were given for two of the members of a Square, first, I find their Roots to be a and b , which being multiplyed the one by the other, produce ab , and that Product being doubled gives $2ab$, for the middle Term of the Compound Square Quantity to make it a compleat square, the Root whereof is a Binomial, viz. $aa - 2ab - bb$, if the said double product be joyned to the said sum of the Squares by the sign—, it will give the Compound Square Quantity $aa - 2ab - bb$ whose Root is $a - b$.

Also if $25aa - 16bb$ were given for two of the members of a Square, whose Root is a Binomial. The said Square being compleated, will be $25aa - 40ab - 16bb$, or $25aa - 40ab - 16bb$, whose Root is either $5a - 4b$, or $5a - 4b$.

V. When the two given members of a Compound Square Quantity, whose Root is a Binomial, are the double product or rectangle, and one of the two affirmative squares, divide half the said double product by the Root of the given square, and square the Quotient, so shall that square be the third member sought, which being joyned to the two given Quantities with the sign—, it will give you a compleat square having for its root a Binomial.

As for Example. Let $aa - 2ba$ be proposed for 2 of the members of a square, whose Root is a Binomial: First, I take half of ($2ba$ the said double product and it is ba , which being divided by (a) the Root of aa the given Square, the Quotient is b , whose square is bb for the third member sought.

Again, Let $25aa - 40a$ be the two proposed terms of such a square, whose Root is a Binomial,

al, and let it be required to find the other square which shall make it a compleat square, raised from a Binomial Root; in order to which, first, I take half ($40a$) the double product, viz. $20a$, and divide it by the Root of ($25aa$) the given square, which is $5a$, and the Quotient is 4, which being squared, gives 16 for the third member required, which being joyned to the rest, gives $25aa + 40a + 16$ for the square compleated.

VI. When the two given members of a square raised from a Binomial Root, are such that one of them is a square affirmative without any Number or Quantity prefixed to it, and the other is the Root of the said square multiplied by some other Quantity, then is that other Quantity by Artists called the Coefficient, and if you square half the said coefficient, or, (which is all one) take $\frac{1}{4}$ of the square of the coefficient, that shall be the third member required, which being joyned to the two given quantities by the sign $+$, it will give you a compleat square raised from a Binomial Root.

Example. Let the two given members of a square be $aa + 2ba$, and let it be required to find out the third member. Here the coefficient is $2b$, half of which is b , which being squared, gives bb for the third member which was sought, so is the square compleated $aa + 2ab + bb$.

In like manner, if the two given members of a square were $aa + ba$, and it were required to find out the third member.

Here the coefficient is b , half of which is $\frac{1}{2}b$, or $\frac{b}{2}$, whose square is $\frac{1}{4}bb$, or $\frac{bb}{4}$ for the mem-

ber

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ber sought. Also let the two given members of a square be $aa+8a$, and let it be required to find out the third member. Here the coefficient is 8, half of which is 4, whose square is 16, for the third member required, so is $aa+8a+16$, a compleat square, whose Root is $a+4$.

Again, if the two given members of a square be $aa-ca$, and the third is required; First, I take half the coefficient c , viz. $\frac{1}{2}c$, and then square it, and it gives $\frac{1}{4}c^2$ or $\frac{cc}{4}$ for the member sought, and so is the Square compleated $aa-ca+\frac{1}{4}cc$, whose Root is $a-\frac{1}{2}c$.

In like manner, if it were required to make $aa+3ba$ a compleat square, take half the coefficient ($3b$) which is $\frac{3}{2}b$, or $\frac{3}{2}b$, whose square is $\frac{9}{4}bb$, or $9\frac{bb}{4}$ which being joyned to the two given Terms with the sign $+$, it gives $aa+3ba+\frac{9}{4}bb$, whose Root is $a+\frac{3}{2}b$.

The same Rule is to be observed for the squaring of half the coefficient when it is a Fraction.

As for Example. Let the two members of a square raised from a Binomial given be $aa+\frac{bd+3c}{f}a$, and let it be required to find the third

member. Here half the coefficient is $\frac{bd+3c}{2f}$ which

being squared, gives $\frac{bbdd+6bdc+9cc}{4ff}$ for the mem-

ber sought, and so the square being compleated, is $aa+\frac{bd+3c}{f}a=\frac{bbdd+6bdc+9cc}{4ff}$, whose Root is

$$a+\frac{bd+3c}{2f}$$

VII. When

VII. When the Root of the given square hath no coefficient, then the number 1 is supposed to be the co-efficient, half whereof, (*viz.* $\frac{1}{2}$) being squared, gives ($\frac{1}{4}$) the third member sought to make it a compleat square.

So $aa - a$ being given for 2 of the members of a square raised from a Binomial, its third member to make the square compleat will be $\frac{1}{4}$, for $aa - a = aa - 1a$, where the Coefficient is 1, whose half is $\frac{1}{2}$, which being squared, gives $\frac{1}{4}$ for the third member sought, so the Square being compleated, is $aa - a + \frac{1}{4}$, whose Root is $a - \frac{1}{2}$.

This Chapter ought to be well understood before any further progress be made, for the manner how to resolve Questions which produce Quadratick (or square) Equations doth principally depend thereupon.

CHAP. XV.

Concerning the Resolution of Questions producing Quadratick Equations.

I. Quadratick (or square) Equations, are such adfected or Compound Equations as consist of three terms, the highest of which is

is a square, and is called the highest term in the Equation, of which three terms two are always unknown, and the third is always known? the first of the three is the square of the Quantity or Number sought, and the second Term is the Product of the Quantity sought, being multiplied by some known Number or Quantity, and is called the Middle Term of an Equation, and the third Term is a Number, or Quantity purely known.

So in this Equation, *viz.* $aa+ba=d$, the first and highest Term or member is aa , which is the square of the Quantity or Number sought, and ba is the middle term of the Equation which is the Product of the Quantity sought, it being drawn into b (which is known) and the third term or member of this Equation is b , which is really known, and is usually called the Absolute Number or Quantity given.

II. The Equations of this kind are of three Forms, which are laid down by Mr. *Kersey*, in the fifteenth Chapter of the first Book of his *Elements of Algebra*, as followeth, *viz.*

Equations of the first Form.

$$\begin{array}{l|l} aa+6a = 55 & aa+ca=b \\ aaaa+8aa = 48 & aaaa+daa=f \\ aaaaaa+4aaa = 837 & aaaaaa+gaaa=b \end{array}$$

Equations of the second Form.

$$\begin{array}{l|l} aa-13a=24 & aa-ba=k \\ aaaa-6aa=27 & aaaa-paa=d \\ aaaaaa-2aaa=48 & aaaaaa-maaa=g \end{array}$$

C c Equa-

Equation of the third Form.

$$10a - aa = 24$$

$$5aa - aaaa = 4$$

$$9aaa - aaaaa = 8$$

$$ca - aa = n$$

$$raa - aaaa = s$$

$$raaa - aaaaaa = t$$

III. The Resolution of Equations which fall under the first Form.

When an Equation is composed after any of the three foregoing Forms, and any known Quantities are mixed with unknown, let it be so reduced by transposition (according to the Rules of the Eleventh Chapter) as that the known quantities may possess one side, and the unknown Quantities the other side of the Equation.

Example. Let this Equation be given, viz.
 $aa - ba = -ba - bdc.$

By the transposition of B on the first part of the Equation, and $-ba$ one the second part, it will be reduced to this Equation, viz. $aa - ba = bdc - b$, which is an Equation of the first Form: And when your Equation is so reduced, add to each part of the Equation the square of half the coefficient, and so will the first part of the Equation be an Exact and compleat square, then according to the 2d and 3d Rule of the Fourteenth Chapter extract the square Root of both parts of the Equation, and from the Square Roots of both parts of the Equation subtract half the coefficient, and then you will discover the value of a . As in the following Examples.

Quest.

Quest. I.

What number is that which being squared, and multiplied by 8 (or b) the sum of the said Square and Product is equal to 384 (or c)?

Resolution.

1. For the number sought put a
2. Whose Square is aa
3. Its Product by 8 (or b) is ba
4. The sum of the second and third steps must be equal to 384 (or c) whence this Equation. $aa + ba = c$
5. To each part of the equation add the square of $(\frac{1}{2}b)$ half of the coefficient, then will it be $aa + ba + \frac{1}{4}bb = c + \frac{1}{4}bb$
6. Then by extracting the square root of both parts of the equation by the second and third Rules of the 14 Chap. it will be reduced to $a + \frac{1}{2}b = \sqrt{c + \frac{1}{4}bb}$
7. By the transposition of $\frac{1}{2}b$ to the second part of the Equation the value of a is discovered to be $a = \sqrt{c + \frac{1}{4}bb} - \frac{1}{2}b$

Which Equation is thus expressed in words, viz. the number sought is equal to the remainder, when $(\frac{1}{2}b)$ 4 is subtracted from the square root of the sum of (c) 384. and $\frac{1}{4}$ of the Square of (b) 8 (added together) which is 16, so that the value of a is 16. For $c + \frac{1}{4}bb = 400$ and $\sqrt{(2) 4000} = 20$. and $20 - 4 = 16$.

Quest. 2.

What number is that whose Square being multiplied by 4 (or b) and its Biquadrate (or fourth Power) multiplied by 6 (or c) and the Products added together, the sum is 3850, (or d)

Resolution.

1. For the number sought put

 a

2. Its Square multiplied by b is

 baa

3. Its Biquadrate multiplied by c is

 $caaaa$

4. The sum of the second and third steps must be equal to 3850 (or d) whence this Equation, viz.

$$caaaa + baa = d$$

5. And because the highest power of the equation is multiplied by c , therefore each part being divided by c the equation is

$$aaaa + \frac{b}{c}aa = \frac{d}{c}$$

6. To each part of the equation add half the square of the coefficient ($\frac{b}{c}$) and the equation will be

$$aaaa + \frac{b}{c}aa + \frac{bb}{4cc} = \frac{d}{c} + \frac{bb}{4cc}$$

7. Then the square Root of each part of the equation in the sixth step, being extracted by the second and third Rules of the 4 Chap. the equation then will be

$$aa + \frac{b}{2c} = \sqrt{\frac{d}{c} + \frac{bb}{4cc}}$$

8. And

8. And by the transposition of $\frac{b}{2c}$ to the second part of the equation the value of aa is found to be

$$aa = \sqrt{(2) \frac{d}{c} - \frac{bb}{4cc} \frac{b}{cc}}$$

9. And because the equation in the 8 step is the value of aa , therefore if the square Root of each part of that equation be extracted, the value of a it self will be discovered to be

$$a = \sqrt{(2) \sqrt{\frac{d}{c} + \frac{bb}{4cc} \frac{b}{cc}}}$$

which in words is as much as to say the Number sought (or a) is equal to the square Root of the remainder when $(\frac{b}{2c})^{\frac{1}{2}}$ is subtracted from the square Root of the sum of $\frac{25}{8}$ (or $\frac{d}{c}$) and $\frac{15}{4}$ (or $\frac{bb}{4cc}$) being added together, so that the value of a , (or the number sought is 5. For

$$\sqrt{\frac{d}{c} + \frac{bb}{4cc}} = \frac{76}{3} \text{ and } \frac{76}{3} - \frac{1}{3} \left(\frac{b}{2c} \right) = \frac{75}{3} = 25 \text{ and } \sqrt{25} = 5, \text{ which is the number sought.}$$

The Proof.

$$4 \times 5 \times 5 + 6 \times 5 \times 5 \times 5 = 3850$$

You must remember always to reduce a Fraction to its lowest Terms before you extract its Root.

III. The Resolution of Equations which fall under the second of the three Forms before mentioned.

Quest. 1.

What number is that which having 8 (or b) times its self subtracted from its square, the remainder is 48 (or c)?

Resolution.

1. For the number sought put a
2. Then will its square be aa
3. The first step multiplied by b is ba
4. If the third step be subtracted from the second, the remainder will be 48 (or c) whence this equation $aa - ba = c$
5. To each part of that equation add the square of $(\frac{1}{2}b)$ half the coefficient, and then it will be $aa - ba + \frac{1}{4}bb = \frac{1}{4}bb + c$
6. Extract the square Root of each part of the last equation by the second and third Rules of the 14th Chapter, and it is $a - \frac{1}{2}b = \sqrt{c + \frac{1}{4}bb}$
7. And by the transposition of $\frac{1}{2}b$ to the second part of the equation, the value of a is discovered to be 12. $a = \sqrt{c + \frac{1}{4}bb} + \frac{1}{2}b$

which in words is as much as to say, the Number sought (or a) is equal to the sum of the universal Square Root of the sum of 48 (or c) and a fourth part of the square of b (or $\frac{1}{4}bb$) being added to

to 4 (or $\frac{b}{2}$) which is 12 for $1=48$ and $\frac{1}{4}bb=16$, and $48+16=64$ and $\sqrt{64}=8$, and $8-4$ ($\frac{1}{2}b$) $=12$.

The Proof.

$$12 \times 12 - 8 \times 12 = 48$$

Quest. 2.

What number is that which having 12 (or b) times its square subtracted from its Biquadrate, or forth power, the remainder is 3328 (or c)?

Resolution.

1. For the number sought put a
2. Then its biquadrate is $aaaa$
3. And its square multiplied by 12 (or b) is baa
4. The difference of the second and third steps must be equal to 3328 (or c), whence this equation, *viz.* $aaaa - baa = c$
5. Square half the coefficient, and add it to each part of the equation, and then it will be $212a - bba + \frac{1}{4} = +\frac{1}{4}bb$
6. Extract the square Root of both parts of the equation by the second and third Rules of the 14 Chap. and then the equation will be $aa - \frac{1}{2}b = \sqrt{c + \frac{1}{4}bb}$
7. By the transposition of $-\frac{1}{2}b$ to the contrary Coast the value of (a) the number sought will be discovered to be $aa = \sqrt{c + \frac{1}{4}bb} + \frac{1}{2}b$

8. By extracting the square root of both parts of the equation in the 17th step the value of a is found to be 8.

$$a = (2)\sqrt{c + \frac{1}{4}bb + \frac{1}{2}b}$$

Which is as much as to say, that the Number sought, (or a) is equal to the universal square root of the sum of 6 (or $\frac{1}{2}b$) being added to the universal square Root of the sum of 3328, (or c) and 36 (or $\frac{1}{4}bb$) which upon tryal you will find to be 8.

For, $c = 3328$, and $b = 12$, and $\frac{1}{4}bb = 36$, wherefore $3328 + 36 = 3364$, and $\sqrt{3364} = 58$, and $58 + (\frac{1}{2}b) 6 = 64$, and $\sqrt{64} = 8$, which is the Number sought.

IV. The manner of resolving Equations which fall under the last of the three forms before-mentioned.

Let the equation proposed (if it falls under the third and last form) be reduced to an equation of the second form, by the transposition of its terms, as in the following questions, viz.

What Number is that whose square being subtracted from 12 (or b) times it self the remainder is 32 (or c)?

Resolution.

1. For the number sought put

2. Its product by 12 (or b) is

3. If from the second step you subtract (aa) the square of the first step, the remainder is

a

ba

$ba - aa$

4. The

4. The remainder in the third
step is equal to 32 (or c) } $ba - aa = c$
whence this equation

Now by transposition I reduce to an equation of the second of the foresaid forms. And First,

5. By transposition of aa to
the contrary part, the e- } $ba = c + aa$
quation is
6. Then by transposition of c } $ba - c = aa$
in the fifth step, the equa- }
tion is
7. And by transposition of ba } $-c = aa - ba$
in the sixth step, the equa- }
tion will be or
 $aa - ba = -c$

So that from a due consideration of the method used in reducing Equations of the third form to equations of the second form you may easily perceive that the work of transposition in the fifth, sixth and seventh steps is performed only by changing the signs of all the Terms of the Equation in the fourth step, viz. by changing $+$ into $-$, and $-$ into $+$.

So the Equation in the fourth step is $ba - aa = c$,

And by changing the signs of $ba - aa$ on the first part of the Equation, and of c in the second part into $-ba + aa$, and $-c$, the Equation will then be $-ba + aa = -c$, or $aa - ba = -c$, which is the same with that in the seventh step; and it is now an Equation of the second of the three foregoing forms, so that I now proceed to the solution of the Equation.

8. The

8. The square of half the coefficient ($\frac{1}{2}b$) in the seventh step to each part of the equation, it will then be } $aa - ba + \frac{1}{4}bb = c + \frac{1}{4}bb$
9. The square root of each part of the last equation being extracted by the second and third Rules of the 14 Chapter the equation will then be } $a - \frac{1}{2}b = \sqrt{-c + \frac{1}{4}bb}$
10. And by the transposition of $\frac{1}{2}b$ in the ninth step to the contrary part, the value of a will then be found to be (8) } $a = \sqrt{-c + \frac{1}{4}bb} + \frac{1}{2}b$

which is as much as to say that the number sought (or a) is equal to the sum of 6 (or $\frac{1}{2}b$) being added to the Square Root of the remainder, when 32 (or c) is subtracted from 36 (or $\frac{1}{4}bb$) which is 8. For, $\frac{1}{4}bb - c = 4$ whose square Root is 2, and $2 + 6$ (or $\frac{1}{2}b$) = 8 which is the number sought.

The Proof.

$$12 \times 8 = 96$$

And $96 - 64$ (aa) = 32 (or c) which was propounded.

V. The Resolution of various Questions producing Quadratick Equations.

Quest. I.

There are two Numbers whose sum is 12 (or b) and the sum of their squares is 80 (or c) I demand what are those numbers?

Reso-

Resolution.

1. For one of the numbers sought put a
2. Then the other will be $b - a$
3. Then the sum of their Squares will be $bb - 2ba + 2aa$
4. Which quantity in the third step is equal to 80 (or c) whence this equation $bb - 2ba + 2aa = c$
5. Which equation being duly reduced by the rules of the eleventh Chapter giveth this equation. $aa - ba = \frac{c}{2} - \frac{bb}{2}$
6. Which Equation being solved according to the third Rule of this Chapter, the value of a is discovered to be $a = \sqrt{\frac{c - bb}{2} + \frac{bb}{4} + \frac{b}{2}}$
7. Wherefore I conclude the numbers sought are 8 and 4, for their sum is 12, and the sum of their squares is 80
8. Moreover the Equation in the sixth step will give this

Canon.

If from half the given sum of the squares you subtract half the square of the given sum, and to the remainder you add half the given sum, the square root thereof being added to the said half sum of the numbers, the sum of this addition will give you the greater number sought, and the greater number being subtracted from the given sum of the numbers, will give the lesser number sought.

Quest.

Question 2.

There are two numbers, the product of whose multiplication is 96 (or b) and the sum of their squares is 208 (or c) I demand what are those numbers?

Resolution.

- | | |
|--|--------------------------|
| 1. For one of the numbers sought put | a |
| 2. Then by dividing 96 (or b) by a , the Quotient will give other which is | $\frac{b}{a}$ |
| 3. The square of the number in the first step is | aa |
| 4. The square of the other number in the second step is | $\frac{bb}{aa}$ |
| 5. And the sum of their squares in the third and fourth steps is | $aa + \frac{bb}{aa}$ |
| 6. Which sum in the fifth step must be equal to the given sum of the Squares 208 (or c) whence followeth this equation, viz. | $aa + \frac{bb}{aa} = c$ |
| 7. Which Equation in the last step being duly reduced by the Rules of the eleventh Chapter the value of a will be discovered to be | |

$$a = \sqrt{(2) \sqrt{\frac{1}{4}cc - bb} + \frac{1}{2}c}$$

8. So that I conclude the numbers sought to be 12 and 8, for their product is 96, and the sum of their squares is 208.

9. More-

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9. Moreover the Equation in the seventh step giveth this

C A N O N.

From $\frac{1}{4}$ of the Square of the given sum of the Squares subtract the Square of the given Product of the Multiplication of the numbers sought, and extract the square Root of the remainder, and to the said Square Root add half the given sum of the said Squares, and then extract the square root of the sum of that Addition, so shall that square Root be one of the Numbers sought, by which if you divide the given Product, the Quotient will be the other Number sought.

Q U E S T I O N 3.

There are two Numbers whose sum is 12 (or b) and the Product of their Multiplication is 20 (or c) what are the Numbers?

R E S O L U T I O N.

- | | |
|--|-----------|
| 1. For one of the Numbers sought put | a |
| 2. Which if you subtract from (12) b the given sum, the remainder will be the other number, viz. | $b - a$ |
| 3. And if the first and second steps be multiplied the one by the other, the Product will be | $bb - aa$ |

4. Which

4. Which Product the Question }
 requires to be equal to 20 (or c) } $ba - aa = c$
 from whence this equation
5. Which equation is of the third and last form, mentioned in the beginning of this Chapter, which being duly reduced by the Rules of the eleventh Chapter, it will be

$$aa - ba = -c$$

6. Which Equation being solved according to the method used in the fourth Rule of this Chapter, the value of a will be discovered to be:

$$a = \sqrt{\frac{1}{4}bb - c} + \frac{1}{2}b.$$

7. So that I conclude the Numbers sought to be 10 and 2, whose sum is 12, and their product 20, according to the conditions of the Question. Moreover the Equation in the sixth step, will present you with this

C A N O N.

From the Square of the half given sum of the Numbers sought, subtracted their given product, and extract the square Root of the remainder, and to its square Root add half the given sum of the numbers sought, so shall the sum of that Addition be the greater number sought, which being subtracted from the said given sum will leave the lesser.

Quest.

Q U E S T. 4.

There are three Numbers which are Geometrical proportionals continued, the mean whereof is 12 (or b) and the two extrems are such, that their difference is 18 (or c) I demand what are those three Numbers?

R E S O L U T I O N.

1. For the lesser extrem put a
2. Then the greater will be $a + c$
3. Then will the product made by the multiplication of the extrems of the first and second steps be $aa + ca$
4. Which Product (or Rectangle) in the third step must be equal to the square of (12 or b) the mean whence this Equation. $aa - ca = bb$
5. Which Equation being solved by the second Rule of this Chapter, the value of a will be found to be $a = \sqrt{bb + \frac{1}{4}cc} - \frac{1}{2}c$
6. I say the extrem proportionals sought are 6 and 24, whose difference is 18: For,

$$6 : 12 :: 12 : 24 \text{ or } a :: b :: b : \frac{bb}{a}$$

7. The Equation in the fifth step being well considered, will present you with this

CANON,

C A N O N.

If to the Square of the given mean you add the square of half the difference of the extreams, or (which is all one) $\frac{1}{4}$ part of the square of the given difference of the extreams, and extract the square root of the sum of that Addition, and then from that square root subtract half the said difference. the remainder will be the lesser extream, and if thereto you add the given difference, that sum will be the greater extream.

Q U E S T. 5.

A Draper sold a piece of Cloth for 24 *l* (or *b*) and gained as much *per Cent.* (or *c*) as the cloth cost him, I demand how much it cost him ?

R E S O L U T I O N.

1. For the price which the cloth cost, put a
2. Then will the gain by its sale be $b - a$
3. Then by the Rule of three find how much is gained *per Cent.* saying, $a : b - a :: c : \frac{cb - ca}{a}$, so that his gain *per cent.* was $\frac{cb - ca}{a}$
4. Which quantity in the 3d step according to the tenure of the question must be equal to what the cloth cost in the first step whence this Equation, *viz.* $a = \frac{cb - ca}{a}$

5. Which

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5. Which being reduced by
the Rules of the eleventh
Chapter, it will be

$$aa+ca=cb$$

6. Which (being an Equation
of the first of the 3 forms
delivered in the beginning
in the fifteenth Chap.) being
solved by the 2d & 3d rule
of the 14 Chapter, the va-
lue of a is discovered to be

$$a=\sqrt{cb+\frac{cc}{4}}-\frac{c}{2}$$

I say the cloth cost 20 $l.$ which is the value of
 a , for $\sqrt{cb+\frac{cc}{4}}=70$ and $70-\frac{c}{2}=20$, so that he
gained 4 $l.$ in laying out 20: For,

$$\begin{array}{cccc} l. & l. & l. & l. \\ 20 & : & 4 & :: 100 & : & 20 \end{array}$$

and so the conditions of the Question are satisfied.

Q U E S T. 7.

A Merchant bought a certain number of pieces
of cloth, and paid 30 pounds (or b) per Cloth,
and sold them again at such a rate per Cloth, that
if the pounds he sold a Cloth for be multiplied
by the pounds he gained per Cloth, the product
will be equal to the Cube of the number of pounds
gained per Cloth, I demand what he gained per
Cloth, and what he sold each Cloth for?

Dd RESO-

RESOLUTION.

1. For the number of pounds gained *per* piece, put a
2. To which if you add 30 or b the sum will be the number of pounds it was sold for *per* piece, *viz.* $a+b$
3. And if (according to the tenure of the Question) the second step be multiplied by the first, the product will be $aa+ba$
4. Which Product in the third step, must (according to the nature of the Question) be equal to the Cube of the pounds gained *per* Cloth in the first step, whence this equation, *viz.* $aa+ba=aaa$
5. Which equation being reduced by the third and sixth Rules of the eleventh Chap. it will then be $aa-a=b$
6. Which equation in the 5th step being solved by the 7th Rule of the 14th Chap. and the sixth Rule of this Chap. the value of a will be discovered to be $a=\sqrt{b+\frac{1}{4}}+\frac{1}{4}$

which is as much as to say in words, a (or the gain *per* Cloth) is equal to the sum when $\frac{1}{2}$ is added to the Square Root of the sum of 30, and $\frac{1}{4}$ added together, (*viz.* the Square Root of $30\frac{1}{4}$) which is $5\frac{1}{2}+\frac{1}{4}=6$.

I say,

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I say he gained 6 pounds *per* Cloth, and he sold it for 36 pounds *per* Cloth, which two numbers will satisfie the conditions of the question.

The Proof.

$$6 \times 36 = 6 \times 6 \times 6 = 216$$

Q U E S T. 8.

A Brick-layer, and a Labourer wrought together at the Building of a certain house 42 days, (or *b*) and the labourer he wrought 4 (or *c*) days more than the Brick-layer did to gain one pound, and at the end of the 42 days the Brick-layer received for his work $1\frac{3}{4}$ pounds (or *d*) more than the Labourer, I demand how many days each of them wrought for *1* l.

R E S O L U T I O N.

1. For the number of days which the Brick-layer wrought for *1* put
2. Then according to the conditions of the question, the number of days that the labourer wrought for *1* l. will be
3. By the Rule of proportion find how many pounds the Brick-layer received for the work of 42 days, as followeth,

$$a : 1 :: b : \frac{b}{a}$$

which is

$$a + c$$

$$\frac{b}{a}$$

D d 2

4 Then

4. Then find out by the Rule of 3 how many pounds the Labourer received for his work in 42 days thus

$$ca + c : 1 :: b : \frac{b}{a+c}$$

which is

5. But the Brick-layer received $1\frac{1}{2}$ (or $\frac{3}{2}$) pounds more than the Labourer for 42 days work, wherefore if to the 4th step you add d (or $\frac{3}{2}$) it will be equal to what the Brick-layer received in the third step, whence this Equation, viz.

6. Which Equation being reduced by the fourth, second, and seventh Rules of the eleventh Chapter, it will then be

$$aa + ca = \frac{bc}{d}$$

7. The Equation in the fifth step being solved by the Rule of this Chapter, the value of a will be discovered, viz.

$$a = \sqrt{\frac{bc}{d}} + \frac{cc}{4} \cdot \frac{c}{2}$$

which is as much as to say a (or the number of days which the Brick-layer wrought) is equal to the difference when $(\frac{c}{2})^2$ is subtracted from the square root of the sum of $(\frac{bc}{d})$ or 66 and $(\frac{cc}{4})$ 4 which is $100 = 10 - 2 = 8$

I say the Brick-layer wrought 8 days for twenty shillings, and the Labourer wrought

$$8 + 4 = 12$$

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$8 + 4 = 12$ days, which two numbers will satisfy the conditions of the question, as will appear by the

P R O O F.

First by the Rule of Three find what the Brick-layer received for the 42 days, saying,

$$\begin{array}{ccccccc} \text{days} & \text{l.} & & \text{days} & \text{l.} & \text{l.} & \text{s.} \\ 8 & : & 1 & : & 42 & : & 4\frac{1}{2} = 5 - 5 \end{array}$$

Then find how much the Labourer received for his 42 days work by the Rule of Three, saying,

$$\begin{array}{ccccccc} \text{days} & \text{l.} & & \text{days} & \text{l.} & \text{l.} & \text{s.} \\ 12 & : & 1 & : & 42 & : & 3\frac{1}{2} = 3 - 10 \end{array}$$

So that I find the Brick-layer for his 42 days work received $5\text{ l.} - 5\text{ s.}$ and the Labourer $3\text{ l.} - 10\text{ s.}$ which is $1\text{ l.} - 15\text{ s.}$ or $1\frac{3}{4}\text{ l.}$ less than the Brick-layer received.

Q U E S T. 9.

A Gentleman bought a House, and sold it again for 280 pounds (or b) and by its sale he gained so many pounds, that their Square being added to the square of the number of pounds it cost him, the sum will amount to 52000 (or c) pounds, now I demand how much the house cost him?

RESOLUTION.

1. For the number of pounds which the house cost, put a
2. Then will the gain by its sale be $b - a$
3. The Square of (a) its first cost is aa
4. The Square of the gain by Sale is $aa - 2ba + bb$
5. The sum of the two quantities in the third and fourth steps is $2a - 2ba + bb$
6. Which quantity in the fifth step is equal to 52000 (or c) whence this Equation $2aa - 2ba + bb = c$
7. Which equation being reduced by the third and seventh Rules of the eleventh Chapter it will be $aa - ba = \frac{c - bb}{2}$
8. Which equation being solved by the third Rule of this Chapter the value of a will be discovered, viz. $a = \sqrt{\frac{c - bb}{2}} + \frac{1}{2}b$

which is as much as to say in words, (a) the price which the house cost is equal to the sum when half what he sold it for is added to the Square Root of the sum of half the given sum of the Squares added to a fourth part of the Square of what it was sold for, that sum being made less by half the Square of what it was sold for; which was 220 $l.$ and he gained by the sale 60 $l.$ For,

$$\frac{c}{2} = 26000, \text{ and } \frac{bb}{2} = 19600, \text{ and } \frac{bb}{2} = 39200, \text{ now}$$

26000

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$26000 + 19600 = 45600$, and $45600 \div 39200 = 6400$, and $\sqrt{(2) 6400} = 80$, and $80 + (\frac{1}{2}b) 140 = 220$, which is the number of pounds the house cost, and $280 - 220 = 60$, which is the number of pounds he gained by the Sale of the house, as you will find by

The Proof.

$$220 \times 220 = 48400, \text{ and}$$

$$60 \times 60 = 3600, \text{ and}$$

$$48400 - 3600 = 52000$$

whereby the conditions of the question are answered.

Q U E S T I O N 10.

A Draper sold 2 pieces of Cloth (whereof one contained a (or b) yards more than the other) for two equal numbers of shillings, the lesser piece he selleth for 2 (or c) shillings, *per* yard more than the other, and the number of shillings which one piece was sold for, did exceed the number of yards in both pieces by 186 (or d) the question is what was the Number of yards in each piece, and what each piece was sold for *per* yard?

R E S O L U T I O N.

- | | | |
|--|---|----------|
| 1. For the number of yards in the least piece put | } | a |
| 2. Then will the yards in the greater piece be | } | $a + b$ |
| 3. Then will the sum of the yards in both pieces be | } | $2a + b$ |

D d 4

4. Then

4. Then if (according to the nature of the Question) to the sum of the yards in the third step, you add 186 } $2a - b - d$
(or d) the sum will be the number of shillings which each piece was sold for, viz.
5. And if the quantity in the fourth step, be divided by (a) the quantity in the first step the Quotient will give the number of shillings that 1 yard of the least piece was sold for. $\frac{2a+b+d}{a}$
6. And if the said Quantity in the fourth step be divided by the number of yards in the biggest piece, (which is the Quantity in the second step, the Quotient will give the number of shillings that a yard of the biggest piece was sold for, which is $\frac{2a+b+d}{a+b}$
7. If to the Quantity in the sixth step you add 2 (or c) } $\frac{2a+b+d+ca+cb}{a+b}$
shillings, it will then be
8. Which Quantity in the seventh step (as the Question requires) is equal to the Quantity in the fifth step, whence this Equation, viz.

$$\frac{2a+b+d+ca+cb}{a+b} = \frac{2a+b+d}{a+b}$$

9. Which

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9. Which equation in the eighth step being reduced by the Rules of the Eleventh Chapter, it will then be

$$aa + \frac{cb - 2b}{c} a = \frac{bb + bd}{c}$$

10. The Equation in the last step being solved by the third rule of this Chapter, the value of a will be discovered to be

$$a = \sqrt{\frac{bb + bd}{c}} + \frac{ccbb - 4cbb + 4bb - cb - 2b}{4cc} = \frac{cb - 2b}{2c}$$

11. But if you consider well the Equation in the ninth step, you will find the coefficient to be 0, for $\frac{cb - 2b}{c} = 0$, and therefore $\frac{cb - 2b}{2c} a = 0$, whence the middle Term in that Equation is 0, and therefore the middle term being removed, the Equation will be $aa = \frac{bb + bd}{c}$ which is a simple Equation, and if the Square root of both parts of that equation be extracted, the value of a will be discovered to be $a = \sqrt{\frac{bb + bd}{c}} = 24$, which is the same with the value of a in the tenth step, as you may easily find upon Tryal, wherefore I say,

The number of yards in the least piece is 24.

And the number of yards in the biggest piece is $24 + 6 = 30$, which two numbers will satisfy the conditions of the questions, as will appear by

Which equation in the eighth step being reduced by the Rule of the Chapter, it will then be

The number of yards in both pieces is $24 + 30 = 44$, which if added to 186 (as the Question requires) will give the number of shillings which one piece was sold for, which is $54 + 186 = 240$, and the least piece was sold at 10 Shillings per yard.

$$\begin{array}{ccccccc} \text{yards} & s. & & \text{yards} & l. & s. & \\ 24 & : & 240 & : & 1 & : & \frac{240}{44} = 10. \end{array}$$

And the price of a yard of the biggest piece was 8s. For,

$\begin{array}{ccccccc} \text{yards} & s. & & \text{yards} & l. & s. & \\ 30 & : & 240 & : & 1 & : & \frac{240}{46} = 8 \end{array}$ which is two shillings per yard less than the lesser piece was sold for per yard, and theretore the answer is true, and the conditions of the question are satisfied.

CHAP. XVI.

The Doctrine of Surd Quantities.

I. **A**LL quantities or Numbers whatsoever, whether Integral, or Fractional, are called Rational, but when the Root of any power cannot be exactly extracted, such Root is called Irrational or Surd, and is expressed by putting the Radical sign before the number out of which the Root proposed ought to be extracted; as $\sqrt{\quad}$ or $\sqrt{(2)}$ placed before any number or quantity signifieth the Square Root of the quantity or Number, and $\sqrt{(3)}$ the Cube Root, and $\sqrt{(4)}$ the Biquadrate Root, &c. So $\sqrt{12}$, or $\sqrt{(2) 12}$ signifieth the Square Root of 12, and $\sqrt{(3) 12}$ its Cube Root, &c.

II. Surd Numbers are two-fold, viz. Simple, and Compound; A Simple Surd quantity is when the Radical sign is prefixed to a Simple quantity, as $\sqrt{(3) 5}$ or $(\sqrt{(4) ab})$.

A Compound Surd quantity consists of several Simple Surds, which are connected together by $+$ or $-$, as $\sqrt{4} + \sqrt{6}$, and $\sqrt{ab} - \sqrt{ac} - \sqrt{d}$, and $\sqrt{(2) \overline{ab+dc}}$ which last Compound Surd is usually called an universal Root.

III. To

III. To Reduce Simple Surd quantities that have different radical signs to a common radical sign.

Let the Indices of the given Powers be reduced to their lowest Terms by their common measurer, and set the quotients under their respective Dividends, and multiply cross-wise, so shall the product be the Index required, before which placing $\sqrt{}$, it shall then be the common radical sign required; Then raise the Powers of the given Roots to the powers signified by the said altern quotients, before which said Powers place the common radical sign found as before, so will you have new surd quantities equal to the given quantities, and having equal Radical signs.

Example. Let it be required to reduce $\sqrt[6]{8}$, and $\sqrt[8]{12}$ to two other Roots equivalent to the former, having a common radical sign.

$$\begin{array}{ccc} \sqrt[6]{8} & \times & \sqrt[8]{12} \\ 3 & \times & 4 \\ \hline \sqrt[24]{4096} & \times & \sqrt[24]{1728} \end{array}$$

First, the exponents 6 and 8 are reduced to 3 and 4, which being placed under the given exponents 6 and 8 as you see, and having multiplied Cross-wise, viz. 3×8 , or 4×6 , you have 24 for a new Index, to which prefix $\sqrt{}$, and it is $\sqrt[24]{}$ for the common radical sign, and then raising 12 to the third power thereof, and 8 to the fourth, you have $\sqrt[24]{4096}$ and $\sqrt[24]{1728}$ equal to $\sqrt[6]{8}$, and $\sqrt[8]{12}$.

So if it were required to reduce $\sqrt[4]{a}$, and $\sqrt[6]{b}$ to Surd Roots equivalent thereto, having a common Radical sign, it will be as followeth.

$$\sqrt[4]{a}$$

$$\sqrt[2]{(4)a}$$

$$\sqrt[3]{(6)b}$$

2

3

$$\sqrt[2]{(12)aaa} \text{ and } \sqrt[3]{(12)bb}$$

IV. Multiplication in Simple Surd Quantities.

1. If the Quantities given to be multiplied have a common radical sign, then multiply them together without any regard to the sign, and to the product prefix the given Radical sign, which new quantity shall be the product sought.

So if $\sqrt{6}$ be to be multiplied by $\sqrt{8}$, the Product will be $\sqrt{48}$, and $\sqrt[3]{(3)4}$ by $\sqrt[3]{(3)8}$ produceth $\sqrt[3]{(3)32}$ and \sqrt{a} by \sqrt{b} , produceth \sqrt{ab} , and $\sqrt[3]{(3)c}$ by $\sqrt[3]{(3)bd}$ produceth $\sqrt[3]{(3)cdb}$. &c.

2. But if the Quantities given to be multiplied have not a common Radical sign, let them be reduced to such by the third Rule foregoing, and then proceed as before.

Example. What is the Product of $\sqrt[2]{(4)a}$ by $\sqrt[3]{(6)b}$? The said quantities being reduced to a common radical sign, will be $\sqrt[2]{(12)aaa}$ and $\sqrt[3]{(12)bb}$, which being multiplied together, produce $\sqrt[2]{(12)aaabb}$ which is the product sought.

So the $\sqrt[2]{12b}$ being multiplied by $\sqrt[3]{(3)c}$ they being Reduced to a common radical sign, are $\sqrt[2]{(6)bbb}$, and $\sqrt[3]{(6)cc}$ which being multiplied produce $\sqrt[2]{(6)bbbcc}$.

3. When a surd quantity is to be multiplied by a rational quantity, then first raise the given rational quantity to the power of the given quantity, whose Root is irrational or surd; and then proceed as before.

So if it were required to multiply $\sqrt{5}$ by 5, the rational number 5 being raised to the second power is 25, and then you will have to multiply

$$\sqrt{5}$$

$\sqrt{5}$ by $\sqrt{25}$, whose Product is $\sqrt{125}$.

Likewise $\sqrt{(3)b}$ being to be multiplied by a , the Product will be $\sqrt{(3)baaa}$, for a being raised to the third power is aaa , and $\sqrt{(3)b}$ by $\sqrt{(3)aaa}$ produceth $\sqrt{(3)baaa}$ as before.

V. Division in Simple Surd Quantities.

1. Reduce the Surd Quantities given to be divided to a common Radical sign by the third Rule of this Chapter, and then divide the Quantity following the Radical sign of the Dividend by the quantity following the radical sign of the Divisor, and to the quotient prefix the said common Radical sign, so shall that Surd quantity be the quotient sought.

Example. There being given $\sqrt{15}$ to be divided by $\sqrt{3}$, the quotient will be $\sqrt{5}$. And \sqrt{b} being to be divided by \sqrt{a} , the quotient will be $\sqrt{\frac{b}{a}}$ and $\sqrt{(2)a}$ being given to be divided by $\sqrt{(3)bc}$, the quotient will be $\sqrt{(6)\frac{aaa}{bbcc}}$, for the given quantities being reduced to a common radical sign, are $\sqrt{(6)aaa}$ and $\sqrt{(6)bbcc}$.

VI. Addition and Subtraction of simple Surd quantities.

1. When the Surd Roots to be added together, are equal, multiply any one of them by the given number of Surd quantities, so shall that product be the sum required, before which prefix the radical sign given, so the sum of $\sqrt{6}$ and $\sqrt{6}$ is $\sqrt{24}$, for the given number of roots is 2, whose square is 4, and $\sqrt{4} \times \sqrt{6} = \sqrt{24}$, so $\sqrt{(3)b}$ being to be added to $\sqrt{(3)b}$, their sum is $\sqrt{(3)8b}$ and $\sqrt{(3)a}$ being to be added to $\sqrt{(3)a}$, and $\sqrt{(3)a}$ their sum will be $\sqrt{(3)27a}$,

$\sqrt[3]{(3)27a}$, for the given Number of Surds is 3 and $\sqrt[3]{(3)a}$ being multiplyed by 3, viz. $\sqrt[3]{(3)27}$ (by the third part of the fourth Rule) the Product is $\sqrt[3]{(3)27a}$ which is the sum of $\sqrt[3]{(3)a}$, $\sqrt[3]{(3)a}$, and $\sqrt[3]{(3)a}$, which was required.

2. When two unequal Surd Roots which have the same Radical sign prefixed to each of them, be to be added together, or when the lesser of them is to be subtracted from the greater, Then you must first try whether they be commensurable, or not; that is, if after they have been divided by their greatest common measurer, the Quotients be rational Quantities, then multiply the sum of those rational quantities by the said common Divisor, and the Product shall be the sum of the Surd Quantities propounded; and if the difference of those Rational Quotients be multiplyed by the said common measurer, then will the Product be the difference of the Surd Quantities propounded.

Example. Let it be required to find the sum and difference of $\sqrt{50}$, and $\sqrt{8}$, their greatest common measurer is $\sqrt{2}$, by which they being divided, the Quotients are $\sqrt{25}$ and $\sqrt{4}$, viz. 5 and 1; whose sum is 7, which being multiplyed by $\sqrt{2}$, the Product is $7\sqrt{2}$ or $\sqrt{98}$, which is the desired sum of the Surd Quantities propounded. And if the difference of the said Rational Quotients, viz. $5-2$ (or 3) be multiplyed by the said common Divisor ($\sqrt{2}$) the Product will be $3\sqrt{2} = \sqrt{18}$, which is the difference of the Surd Quantities given, the lesser being subtracted from the greater.

But if the simple Surd quantities given to be added, or subtracted, be incommensurable, neither their sum nor difference can be express'd by any sim-

simple Term, or Root, but their sum and difference must be exprest by $+$ and $-$ as suppose you were to add $\sqrt{10}$ and $\sqrt{13}$ together, their sum would be $\sqrt{13} + \sqrt{10}$, and their difference $\sqrt{13} - \sqrt{10}$. The like of other quantities exprest by letters.

CH A P. XVII.

The Parts of Numeration in Compound Surd Quantities.

1. Addition and Subtraction in Compound Surd Quantities.

THE Addition and Subtraction of Compound Surd quantities is the same with the simple Surds, having respect to the signs of Affirmation and Negation, viz. $+$ and $-$

So if to $6 + \sqrt{18}$ ($3\sqrt{2}$) you add $4 + \sqrt{8}$ ($2\sqrt{2}$) the sum will be $10 + \sqrt{50}$ ($5\sqrt{2}$) and if from $6 + \sqrt{18}$ ($3\sqrt{2}$) you subtract $4 + \sqrt{8}$ ($2\sqrt{2}$) the difference will be $2 - \sqrt{2}$.

Likewise if to $\sqrt{320} - \sqrt{108}$ ($8\sqrt{5} - 6\sqrt{3}$) you are to add $\sqrt{80} - \sqrt{27}$ ($4\sqrt{5} - 3\sqrt{3}$) the sum is $\sqrt{720} - \sqrt{27}$ ($12\sqrt{5} - 3\sqrt{3}$) and if you subtract the

the latter from the former, the remainder will be $\sqrt{80} + \sqrt{243}(4\sqrt{5} + 9\sqrt{3})$

These two examples are of Compound Surd quantities which are commensurable, and the next is of Compound Surd quantities, partly commensurable, and partly incommensurable. As

Let it be required to add $\sqrt{12}(2\sqrt{3}) + \sqrt{5}$, to $\sqrt{27}(3\sqrt{3}) + \sqrt{8}$ the sum will be $\sqrt{75}(5\sqrt{3}) + \sqrt{8} + \sqrt{5}$, and if the former be subtracted from the latter, the remainder will be $\sqrt{3} + \sqrt{8} - \sqrt{5}$.

The same is to be observed in Addition and Subtraction of Compound Surd quantities altogether incommensurable. As in the following Examples.

| | |
|------------------|---|
| To and from | $\sqrt{10} + \sqrt{7}$ |
| Add and Subtract | $\sqrt{3} + \sqrt{2}$ |
| Sum is | <hr/> $\sqrt{10} + \sqrt{7} + \sqrt{3} + \sqrt{2}$ <hr/> |
| Or, | $\sqrt{17} + \sqrt{280} : \sqrt{5} + \sqrt{24}$ |
| Difference is | <hr/> $\sqrt{10} + \sqrt{7} - \sqrt{3} - \sqrt{2}$ <hr/> |
| Or, | $\sqrt{17} + \sqrt{280} : -\sqrt{5} + \sqrt{24}$ |
| To and from | $\sqrt{(3)10} + \sqrt{(3)7}$ |
| Add and Subtract | $\sqrt{(3)3} - \sqrt{(3)2}$ |
| Sum is | <hr/> $\sqrt{(3)10} + \sqrt{(3)7} + \sqrt{(3)} - \sqrt{(3)2}$ <hr/> |
| Difference is | $\sqrt{(3)10} + \sqrt{(3)7} - \sqrt{(3)3} + \sqrt{(3)2}$ |

II. Multiplication in Compound Surd Quantities.

$$\text{Multiplicand } \sqrt{180} + \sqrt{48} \quad (6\sqrt{5} + 4\sqrt{3})$$

$$\text{Multiplier } \sqrt{125} + \sqrt{12} \quad (5\sqrt{5} + 2\sqrt{3})$$

$$\begin{array}{r} 150 + 20\sqrt{15} \\ + 12\sqrt{15} + 24 \\ \hline \end{array}$$

$$\text{Product } 150 + 32\sqrt{15} + 24$$

$$\text{Product contracted } 174 + 32\sqrt{15}$$

$$\text{Multiplicand } \sqrt{abb} + \sqrt{cff} \quad (b\sqrt{a} + f\sqrt{c})$$

$$\text{Multiplier } \sqrt{add} + \sqrt{caa} \quad (d\sqrt{a} + a\sqrt{c})$$

$$\begin{array}{r} bda + fd\sqrt{ca} \\ + ba\sqrt{ca} + fac \\ \hline \end{array}$$

$$\text{Product } bda + fd + bax\sqrt{ca} + fac$$

$$\text{Multiplicand } \sqrt{bc} + a$$

$$\text{Multiplier } \sqrt{bc} - a$$

$$\text{Product } \underline{ba - aa}$$

III. Di-

III. Division in Compound Surd Quantities.

$$\begin{array}{r|l}
 \text{Dividend } \sqrt{21} + \sqrt{15} & \sqrt{(3)14} - \sqrt{(3)28} \\
 \text{Divisor } \sqrt{3} & \sqrt{(3)7} \\
 \hline
 \text{Quotient } \sqrt{7} + \sqrt{5} & \sqrt{(3)2} - \sqrt{(3)4} \\
 \hline
 \end{array}$$

Divisor Dividend
 $a + \sqrt{bc}$ $ab + b\sqrt{bc}$ (b Quotient.

$$\begin{array}{r}
 ab + b\sqrt{bc} \\
 \hline
 \circ \quad \circ
 \end{array}$$

$$\begin{array}{r}
 a + \sqrt{bc} \quad aaa + bc\sqrt{bc} \quad (aa + bc - a\sqrt{bc} \\
 aaa + a\sqrt{bc}
 \end{array}$$

$$\begin{array}{r}
 + bc\sqrt{bc} - a\sqrt{bc} \\
 + bc\sqrt{bc} + abc
 \end{array}$$

$$\begin{array}{r}
 - a\sqrt{bc} - abc \\
 - a\sqrt{bc} - abc
 \end{array}$$

$$\begin{array}{r}
 \circ \quad \circ
 \end{array}$$

These Examples will not seem difficult to the ingenious, if what is before delivered concerning Surd quantities be duly considered.

C H A P. XVII.

The Parts of Numeration in
Universal Surd Roots.

WHEN it is required to extract the Root of any Compound quantity, whether Square, Cube, Biquadrat, &c. if they cannot be exactly extracted without any remainder; then if to such given compound quantity you prefix the Radical sign, such Roots are called Universal Surd Roots, and first, concerning

I. Multiplication in Universal Surds.

1. When any Universal Root is to be multiplied by a Rational quantity, or by any Surd, multiply the Square of the Multiplicand by the Square of the Multiplier, when the Universal Radical sign is quadratick, or the Cube of the Multiplicand by the Cube of the Multiplier, when the Universal Radical sign is Cubical, and before that Product prefix the given Universal Radical sign, so shall that new Universal Root be the Product sought.

Example.

Example. Let it be required to multiply by 2 this universal Square Root, viz. $\sqrt{10} + \sqrt{40}$: I take the square of 2, which is 4, and the square of $\sqrt{10} + \sqrt{40}$, which is $\sqrt{10} + \sqrt{40}$, and multiply it by 4 and the Product is $\sqrt{40} + 4\sqrt{40}$, whose universal square Root is the Product sought, viz. $\sqrt{40} + 4\sqrt{40}$:

Also if $\sqrt{(3)64} + \sqrt{(3)27}$ were to be multiplied by 2, or doubled, take the Cube of the universal Root given, which is $\sqrt{(3)64} + \sqrt{(3)27}$, and multiply the same by the Cube of 2, which is 8, and the Product is $8\sqrt{(3)64} + 8\sqrt{(3)27}$, the Cube Root of which is the Product sought, viz. $\sqrt{(3)8}\sqrt{(3)64} + 8\sqrt{(3)27}$, and it is double to $\sqrt{(3)}\sqrt{(3)64} + \sqrt{(3)27}$ the Surd Root given.

In like manner, if it were required to multiply $\sqrt{12} + \sqrt{6} + \sqrt{12} - \sqrt{6}$ into its self, or to find its square; the squares of the parts are $12 + \sqrt{6}$ and $12 - \sqrt{6}$ the sum of which is 24 and the Product made by the Multiplication of the parts one into the other, viz. $\sqrt{12} + \sqrt{6}$ into $\sqrt{12} - \sqrt{6}$ is $\sqrt{138}$, (for the difference of the Squares of 12 and $\sqrt{6}$ is 138, whose square Root is $\sqrt{138}$, and the double of the said Product is $2\sqrt{138}$, which added to 24 (the sum of the squares of the parts) makes $24 + 2\sqrt{138}$, which is the square of $\sqrt{12} + \sqrt{6} + \sqrt{12} - \sqrt{6}$.

Likewise if $6 + \sqrt{20} - \sqrt{16}$ is to be multiplied by $6 - \sqrt{20} - \sqrt{16}$, the Product will be found to be 20, for if $23 - \sqrt{16}$ (which is the square of $\sqrt{20} - \sqrt{16}$) be subtracted from 36 (the square of 6) there will remain $16 - \sqrt{16}$ which is 20, the

Product sought. Also $6 - \sqrt{20} - \sqrt{10} = 10$, and $6 - \sqrt{20} - \sqrt{16} = 2$ and $2 \times 10 = 20$ as before.

Again, if it be required to multiply $\sqrt{aa + bb}$ by a , the squares of the given quantities are $aa - bb$ and aa , which being multiplyed the one into the other, the Product will be $aaaa - bbaa$, the universal Square Root of which is the Product sought, viz. $\sqrt{aaaa - bbaa}$: which may be more compendiously exprest thus, $a\sqrt{aa - bb}$.

II. Division in Universal Surds.

As in Multiplication you multiplyed the Square of the Multiplicand by the Square of the Multiplier, the given Radical sign being Quadratick, &c. So in Division of Universal Surd Roots you are to divide the square of the Dividend by the square of the Divisor, when the universal Radical sign is Quadratick, and Divide the Cube of the Dividend by the Cube of the Divisor, when the universal Radical sign is Cubical, &c: so shall the Quotient, when the universal radical sign given is prefixed thereto, be the Quotient required.

Example. What is the Quotient when $\sqrt{40} - 4\sqrt{30}$ is divided by 2? Here I divide $40 - \sqrt{40} - 4\sqrt{40}$ (which is the square of $\sqrt{40} - 4\sqrt{40}$: the dividend) by 4 (the square of the given Divisor) and there ariseth $\sqrt{10} - \sqrt{40}$: the universal square Root of which, viz. $\sqrt{10} - \sqrt{40}$: is the Quotient required.

Also if it were required to divide $\sqrt{(3)8\sqrt{(3)64} - 8\sqrt{(3)27}}$ by 2, the Quotient would be found; to be $\sqrt{(3)}\sqrt{(3)64} - \sqrt{(3)27}$: here the Cube of the given

given Dividend is $8\sqrt{(3)}64 + 8.(3)27$ which being divided by 8 (the Cube of 2) there will arise $\sqrt{(3)}64 + \sqrt{(3)}27$, to which if you prefix the universal radical sign of its Cube Root, it will be $\sqrt[3]{(3)\sqrt{(3)}64 + \sqrt{(3)}27}$: which is the Quotient sought.

Likewise if it be required to divide $\sqrt{aaaa + bbaa}$ by a , the Quotient will be found to be $\sqrt{aa + bbb}$: for, the square of the Dividend is $aaaa + bbaa$, and the square of the Divisor is aa , and when the Division is ended, there will arise $aa + bb$ the universal square Root of which is $\sqrt{aa + bb}$: which is the Quotient sought.

But when the work of Division in universal Surd Quantities happens to be intricate, and its operation cannot be finished without a remainder, you may set the power of the Dividend for a Numerator, and the power of the Divisor for a Denominator, and against the line of Separation, place, or prefix the universal radical sign, which universal Root so signified shall be the Quotient sought.

As if it were required to divide $\sqrt{\sqrt{ab} + bc}$ by $\sqrt{\sqrt{a} + c}$: the quotient will be $\sqrt{\frac{\sqrt{ab} + bc}{\sqrt{a} + c}}$.

III. *Addition and Subtraction in Universal Surd Quantities.*

1. If two Universal Surd quantities that are commensurable are proposed to be added together, or subtracted, the operation may be performed like simple Surds. As for Example. If the sum and difference of $\sqrt{8} + 4\sqrt{3}$: and $\sqrt{2} + \sqrt{3}$: were required.

Here each of the said quantities being divided by their greatest common measurer, $\sqrt{2} + \sqrt{3}$: the Quotients are $\sqrt{4}$ and $\sqrt{1}$, viz. 2 and 1, which are rational Numbers expressing the proportion of the Surds propounded, therefore if their common Divisor be multiplied $2 - 1$ (viz. 3.) it giveth $3\sqrt{2} - \sqrt{3}$: for the sum required, and the said common Divisor being multiplied by $(2 - 1)$ the difference of the said 2 and 1, it will produce $\sqrt{2} - \sqrt{3}$: for the difference of the Roots proposed.

Likewise if it were required to find the sum and difference of $aaaa - aabb$: and $\sqrt{aabb} - bbbb$.

The said Quantities being reduced, are $a\sqrt{aa} - bb$: and $b\sqrt{aa} - bb$:

Therefore is their sum $a + b\sqrt{aa} - bb$: and their difference is $a - b\sqrt{aa} - bb$.

2. When the Root of a residual is to be added to, or subtracted from, the Root of its correspondent Binomial, then may those Roots be connected together by the signs $+$ and $-$; and then the whole being multiplied by it self, the universal Root of the Product shall be the sum or difference of the Roots propounded.

As suppose $\sqrt{12} + \sqrt{6}$: were propounded to be added to $\sqrt{12} - \sqrt{6}$: the given Roots being connected together by $+$, make $\sqrt{12} + \sqrt{6}$: $12 + \sqrt{6}$: which composed Quantity being multiplied by it self, produced $24 + 2\sqrt{128}$, whose universal Square Root ($\sqrt{24 + 2\sqrt{128}}$) shall be the sum of the Quantities proposed to be added.

But if $\sqrt{12} - \sqrt{6}$: $-\sqrt{12} - \sqrt{6}$: be multiplied into it self, the product will be $24 - 2\sqrt{128}$, whose

whose universal square Root is the difference of the two given Roots.

3. But if the universal Roots to be added or subtracted are not commensurable, &c. then they are to be added by $+$, and subtracted by $-$

So if it were required to add $\sqrt{5} + \sqrt{3}$ to $\sqrt{3} - \sqrt{2}$ their sum would be $\sqrt{5} + \sqrt{3} + \sqrt{3} - \sqrt{2}$:

And the latter being subtracted from the former, the remainder would be $\sqrt{5} + \sqrt{3} - \sqrt{3} - \sqrt{2}$:

And the sum of $\sqrt{ab} + c$ being added to $\sqrt{d} + b$ will be $\sqrt{ab} + c + \sqrt{d} + b$ and the latter being subtracted from the former, the remainder will be $\sqrt{ab} + c - \sqrt{d} - b$:

IV. *The Extraction of the Square Root out of Binomials, and Residuals.*

Subtract the Square of the lesser part of the given Binomial, from the Square of the greater part, and add the Square Root of the remainder to the greater part, and also subtract it therefrom, and then extract the square Roots of the Sum and remainder, and joyn them together by $+$ if the quantity proposed be a Binominal, but by $-$ if it be a Residual, which Roots so joyned, are the square Root of the given Binomial, or Residual.

Example 1.

Extract the Square Root of this Binomial, viz. }

1. From the square of the greater part 38, viz. from }

$$38 + \sqrt{1300}$$

$$1444$$

2. Sub-

2. Subtract the square of the
lesser part, viz. $\sqrt{1300}$
which is } 1300
3. The remainder is } 144
4. The square root of the re-
mainder is } 12
5. To which root if you add
the greater part 38, the sum
is } 50
6. The half of which sum is } 25
7. The square root of the said
half sum is the greater part
of the root sought, which is } 5
8. From the greater part of
the given Binomial, viz. 38,
subtract the square root in
the fourth step, viz. 12, the
remainder is } 26
9. The half of which is } 13
10. The square root of the
said half remainder is the
lesser part of the root
sought. } $\sqrt{13}$
11. To which if you add the
quantity in the seventh step,
the sum will be the square
root sought, viz. } $5 + \sqrt{13}$

which is the square root of the given Binomial,
but if the given surd quantity had been a Resi-
dual, viz. if it had been required to extract the
square root of $38 - \sqrt{1300}$, then the root would
have been $5 - \sqrt{13}$.

Example.

Example 2.

- Extract the square root of
this Binomial, viz.
1. The square of the greater
part 7, is 49
 2. From which subtract the
square of the lesser part,
(viz. $\sqrt{20}$), which is 20
 3. The remainder is 29
 4. The square root of that re-
mainder is $\sqrt{29}$
 5. To which square root add
the greater part of the gi-
ven Binomial, viz. 7, and the
sum is $7 + \sqrt{29}$
 6. The half of which sum is $\frac{7}{2} + \sqrt{\frac{29}{4}}$
 7. The square root of the said
sum is the greater part of the
root sought, which is $\sqrt{\frac{7}{2}} + \sqrt{\frac{29}{4}}$
 8. From the greater part of
the given Binomial, (viz.
from 7) subtract $\sqrt{29}$ in
the fourth step, and the re-
mainder is $7 - \sqrt{29}$
 9. The half of which remain-
der is $\frac{7}{2} - \sqrt{\frac{29}{4}}$
 10. The square root of the
said half remainder is the
lesser part of the root
sought, which is $\sqrt{\frac{7}{2}} - \sqrt{\frac{29}{4}}$

II. Which

11. Which being joyned to the greater part of the root sought in the seventh step by the sign $+$, the sum will be the square root sought, which is

$$\sqrt{\frac{7}{2} + \sqrt{\frac{2}{4}}} + \sqrt{\frac{7}{2} - \sqrt{\frac{2}{4}}}$$

But if the lesser part of the said root found in the tenth step be joyned to the greater part found in the seventh step by interposing the sign $-$ instead of $+$, it will then be the square root of the residual $7 - \sqrt{20}$.

Example. 3.

Let it be required to extract the square root of this Binomial, viz. $aa - d$ added to $2avd$, supposing the greater part of the given Binomial, to be $aa - d$. Then,

1. The square of the greater part is $\left. \begin{array}{l} aaaa - bdd - bb \end{array} \right\}$
2. From which subtract the square of the lesser part $\left. \begin{array}{l} (2avd) \text{ viz. } 4daa, \text{ and the remainder is } aaaa - 2aid - dd \end{array} \right\}$
3. The square root of that remainder is $\left. \begin{array}{l} aa - d \end{array} \right\}$
4. To which square root add the greater part of the given Binomial, viz. $aa - d$, and the sum is $\left. \begin{array}{l} 2aa \end{array} \right\}$
5. The half of which sum is aa
6. The square root of which half sum is the greater part of the root sought, which is $\left. \begin{array}{l} a \end{array} \right\}$

7. From

7. From $(aa - d)$ the greater part of the given Binomial subtract the square root found in the third step $(aa - d)$ and the remainder is

$$2d$$

8. The half of which remainder is

$$d$$

9. The square root of which half remainder, is the lesser part of the root sought, viz.

$$\sqrt{d}$$

10. Which said root being joyned to the greater part found in the sixth step by the sign $-$, it will be the root sought, viz.

$$a - \sqrt{d}$$

but if the quantity in the ninth step be joyned to the quantity in the sixth step, by interposing the sign $+$, it will then be the square root of the residual, $aa - d$ less $2a\sqrt{d}$.

Example. 4.

Let it be required to extract the square root of $e - d\sqrt{ed}$ more $2ed$ supposing the greater part to be

$$\overline{e - d\sqrt{ed}}, \text{ then}$$

1. The square of the greater part is

$$eed - 2eedd - eddd$$

2. From which subtract the square of the lesser part, which is

$$4eedd$$

3. And the remainder is

$$eed - 2eedd - eddd$$

4. The square root of that remainder is

$$\overline{e - d\sqrt{ed}}$$

5. To

5. To which if you add the greater part of the given binomial, the sum is } $2e\ddot{v}ed$
6. The half of which sum is } $e\ddot{v}ed$
7. The square root of the said half sum is the greater part of the root sought, which is } $\overline{v:e\ddot{v}ed:}$
8. From the greater part of the given binomial subtract the square root found in the fourth step, and the remainder is } $2d\ddot{v}ed$
9. The half of which remainder is } $d\ddot{v}ed$
10. The square root of the said half remainder is the lesser part of the root sought, which is } $\overline{v:d\ddot{v}ed:}$
11. If to the greater part of the root sought in the seventh step, you join the lesser part in the eleventh step, by interpoling the sign +, it will then be the root sought, which is

$$\overline{v:e\ddot{v}ed:} + \overline{v:d\ddot{v}ed:}$$

But if the two said quantities are joyned together by the interposition of the sign —, it will then be the square root of the residual $e + d\ddot{v}ed$ less $2ed$.

V. *Some Questions to exercise the Rules of this and the foregoing Chapters.*

Q U E S T. 1.

Let it be required to divide 100 (or e) into two such unequal parts, that 100 multiplied by the

the lesser part may be equal to the square of the greater.

R E S O L U T I O N.

1. For the greater number put a
 2. Then will the lesser be $c - a$
 3. By which if you multiply }
100 (or c) the product will be } $cc - ca$
 4. Which quantity in the 3d }
step must be equal to the }
square of the Quantity in }
the first step, whence this }
equation. } $aa = cc - ca$
 5. Which Equation being re- }
duced by the rules of the 11 }
Chap. and solved, the value }
of a will be discovered to be } $a = \sqrt{cc + \frac{cc}{4}} - \frac{c}{2}$
- which Equation in the last step being duly considered, will present you with this

Theorem.

To the Square of the given line or number add a fourth part of its Square, and extract the Square root of that sum; then from the said square root subtract half the given line, so shall the remainder be the greater segment, or number sought.

Q U E S T. 2.

What Number is that whose square being made less by the Rectangle of it self drawn into 12 (or b) the remainder is equal to f ?

1. For

1. For the number sought put a
2. The square of which is aa
3. The Rectangle of a in b is ba
4. If the quantity in the third step be subtracted from the quantity in the second step, the remainder is equal to f , whence this Equation. $aa - ba = f$
5. Which Equation being solved by the rules of the 11th Chapter the value of a will be found to be $a = +\frac{1}{2}b + \sqrt{f + \frac{1}{4}bb}$

The Proof.

6. If $a = \frac{1}{2}b + \sqrt{f + \frac{1}{4}bb}$
 7. Then by subtracting $\frac{1}{2}b$ from each part of the equation there remaineth. $a - \frac{1}{2}b = \sqrt{f + \frac{1}{4}bb}$
 8. Then by squaring each part of the equation you have $aa - ba + \frac{1}{4}bb = f + \frac{1}{4}bb$
 9. And by subtracting $\frac{1}{4}bb$ from both sides of the equation there remaineth $aa - ba = f$
- which was to be proved

QUEST. 10.

1. Let c and d be put for two such known Quantities that d not $\mid \frac{1}{4}cc$, and let a be put for a quantity unknown, and let it be granted that $ca - aa = d$ what is the value of a ?

2. The given equation in the first step is one of the third form mentioned in the beginning of the fifteenth Chapter, and it will be found that the 2 values of a are

$$a = \frac{1}{2}c +$$

$$a = \frac{1}{2}c + \sqrt{\frac{1}{4}cc - d}:$$

And

$$a = \frac{1}{2}c - \sqrt{\frac{1}{4}cc - d}:$$

By either of which values of a the Equation propounded in the first step may be expounded, as will appear by the

D E M O N S T R A T I O N.

1. If $a = \frac{1}{2}c + \sqrt{\frac{1}{4}cc - d}:$
2. Then by the transposition of $\frac{1}{2}c$ to the contrary coast, it is $a - \frac{1}{2}c = \sqrt{\frac{1}{4}cc - d}:$
3. And by squaring each part of the last Equation, it is $aa - ca + \frac{1}{4}cc = \frac{1}{4}cc - d$
4. And by subtracting $\frac{1}{4}cc$ from each part of the equation, it is $aa - ca = -d$
5. And by changing the signs on the quantities on each side of the equation, it is $ca - aa = d$

Which was to be demonstrated.

6. Again, if $a = \frac{1}{2}c - \sqrt{\frac{1}{4}cc - d}:$
7. Then by transposition of $\sqrt{\frac{1}{4}cc - d}$ to the other side it is $a + \sqrt{\frac{1}{4}cc - d} = \frac{1}{2}c$
8. And by transposition of a it is $\sqrt{\frac{1}{4}cc - d} = \frac{1}{2}c - a$
9. And by squaring each part of the equation it will then be $\frac{1}{4}cc - d = \frac{1}{4}cc - ca + aa$

F f

10. And

10. And the subtracting $\frac{1}{2} ca$ from both parts of the equation, it is $\left. \begin{array}{l} \text{from both parts of the equation, it is} \end{array} \right\} -d = -ca + aa$
11. And the quantities on both sides of the equation being transposed to the contrary coast, and the signs of each thereby changed, the equation will then be $\left. \begin{array}{l} \text{being transposed to the contrary coast, and the signs of each thereby changed, the equation will then be} \end{array} \right\} ca - aa = d$
which was likewise to be proved.

Q U E S T. 2.

Let it be required to divide 100 into two such parts that if each part be divided by the other, the sum of the Quotients may be 3. This is *Quest. 1.* of the ninth Chapter of the second Book of *Kersey's Elements of Algebra*, and it is thus wrought, *viz.*

1. For one of the parts sought put a
2. Then will the other be $100 - a$
3. Each of which quantities in the first and second steps being mutually divided by each other (according to the import of the question) this equation ariseth $\left. \begin{array}{l} \text{in the first and second steps} \\ \text{being mutually divided by} \\ \text{each other (according to} \\ \text{the import of the question)} \end{array} \right\} \frac{a}{100 - a} + \frac{100 - a}{a} = 3$
4. Which equation being duly reduced, gives $\left. \begin{array}{l} \text{Which equation being duly} \\ \text{reduced, gives} \end{array} \right\} 100a - aa = 2000$
5. Which is an equation of the third form mentioned in Chap. 15. and being solved according to the method there given, the two values of a will be found to be $\left. \begin{array}{l} \text{Which is an equation of} \\ \text{the third form mentioned} \\ \text{in Chap. 15. and being} \\ \text{solved according to the} \\ \text{method there given, the two} \\ \text{values of } a \text{ will be found to} \\ \text{be} \end{array} \right\} a = \left\{ \begin{array}{l} 50 + 10\sqrt{5} \\ 50 - 10\sqrt{5} \end{array} \right.$

Which you may easily prove at your leisure.

CHAP.

C H A P. XVIII.

Algebraical Questions Resolved by various Positions.

MR. Kersey in the Twelfth Chapter of the second Book of his Elements of Algebra, hath laid down Rules for the solution of Questions Algebraically by various Positions; assuming a peculiar letter to represent every one of the Quantities sought, viz. *a* for one unknown Quantity, *e* for another, and *y* for a third, &c. and for the performance of the work he hath laid down 3 Rules which are as followeth, viz.

R U L E 1.

When many Quantities are sought in a Question, let them be represented by various letters, and let the tenor of the Question be represented by Equations; which done by Transposition find what any single letter in the first equation is equal to; Then wheresoever that Letter is found in the other equations, instead thereof, take what it is found equal to, so will that letter quite vanish out of the following Equations; Then by Transposition set a second letter alone in one of those equations out of which the first letter was cancelled and proceed as before, so at

length one of the letters will be made known, by help of which the rest will be easily discovered.

R U L E 2.

When the same Quantity (suppose a) is found in two several Equations, and equal Numbers are prefixed to those Quantities, then if their signs be both $+$, or both $-$, subtract the lesser Equation from the greater; but if the signs be one $+$, and the other $-$, then add those two Equations together, so will the said Quantity a quite vanish.

R U L E 3.

When the same Quantity (suppose a) is found in two several Equations, but the Numbers prefixed to those equal Quantities are unequal, those two Equations may be reduced to two others which shall have equal Numbers prefixed to the said Quantity a thus, viz, Multiply all the Quantities in the first Equation by the number prefixed to a in the second Equation; and also multiply all the Quantities in the second Equation by the number prefixed to the same quantity a in the first Equation, so by such alternate multiplication two new Equations will be produced, wherein the Numbers prefixed to the said quantity a will be equal to one another, and then proceed according to the second Rule, and expel the same quantity out of the rest of the Equations; proceed in like manner with a second quantity, until at length some one quantity be made known; by which all the rest may be found

found out. The three foregoing Rules will be exercised in the Resolution of the following *Questions.*

Q U E S T. 1.

Divide 100 (or c) into two such Numbers, (*viz.* a and e) that $\frac{a}{3} + \frac{e}{5}$ may be equal to 30 (or d) I demand the numbers a and e ?

R E S O L U T I O N.

1. If $a + e = c$
2. And $\frac{a}{3} + \frac{e}{5} = d$
3. Then by transposition of e in the first step, you will have $a = c - e$
4. By reducing the Equation in the second step, so as a may solely possess one side thereof, you will have $a = \frac{15d - 3e}{5}$
5. If instead of a in the 4th step, you take what a is equal to in the third step, you will have this equation, *viz.* $c - e = \frac{15d - 3e}{5}$
6. The first part of the equation in the fifth step being multiplied by 5, will give $5c - 5e = 15d - 3e$
7. By the transposition of $-5e$ it is $5c = 15d + 2e$
8. And by the transposition of $15d$ in the last step, you have $5c - 15d = 2e$

9. Each part of the Equation in the last step being divided by 2, will give the value of e , viz.

$$\frac{5c - 15d}{2} = e$$

I say the value of e is 25, and the value of a is $100 - 25 = 75$, which will answer the Conditions of the Question. As appears by

The Proof.

$$25 + 75 = 100; \text{ and } 25^2 + 75^2 = 30$$

QUEST. 2.

There are two Numbers (a the greater, and e the lesser) whose difference 4 (or b) and the difference of their Squares is 64 (or c) what are the Numbers?

RESOLUTION.

1. If $a - e = b$
2. Then by the transposition of e you have $a = b + e$
3. And if $aa - ee = c$
4. Then by transposition of ee it is $aa = c + ee$
5. If both parts of the equation in the second step be squared, it will be $aa = bb + 2be + ee$
6. And if instead of aa in the fifth Equation, you place what it is equal to in the fourth step, the Equation will then be

$$c + ee = bb + 2be + ee$$

7. By

7. By subtracting ce from both parts of the last equation } $c = bb + 2be$
you will have

8. By dividing both parts of the last equation by b , you will then have } $\frac{c}{b} = b + 2e$

9. By transposition of b in the last step, the Equation will then be } $\frac{c}{b} - b = 2e$

10. And if both parts of the equation in the last step be divided by 2, the value of e will then be discovered to be } $\frac{c}{2b} - \frac{b}{2} = e$

I say the value of e (the lesser number) is 6, and by the second step (*) the greater number is $e + b = 6 + 4 = 10$, which two numbers, (*viz.* 10 and 6) will satisfy the Conditions of the Question, as will appear by

The Proof.

$$10 - 6 = 4, \text{ and } 10 \times 10 - 6 \times 6 = 64$$

Q U E S T. 3.

A Maid being at Market sold 10 dozen of Eggs, and twelve Pounds of Butter for thirteen Shillings. And at another time, and at the same rate, she selleth Eight dozen of Eggs, and 18 pounds of Butter for 16 shillings, I demand how she sold her Eggs *per* dozen, and her butter *per* pound?

Let a represent the desired value of a dozen of Eggs, and for the price of a pound of Butter

F f 4

put

put e , and then may the question, being abstracted from words, be stated thus, *viz.*

1. If

$$10a - 12e = 13$$

2. And

$$8a - 18e = 16$$

What are the values of a and e ?

R E S O L U T I O N.

3. By transposition of $12e$ in the first step, that equation will be

$$10a = 13 - 12e$$

4. And both parts of the last equation being divided by 10, it is

$$a = \frac{13 - 12e}{10}$$

5. By transposition of $8e$ in the second step, that equation will be

$$8a = 16 - 18e$$

6. Each part of the last equation being divided by 8, will give

$$a = \frac{16 - 18e}{8}$$

7. If instead of a in the sixth step, you place what it is equal to in the fourth step, the equation will then be

$$\frac{13 - 12e}{10} = \frac{16 - 18e}{8}$$

8. Both parts of the last equation being reduced to Integers will give

$$104 - 96e = 160 - 180e$$

9. By transposition of $180e$ and 104 in the last equation each to the contrary coast, the equation will then be

$$84e = 56$$

10. If

10. If each part of the last Equation be divided by 84, }
 the value of e will be discovered to be }
 which is 8 $d.$ for the price of one pound of Butter.

$$e = \frac{56}{84} = \frac{2}{3}$$

11. By the tenth step the value of e is discovered to be $\frac{2}{3} s.$ by which means the value of a (by the quantities in the fourth and sixth steps.) is found to be 6 $d.$ for the 4th step, is

$$a = \frac{13 - 12}{10}$$

And it hath been found before, that $e = \frac{2}{3} s.$ so that $12e = 12 \times \frac{2}{3} s = 8 s.$ and $\frac{13 s. - 8 s.}{10} = \frac{5}{10} = 6 d.$ so the Maid sold her Eggs at 6 $d.$ per dozen, and her Butter at 8 $d.$ per pound, which will answer the conditions of the question.

Q U E S T. 4.

Three Men, viz. $A, B,$ and C discourse thus together concerning their Age; quoth B to A , your age added to mine is 54 (or b) years; quoth C to B , and my age added to yours makes 78 (or c) years? and quoth A to C , my age added to yours is 72 (or d) years. I demand the age of each person?

Let the age of each Person be represented by the letters a, e, y , viz. for the age of A put a , for the age of B put e , and for the age of C put y ; and the *Question* being abstracted from words, will be as followeth, viz.

1. If $a + e = b (= 54)$
2. And $e + y = c (= 78)$
3. And $y + a = d (= 72)$

What are the values of a , e and y ?

R E S O L U T I O N.

4. By transposition of e in the first step there will arise $a = b - e$
5. If instead of a in the third step you put what a is equal to in the fourth step, there will arise $y + b - e = d$
6. By the transposition of d and e in the last step, there ariseth $e = y + b - d$
7. And if instead of e in the second step you take the latter part of the sixth step, there will then arise $2y + b - d = c$
8. In which last equation there is no unknown quantity but y , and therefore the equation being duly reduced, will discover the value of y to be $y = \frac{c + d - b}{2}$
9. If in the sixth step instead of y you take the latter part of the equation in the eighth step, the value of e will be found to be $e = \frac{c + b - d}{2}$
10. And if instead of e in the fourth step, you take the latter part of the Equation in the ninth step, the value of a will be discovered, viz. $a = \frac{b + d - c}{2}$

And

And thus the work is finished, and the equations in the eighth, ninth, and tenth steps present you with this

C A N O N.

From the sum of every two of the three given Numbers subtract the third number remaining, so shall the three remaining numbers being divided by 2 be the Numbers sought. So the Number sought in the Question, *viz.* a , e , and y , are found to be 24, 30, and 48, *viz.* the age of A is 24, the age of B is 30, and the age of C is 48, which three Numbers will satisfy the Conditions of the Question for $24 + 30 = 54$, and $30 + 48 = 78$, and $48 + 24 = 72$:

Q U E S T I O N 5.

What two Numbers are those whose sum is 20, (or b) and their difference 4 (or c)?

Let a be put for the greater Number sought, and e for the lesser, and then the Question being extracted from words may be stated thus, *viz.*

1. If

$$a + e = b (20)$$

2. And

$$a - e = c (4)$$

What are a and e ?

R E S O L U T I O N.

3. Forasmuch as $+1a$ is found in each of the equations in the first and second steps, therefore (by the second Rule) they being subtracted, do give this Equation, *viz.*

$$2e = b - c$$

4. And

4. And by dividing both parts
of the Equation in the third
step by 2, the value of c will
be discovered, viz. $c = \frac{1}{2}b - \frac{1}{2}e$

5. And if instead of e in the
second step you put what e
is equal to in the fourth
step, you will have this E-
quation, viz. $a - \frac{1}{2}b + \frac{1}{2}e = c$

6. By the transposition of $-\frac{1}{2}b$
and $+\frac{1}{2}e$ to the contrary
coast, the value of a will be
discovered, viz. $a = \frac{1}{2}b - \frac{1}{2}e$

From the fourth and sixth steps is raised this

C A N O N.

If from half the sum of two numbers you sub-
tract half their difference, the remainder will
be the lesser number; and if to half their sum you
add half their Difference, that sum will be the
greater number, whereby the two numbers sought
in this *Question* are found to be 12 and 8; for
 $12 + 8 = 20$ and $12 - 8 = 4$.

Q U E S T. 6.

What 3 Numbers are those, that if to the first
there be added 121 (or b) the sum will be equal
to the sum of the first and second; and if to the
second there be added 121, the sum will be equal
to double the sum of the first and third; and if to
the third there be added 121, their sum will be
triple the sum of the first and second?

If for the number sought you put a , e , and y ,
viz. for the first number a , for the second e ,
and

and for the third y , then the Question being abstracted from words, may be stated thus, *viz.*

1. If

$$a + b = e + y$$

2. And

$$e + b = 2a + 2y$$

3. And

$$y + b = 3a + 3e$$

What are the Numbers a & y ?

R E S O L U T I O N.

4. By the transposition of y in the first Equation, there ariseth

$$a + b - y = e$$

5. And if instead of e in the second Equation you take what is equal thereto in the fourth equation, there ariseth

$$a + b - y + b = 2a + 2y.$$

6. The last Equation after due reduction will be

$$2b = a + 3y$$

7. And if instead of $3e$ in the third Equation you take the triple of what e is found equal to, in the fourth step, you will find the following Equation to arise, *viz.*

$$y + b = 3a + 3a + 3b - 3y.$$

8. Which Equation after due reduction by transposition the quantities will be found to be

$$4y = 6a + 2b$$

9. And both parts of the last Equation being divided by 4, there ariseth

$$y = \frac{3}{2}a + \frac{1}{2}b$$

10. Then

10. Then if instead of $3y$ in the sixth equation there be taken the triple of the latter part of the ninth equation, there ariseth
- $$2b = a + \frac{2}{3}a + \frac{2}{3}b$$
11. After due Reduction of the equation in the tenth step, the value of a will be discovered, viz.
- $$a = \frac{b}{11}$$
12. Again, if instead of a the sixth Equation you put the latter part of the Eleventh Equation, there ariseth
- $$2b = \frac{b}{11} + 2y$$
13. After due reduction of the equation in the twelfth step, the value of y will be discovered to be
- $$y = \frac{7b}{11}$$
14. And if for a and y in the fourth step there be put their equals in the 11th and 13th steps there will arise
- $$\frac{b}{11} + \frac{7b}{11} = e$$
15. The Equation in the last step being duly reduced, will discover the value of e , viz.
- $$e = \frac{8b}{11}$$

From the eleventh, thirteenth, and fifteenth steps is gathered this

C A N O N.

If the number given to be added to the three numbers required be divided by 11, the Quotient will give the first number, and its Quintuple (or Product by 5) being divided by 11, will give the second number, and its septuple (or product

duct by 7) being divided by 11, will give the third number.

By which Canon the numbers required in the Question are 11, 55, and 77, (the second being 5 times as much as the first, and the third is 7 times as much as the first) which said numbers will satisfy the conditions of the Question, as will appear by

P R O O F.

$$11 + 121 = 55 + 77 = 132.$$

$$\text{And } 55 + 121 = 2 \times 11 + 77 = 176.$$

$$\text{And } 77 + 121 = 3 \times 11 + 55 = 198.$$

which was to be done.

Q U E S T. 7.

What two numbers are those that if to 10 times the greater there be added six times the lesser, the sum will be 228 (or *b*) and if from 4 times the greater you subtract 2 times the lesser, the remainder will be 56 (or *c*)? For the two numbers put *a* and *e*, and then the foregoing question being abstracted from words, may be stated thus. *viz.*

1. If

$$10a + 6e = b$$

2. And

$$4a - 2e = c$$

What are the numbers *a* and *e*?

R E S O L U T I O N.

3. The first equation (according to the third Rule) being multiplied by 4, which is prefixed to *a* in the second equation, produceth

$$40a + 24e = 4b$$

4. And

4. And the second Equation being multiplyed by 10, which is prefixed to a in the first, it produceth

$$40a - 20c = 10c$$

5. And if from the Equation in the third step you subtract the equation in the fourth step, because $40a$ is found in both, (according to the second Rule) there ariseth this equation, viz.

$$44e = 4b - 10c$$

6. Both parts of the equation in the fifth step, being divided by 44 the value of e will be discovered to be

$$e = \frac{4b - 10c}{44}$$

7. If instead of $-2c$ in the second step you put double the latter part of the equation in the sixth step, you will have this equation

$$4a - \frac{2b - 5c}{11} = c$$

8. The seventh equation being duly reduc'd, the value of a will be discover'd to be

$$a = \frac{6c + 2b}{44}$$

By the sixth and eighth steps the numbers sought are 18 and 8, which will answer the conditions of the *Question*, as you may perceive by

The Proof.

$$10 \times 18 - 6 \times 8 = 228$$

And

$$4 \times 18 - 2 \times 8 = 56$$

Soli Deo Gloria.

F I N I S

